On The Uniqueness Problems of Entire Functions Concerning Differential Polynomials

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Abstract: We discuss the uniqueness of entire functions concerning differential polynomials. The results extend and generalise recent results of Jiang-Tao Li and Ping Li.

Keywords: Entire functions, Uniqueness, Differential Polynomials, Deficient value, Weighted sharing.

1. Introduction

Let f and g be two non-constant meromorphic functions defined in the open complex plane C. For $a \in C \cup \{\infty\}$ we say that f and g share the value aCM (counting multiplicities) if the a-points of f and g coincide in locations and multiplicities. If we do not consider the multiplicities, we say that f and g share the value aIM(ignoring multiplicities). For standard definitions and notations of the value distribution theory we refer to [1].

We denote by S(r, f) any quantity satisfying $S(r, f) = o\{T(r, f)\}$ as $r \to \infty$ possibly outside a set of finite linear measure. A meromorphic function a = a(z) is called small function of f if T(r, a) = S(r, f).

In [2] and [3] the idea of weighted sharing is introduced which measures how close a shared value is to being shared IM or to being shared CM. We now explain the idea of weighted sharing of values.

Definition 1.1. [2, 3]Let k be a nonnegative integer or infinity. For $a \in C$ we denote by $E_k(a, f)$ the set of all apoints of f where an a-point of multiplicity m is counted m times if $m \leq k$ and k+1 times if $m \geq k$. If $E_k(a, f) = E_k(a, g)$ we say that f, g share the value a with weight k.

The definition implies that if f, g share a value a with weight k then z_0 is a zero of f - a with multiplicity $m(\leq k)$ if and only if it is a zero of g - a with multiplicity $m(\leq k)$ and z_0 is a zero of f - a with multiplicity m(> k) if and only if it is a zero of g - a with multiplicity m(> k), where m is not necessarily equal to n.

We write f, g share (a, k) to mean that f, g share the value a with weight k. Clearly if f, g share (a, k) then f, g share (a, p) for all integers p, $0 \le p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share (a, 0) or (a, ∞) respectively.

Definition 1.2. Let f be a non-constant meromorphic function and $a \in C \cup \{\infty\}$. For a positive integer k we denote by $N_{k}(r, a; f)$ $(N_{(k}(r, a; f))$ the counting function of those a-points of f whose multiplicities are less than or

equal to k (greater than or equal to k), where an a-point is counted according to its multiplicity.

Also by $\overline{N}_{k}(r, a; f)$ and $\overline{N}_{(k}(r, a; f))$ we denote the corresponding reduced counting functions. We put

$$\lambda(f) = \limsup_{r \to \infty} \frac{\log^+ T(r, f)}{\log r}$$

and

$$\delta(a, f) = 1 - \limsup_{r \to \infty} \frac{N(r, a; f)}{T(r, f)}.$$

 $\lambda(f)$ and $\delta(a, f)$ are called the order of f(z) and the deficiency of a with respect to f(z) respectively.

Definition 1.3. Let f and g be two non-constant meromorphic functions for anypositive integer k we denote by $N_{(k}^{L}(r, 1; f)$ the counting function of zeros of f - 1and g - 1 whose multiplicities are not less than k and about which f - 1 has larger multiplicity than g - 1. By $N_{(k}^{E}(r, 1; f)(N_{k}^{E}(r, 1; f))$ we denote the counting function of zeros of f - 1 and g - 1 whose multiplicities are not less (greater) than k and about which f - 1 and g - 1 have equal multiplicities. By $\overline{N}_{(k}^{L}(r, 1; f), \overline{N}_{(k}^{E}(r, 1; f))$ and $\overline{N}_{k}^{E}(r, 1; f)$ we denote the corresponding reduced counting functions.

Definition 1.4. Let f be a non-constant meromorphic function we denote by $N_0(r, 0; f')$ the counting function of zeros of f' which are not zeros of f(f - 1).

In 1976 C. C. Yang [7] asked: If two transcendental entire functions f and g assume the same zeros with the same multiplicities and that their first derivatives assume the same 1-points with the same multiplicities then what can be said about therelationship between f and g?

To solve the above question in 1981 K. Shibazaki [6] proved the following:

Theorem A. [6] Let f and g be two entire functions of finite order. If f' and g' share the value 1 CM with $\delta(0, f) > 0$ and 0 being lacunary for g then either $f \equiv g$ or $f'g' \equiv 1$.

In 1990 H. X. Yi [10] proved the following theorem:

Theorem B. [10] Let f and g be two non-constant entire functions and let k be a non-negative integer. If f and g share the value 0 CM, $f^{(k)}$ and $g^{(k)}$ share the value 1CM and $\delta(0, f) > \frac{1}{2}$, then either $f \equiv g$ or $f^{(k)}g^{(k)} \equiv 1$.

Let f be a non-constant meromorphic function. We denote by $P(f) = \sum_{i=0}^{k} a_i f^{(i)}$ the differential polynomial of f, where $a_i, i = 0, 1, 2, \dots, k$ are finite complex numbers and kis a positive integer.

Recently Jiang-Tao Li and Ping Li [4] proved the following theorem:

Theorem C. [4] Let f and g be two non-constant entire functions. Suppose that f and g share the value 0 CM, P(f) and P(g) share the value 1 CM and $\delta(0, f) > \frac{1}{2}$. If $\lambda(f) \neq 1$, then $f \equiv g$ unless $P(f)P(g) \equiv 1$.

Theorem D. [4]. Let f and g be two non-constant entire functions. Suppose that f and g share the value 0 CM, P(f) and P(g) share the value 1IM and $\delta(0, f) > \frac{4}{5}$. If $\lambda(f) \neq 1$, then $f \equiv g$ unless $P(f)P(g) \equiv 1$.

In this paper we prove the following theorem which extend and generalise Theorem C and Theorem D.

Theorem 1.1. Let f and g be two non-constant entire functions. Suppose that f and g share the value 0 CM, P(f) and P(g)share (1, l), where $l = 0, 1, 2, \dots, \infty$. If $\lambda(f) \neq 1$, then $f \equiv g$ unless $P(f)P(g) \equiv 1$ if one of the following holds:

(i)
$$l = 0$$
 and $\delta(0, f) > \frac{4}{5}$
(ii) $0 < l < \infty$ and $\delta(0, f) > \frac{l+1}{2l+1}$
(iii) $l = \infty$ and $\delta(0, f) > \frac{1}{2}$

2. Lemmas

Lemma 2.1. [5] Let f be a non-constant meromorphic function and let k be a nonnegative integer. Then $T(r, P(f)) \le T(r, f) + k \overline{N}(r, \infty; f) + S(r, f)(2.1)$

Lemma 2.2. [4] Suppose that f(z) is a non-constant meromorphic function in the complex plane and a(z) is a small function of f(z). If f(z) is not a polynomial, then

$$N(r, 0; P(f) - P(a)) \le T(r, P(f)) - T(r, f) + N(r, a; f) + S(r, f)$$
(2.2)
and
$$N(r, 0; P(f) - P(a)) \le N(r, a; f) + k\overline{N}(r, \infty; f) + S(r, f) (2.3)$$

Lemma 2.3. [4] Let f and g be two non-constant meromorphic functions such that f and g share the value 1 IM. Let

$$\begin{split} h &= \frac{f''}{f'} - \frac{2f'}{f-1} - \frac{g''}{g'} + \frac{2g'}{g-1} \\ \text{If } h &\equiv 0, \text{ then} \\ T(r; f) &\leq N(r, 0; f) + 2 \, \overline{N}(r, \infty; f) + 2 \, \overline{N}_{(1}^{L}(r, 1; f) + \\ N(r, 0; g) + 2 \, \overline{N}(r, \infty; g) + \overline{N}_{(1}^{L}(r, 1; g) + S(r, f) + S(r, g). \end{split}$$

Lemma 2.4. Let f and g be two non-constant meromorphic functions such that f and g share (1, l) where l is a positive integer. Let

$$h = \frac{f''}{f'} - \frac{2f'}{f-1} - \frac{g''}{g'} + \frac{2g'}{g-1} (2.4)$$

 $\begin{array}{l} \text{If } h \not\equiv 0, \text{then} \\ T(r,f) \leq N(r,0;\,f) + 2\,\overline{N}(r,\infty;\,f) \\ +\,\overline{N}_{(l+1}^L(r,1;\,f) + N(r,0;\,g) \\ +\,2\,\overline{N}(r,\infty;\,g) + \,\overline{N}_{(l+1}^L(r,1;\,g) \\ +\,S(r,f) + \,S(r,g)\,(2.5) \end{array}$

Proof. We get from Nevanlinna's second fundamental theorem

$$\begin{split} T(r,f) &+ T(r,g) \leq \overline{N}(r,0;\,f) + \overline{N}(r,1;\,f) \\ &+ \overline{N}(r,\infty;\,f) - N_0(r,0;\,f') \\ &+ \overline{N}(r,0;\,g) + \overline{N}(r,1;\,g) \\ &+ \overline{N}(r,\infty;\,g) - N_0(r,0;\,g') \\ &+ S(r,f) + S(r,g).\,(2.6) \end{split}$$

Since f and g share (1, l), we get

$$\begin{split} \overline{N}(r,1;f) &= N_{l_{j}}^{E}(r,1;f) + \overline{N}_{(l+1}^{L}(r,1;f) \\ &+ \overline{N}_{(l+1}^{L}(r,1;g) + \overline{N}_{(l+1}^{E}(r,1;f) \\ &+ S(r,f) + S(r,g) \\ &= \overline{N}(r,1;g) + S(r,f) + S(r,g). \end{split}$$

Hence

$$\begin{split} &\overline{N}(r,1;f) + \overline{N}(r,1;g) \\ &= \overline{N}_{1j}(r,1;f) + \overline{N}_{(2}(r,1;f) + \overline{N}(r,1;g) \\ &+ S(r,f) + S(r,g) \\ &\leq \overline{N}_{1j}(r,1;f) + \overline{N}_{(2}(r,1;g) + \overline{N}(r,1;g) \\ &+ S(r,f) + S(r,g) \\ &\leq \overline{N}_{1j}(r,1;f) + N(r,1;g) + S(r,f) + S(r,g) \\ &\leq \overline{N}_{1j}(r,1;f) + T(r,g) \\ &+ S(r,f) + S(r,g) \quad (2.7) \end{split}$$

From (2.6) and (2.7) we get

$$\begin{split} T(r,f) + T(r,g) &\leq \overline{N}(r,0;f) + \overline{N}(r,1;f) \\ &+ \overline{N}(r,\infty;f) - N_0(r,0;f') \\ &+ \overline{N}(r,0;g) + \overline{N}(r,1;g) \\ &+ \overline{N}(r,\infty;g) - N_0(r,0;g') \\ &+ S(r,f) + S(r,g) \\ &\leq \overline{N}(r,0;f) + \overline{N}(r,\infty;f) \\ &- N_0(r,0;f') + \overline{N}(r,0;g) \\ &+ \overline{N}(r,\infty;g) - N_0(r,0;g') \\ &+ \overline{N}_{1}(r,1;f) + T(r,g) \\ &+ S(r,f) + S(r,g) \end{split}$$

Therefore

 $\begin{array}{l} T(r,f) \leq \overline{N}(r,0;f) + \overline{N}(r,\infty;\,f) - N_0(r,0;\,f') \\ + \overline{N}(r,0;\,g) + \,\overline{N}(r,\infty;\,g) - N_0(r,0;\,g') \end{array}$

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$$+ \overline{N}_{1}(r, 1; f) + S(r, f) + S(r, g) (2.8)$$

From (2.4) it can be easily calculated that the possible poles of h occur at (i) multiple zeros of f and g, (ii) those 1 points of f and g whose multiplicities are different (iii) those poles of f and g whose multiplicities are different, (iv) zeros f'(g') which arenot the zeros of f(f - 1)(g(g - 1)).

Since h has only simple poles, we have

$$\begin{split} N(r,\infty; h) &\leq \overline{N}(r,\infty; f) + \overline{N}(r,\infty; g) \\ &+ \overline{N}_{(l+1}^{L}(r,1; f) + \overline{N}_{(l+1}^{L}(r,1; g) \\ &+ \overline{N}_{(2}(r,0; f) + \overline{N}_{(2}(r,0; g) \\ &+ N_{0}(r,0; f') + N_{0}(r,0; g') \\ &+ S(r,f) + S(r,g). (2.9) \end{split}$$

Let z_0 be a common zero of f - 1 and g - 1 then by (2.4), we have z_0 is a zero of h. Also from (2.4), we have m(r, h) = S(r, f) + S(r, g).

Hence by (2.9) we have

$$\begin{split} &\bar{N}_{1)}(r,1;\,f) \leq N(r,0;\,h) \leq T(r,h) + \,O(1) \\ &\leq N(r,\infty;\,h) + \,S(r,f) + \,S(r,g) \\ &\leq \bar{N}(r,\infty;\,f) + \,\bar{N}(r,\infty;\,g) \\ &+ \bar{N}_{(l+1}^{L}(r,1;\,f) + \,\bar{N}_{(l+1}^{L}(r,1;\,g) \\ &+ \,\bar{N}_{(2}(r,0;\,f) + \,\bar{N}_{(2}(r,0;\,g) \\ &+ N_{0}(r,0;\,f') + N_{0}(r,0;\,g') \\ &+ \,S(r,f) + \,S(r,g).\,(2.10) \end{split}$$

From (2.8) and (2.10) we have

$$\begin{split} T(r,f) &\leq \overline{N}(r,0;f) + \overline{N}(r,\infty;f) - N_0(r,0;f') \\ &+ \overline{N}(r,0;g) + \overline{N}(r,\infty;g) - N_0(r,0;g') \\ &+ \overline{N}(r,\infty;f) + \overline{N}(r,\infty;g) + \overline{N}_{(l+1}^L(r,1;f) \\ &+ \overline{N}_{(l+1}^L(r,1;g) + \overline{N}_{(2}(r,0;f) \\ &+ \overline{N}_{(2}(r,0;g) + N_0(r,0;f') + N_0(r,0;g') \\ &+ S(r,f) + S(r,g) \\ &\leq \overline{N}(r,0;f) + 2\overline{N}(r,\infty;f) + \overline{N}(r,0;g) \\ &+ 2\overline{N}(r,\infty;g) + \overline{N}_{(l+1}^L(r,1;f) \\ &+ \overline{N}_{(2}^L(r,0;g) + S(r,f) + S(r,g) \\ &\leq N(r,0;f) + 2\overline{N}(r,\infty;f) + N(r,0;g) \\ &+ 2\overline{N}(r,\infty;g) + \overline{N}_{(l+1}^L(r,1;f) \\ &+ \overline{N}_{(l+1}^L(r,1;g) + S(r,f) + S(r,g) \\ & \text{This completes the proof.} \end{split}$$

Lemma 2.5. [8] Let $f_j (j = 1, 2, ..., m + 1)$ and $g_j (j = 1, 2, ..., m)$ are entire functions satisfying the following conditions:

(i) $\sum_{j=1}^{m} f_j(z) e^{g_j(z)} \equiv f_{m+1}(z);$

(ii) The order of $f_j(z)$ is less than the order of $e^{g_k(z)}$ for $1 \le j \le m+1, 1 \le k \le m$ and the order of $f_j(z)$ is less than the order of $e^{g_l(z)-g_k(z)}$ for $m \ge 2$ and $1 \le j \le m+1, 1 \le l, k \le m, l \ne k$. Then $f_j \equiv 0 \ (j = 1, 2, \dots, m+1)$. **Lemma 2.6.** Let f and g be two non-constant entire functions. Suppose that f and gshare the value 0 CM, P(f) and P(g) share the value 1IM. If $\lambda(f) \neq 1$ and $P(f) \equiv P(g)$, then $f \equiv g$.

Proof. It can be proved easily with the help of the proof of the Theorem D.

3. Proof of the Theorem 1.1

Proof. If $l = 0, \infty$ then by Theorem D and Theorem C we get the result.

Now suppose that $0 < l < \infty$. Then we have to consider the following two cases:

Case 1 Any one or both of P(f) and P(g) is constant. Suppose that P(f) = c; where c is a finite complex constant. Then

$$f \equiv d + \sum_{i=1}^{m} p_i(z) e^{\alpha_i z},$$

where *d* is a finite complex constant, $p_i(i = 1, 2, ..., m)$ are polynomials and $\alpha_i(i = 1, 2, ..., m)$ are distinct finite complex constants. Since $\lambda(f) \neq 1$ therefore $f \equiv d + \sum_{i=1}^{m} p_i(z)$, that is *f* is apolynomial. Suppose the degree of *f* is *n*. Then $N(r, 0; f) = n \log r$ and $T(r, f) = n \log r + O(1)$.

Therefore

 $\delta(0, f) = 1 - \limsup_{r \to \infty} \frac{N(r, 0; f)}{T(r, f)} = 0,$ which is a contradiction.

Case $2P(f), P(g) \neq c$ where c is a finite complex constant. Let $F = P(f), \quad G = P(g)$ and $H = \frac{F''}{F'} - \frac{2F'}{F-1} - \frac{G''}{G'} + \frac{2G'}{G-1}$ then F and G share (1, l).

By Milloux's basic result we have

 $\begin{array}{l} T(r,f) \leq N(r,0;\,f) + \overline{N}(r,\infty;\,f) \\ + \,\overline{N}(r,1;\,P(f)) + S(r,f) \\ \leq N(r,0;\,g) + \,\overline{N}(r,1;\,P(g)) + S(r,f) \\ \leq T(r,g) + \,T\!\left(r,P(g)\right) + S(r,f). \end{array}$

Hence by Lemma 2.1 we have $T(r,f) \leq 2T(r,g) + S(r,f) + S(r,g) \quad (3.1)$

Similarly we have $T(r,g) \leq 2T(r,f) + S(r,f) + S(r,g) \quad (3.2)$

From (3.1) and (3.2) we have S(r, f) = S(r, g) (3.3)

Now we have to consider the following two subcases:

Subcase1H $\equiv 0$. In this case we have $F = \frac{PG + Q}{RG + 5}$, (3.4) where P, Q, R and S are finite complex constants such that $PS - QR \neq 0$: Now we have to consider the following three cases:

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Case IP = 0 and $R \neq 0$. In this case $F = \frac{Q}{RG+S}$ If $S \neq 0$; then $N\left(r, -\frac{S}{R}; G\right) = N(r, \infty; F)$. By Nevanlinna's Second Fundamental Theorem we have $T(r, G) \leq N(r, 0; G) + N\left(r, -\frac{S}{R}; G\right)$ $+ N(r, \infty; G) + S(r, G)$ $= N(r, 0; G) + N(r, \infty; F)$ $+ N(r, \infty; G) + S(r, G)$ = N(r, 0; G) + S(r, G) = N(r, 0; G) + S(r, G)= N(r, 0; G) + S(r, G) (3.5)

By Lemma 2.2 from (3.5) and (3.3) we have

$$T(r, P(g)) \le T(r, P(g)) - T(r, g) + N(r, 0; g) + S(r, f)$$

Hence

 $T(r,g) \le N(r,0; g) + S(r,f)$ = N(r,0; f) + S(r,f) (3.6)

By (3.1) and (3.6) we have

 $T(r, f) \leq 2N(r, 0; f) + S(r, f),$ which contradicts the condition $\delta(0, f) > \frac{l+1}{2l+1}.$ Hence S = 0 and so $FG = \frac{Q}{R}$. If 1 is a Picard exceptional value of F then $\frac{Q}{R} = 1$ otherwise it contradicts the Deficiency Theorem [9]. So $FG \equiv 1$.

If 1 is not a Picard exceptional value of F, then there exist a complex number z_0 such that $F(z_0) = G(z_0) = 1$. So $\frac{Q}{R} = 1$. Hence $FG \equiv 1$.

Case IIP $\neq 0$ and R = 0In this case we have $F = \frac{P}{s}G + \frac{Q}{s}$.

If $Q \neq 0$, then $N\left(r, \frac{Q}{S}; F\right) = N(r, 0; G).$

Hence by Nevanlinna's Second Fundamental Theorem we have

 $T(r,F) \le N(r,0; F) + N\left(r,\frac{Q}{S}; F\right)$ $+ N(r,\infty; F) + S(r,F)$ = N(r,0; F) + N(r,0; G) + S(r,F)(3.7)

By Lemma 2.2 from (3.7) and (3.3) we have

 $T(r, P(f)) \leq T(r, P(f)) - T(r, f) + N(r, 0; f)$ + N(r, 0; g) + S(r, f)Hence $T(r, f) \leq N(r, 0; f) + N(r, 0; g) + S(r, f)$ = 2N(r, 0; f) + S(r, f), which contradicts the condition $\delta(0, f) > \frac{l+1}{2l+1}$, Hence Q = 0 and so $F = \frac{p}{s}G$. If 1 is a Picard exceptional value of F then $\frac{p}{r} = 1$ of

If 1 is a Picard exceptional value of F then $\frac{F}{s} = 1$ otherwise it contradicts the Deficiency Theorem [9]. So $F \equiv G$. If 1 is not a Picard exceptional value of F, then there exist a complex number z_0 such that $F(z_0) = G(z_0) = 1$. Hence $F \equiv G$. By Lemma 2.6 we have $f \equiv g$.

Case IIIPR $\neq 0$

From (3.4) it is clear that $\frac{P}{R}$ is a Picard exceptional value of F. Hence by Nevanlinna's Second Fundamental Theorem we have

$$T(r,F) \le N(r,0;F) + N\left(r,\frac{t}{R};F\right) + N(r,\infty;F) + S(r,F) = N(r,0;F) + S(r,F)$$
(3.8)

By Lemma 2.2 and (3.8) we have $T(r, f) \le N(r, 0; f) + S(r, f)$, which contradicts the condition $\delta(0, f) > \frac{l+1}{2l+1}$. Subcase 2H $\not\equiv 0$. By Lemma 2.4 we have

$$\begin{split} T(r,F) &\leq N(r,0;\,F) + \bar{N}_{(l+1}^{L}(r,1;\,F) \\ &+ N(r,0;\,G) + \bar{N}_{(l+1}^{L}(r,1;\,G) \\ &+ S(r,F) + S(r,G) \quad (3.9) \\ &\text{Hence by (2.3) we have} \end{split}$$

$$\begin{split} &\overline{N}_{(l+1}^{L}(r,1;\,F) + \overline{N}_{(l+1}^{L}(r,1;\,G) \\ &\leq \frac{1}{l}N(r,0;\,F') \\ &\leq \frac{1}{l}N(r,0;\,F) + \frac{1}{l}\overline{N}(r,\infty;\,F) + S(r,F) \ (3.10) \end{split}$$

From (3.9) and (3.10) we have $T(r,F) \le \frac{l+1}{l} N(r,0;F) + N(r,0;G) + S(r,F) + S(r,G) \quad (3.11)$

Hence by Lemma 2.2 and (3.11) we have

$$T(r, P(f)) \le T(r, P(f)) - T(r, f) + N(r, 0; f) + \frac{1}{l}N(r, 0; f) + N(r, 0; g) + S(r, f) + S(r, g)$$

Therefore

$$T(r, f) \le N(r, 0; f) + \frac{1}{l}N(r, 0; f) + N(r, 0; g) + S(r, f) + S(r, g) \le \frac{2l+1}{l}N(r, 0; f) + S(r, f),$$

which contradicts the condition $\delta(0, f) > \frac{l+1}{2l+1}$. This completes the proof.

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