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Dynamic Analysis of Flexible Mechanisms by Multibody Dynamics

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Abstract: ***In modeling, analysis & control of flexible mechanisms, four bar mechanisms are generally preferred due to their manufacturing techniques & low cost. Modeling, analysis and control of flexible mechanisms have been researched since the early 70s. The investigation has been focused mostly on the definition of accurate mathematical models both for single flexible bodies and multi-body systems. A general approach is presented for modeling of a flexible multi-body system by using a lumped mass finite element method. With ANSYS theoretical and practical knowledge of the finite element method and analyzing engineering problems can be gained. In this study, vibration characteristics of flexible four-bar mechanisms & natural frequencies and corresponding modes shapes of the flexible mechanisms are investigated by using the procedure developed in ADAMS.***

Keywords: Finite element method (FEM), Automatic dynamic analysis of mechanical systems (ADAMS).

1. Introduction

The high productivity & technology system demanded by modern mechanical industry requires high operating speeds, superior reliability, accurate performance, light weights and high-precision machinery. In order to achieve high speed operation with increased efficiency, weights of machine components are reduced. As operating speed increases & weight of components decreases, rigid body is not enough so, a rigid body links becomes flexible. High speed light weight manipulator is an example of flexible multi-body system. We already know that modeling of flexible multi-body systems can be done by using lumped mass finite element method. Now we will perform modeling of flexible multi-body systems considering lumped masses by ADAMS. Finite element method is used for modeling of flexible links which behave like both continuous systems with infinite degrees of freedom and discrete systems. A general model to describe the elastic motion of a mechanism can be established with the use of finite element methods resulting in a set of second order differential equations. A common assumption in this procedure is that the total motion is comprised of an elastic motion superposed onto the rigid body motion. As a result the equations of this motion have a significant feature time dependent coefficients. If the effects of nonlinear elastic deflections and/or nonlinear joint characteristics are considered, the equations of motion will be nonlinear. The dynamic response is viewed as a transient response and a steady state response.

2. Kinematics of Rigid Four-Bar Mechanism

A four-bar mechanism of which links are drawn as position vectors is shown inFigure 1.1.

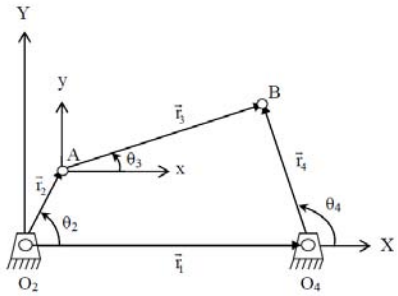

Figure 1.1: A four-bar linkage mechanism showing position vectors

The vector loop closure equation shown in Figure 1.1 is written as:

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0 \tag{1.1}$$

Equation 1.1 is expressed in terms of complex numbers as follows:

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_4 e^{i\theta_4} - r_1 = 0 \tag{1.2}$$

By using the Euler expansion, Equation 1.2 is written as:

$$r_2(\cos\theta_2 + i\sin\theta_2) + r_3(\cos\theta_3 + i\sin\theta_3) - r_4(\cos\theta_4 + i\sin\theta_4) - r_1 = 0 \tag{1.3}$$

Equation 1.3 can be resolved into real and imaginary parts as:

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1 = 0 \tag{1.4}$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 = 0 \tag{1.5}$$

Taking the square of both sides of Equations 1.4 and 1.5 and summing them, the following equation is found:

$$r_3^2 = (-r_2\sin\theta_2 + r_4\sin\theta_4)^2 + (-r_2\sin\theta_2 + r_4\cos\theta_4 + r_1)^2 \tag{1.6}$$

To simplify the Equation 1.6, the constants K_1, K_2, and K_3 are defined interms of the constant link lengths in the equations:

$$K_1 = \frac{r_1}{r_2} \tag{1.7}$$

$$K_2 = \frac{r_1}{r_4} \tag{1.8}$$

$$K_3 = \frac{r_2^2 - r_3^2 - r_4^2 + r_1^2}{2r_2 r_4} \quad (1.9)$$

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos\theta_2 \cos\theta_4 + \sin\theta_2 \sin\theta_4 \quad (1.10)$$

Then Freudenstein's equation (Todorov 2002) is obtained as follows:

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4) \quad (1.11)$$

In order to reduce the Equation 1.11 to a more tractable form for solution, thefollowing half angle identities are substituted:

$$\sin\theta_4 = \frac{2\tan(\frac{\theta_4}{2})}{1 + \tan^2(\frac{\theta_4}{2})} \quad (1.12)$$

$$\cos\theta_4 = \frac{1 - \tan^2(\frac{\theta_4}{2})}{1 + \tan^2(\frac{\theta_4}{2})} \quad (1.13)$$

Then, the following equation which is quadratic in terms of $\tan(\frac{\theta_4}{2})$ is found:

$$A\tan^2(\frac{\theta_4}{2}) + B\tan(\frac{\theta_4}{2}) + C = 0 \quad (1.14)$$

where

$$A = \cos\theta_2 - K_1 \cos\theta_2 + K_2 \cos\theta_2 + K_3 \quad (1.15)$$

$$B = -2\sin\theta_2 \quad (1.16)$$

$$C = K_1 - (K_2 + 1)\cos\theta_2 + K_3 \quad (1.17)$$

θ_4 expressedin Equation 1.14 can be found by solving the quadratic equation as:

$$\theta_4 = 2\tan^{-1}(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}) \quad (1.18)$$

Where the plus and minus sign refers to two different configuration of the mechanism.

In order to make velocity analysis of four-bar linkage, the derivation of Equation 1.2 is considered since r_1 is constant and $\omega_2 = 0$, the aforementioned equation is found:

$$ir_2\omega_2 e^{i\theta_2} + ir_3\omega_3 e^{i\theta_3} - ir_4\omega_4 e^{i\theta_4} = 0 \quad (1.19)$$

Resolving into real and imaginary parts, the following equations are written:

$$r_2\omega_2 \cos\theta_2 + r_3\omega_3 \cos\theta_3 - r_4\omega_4 \cos\theta_4 = 0 \quad (1.20)$$

$$-r_2\omega_2 \sin\theta_2 - r_3\omega_3 \sin\theta_3 + r_4\omega_4 \sin\theta_4 = 0 \quad 1.21)$$

Then velocity equations are obtained as follows:

$$\omega_3 = \frac{r_2\omega_2}{r_3}\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad (1.22)$$

$$\omega_4 = \frac{r_2\omega_2}{r_4}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad (1.23)$$

In order to make acceleration analysis of four-bar linkage, the derivation of Equation 1.19 is taken and the following equation is found:

$$-r_2\omega_2^2 e^{i\theta_2} + ir_2\alpha_2 e^{i\theta_2} - r_3\omega_3^2 e^{i\theta_3} + ir_3\alpha_3 e^{i\theta_3} + r_4\omega_4^2 e^{i\theta_4} - ir_4\alpha_4 e^{i\theta_4} = 0 \quad (1.24)$$

The following parameters are defined:

$$D_1 = -r_2\omega_2^2 \cos\theta_2 - r_2\alpha_2 \sin\theta_2 - r_3\omega_3^2 \cos\theta_3 + r_4\omega_4^2 \cos\theta_4 = 0 \quad (1.25)$$

$$D_2 = r_3 \sin\theta_3 \quad (1.26)$$

$$D_3 = r_4 \sin\theta_4 \quad (1.27)$$

$$E_1 = -r_2\omega_2^2 \sin\theta_2 + r_2 \cos\theta_2 - r_3\omega_3^2 \sin\theta_3 + r_4\omega_4^2 \sin\theta_4 \quad (1.28)$$

$$E_2 = r_3 \cos\theta_3 \quad (1.29)$$

$$E_3 = -r_4 \cos\theta_4 \quad (1.30)$$

Therefore, Equation 1.25 and 1.26 are written as:

$$D_1 + D_2\alpha_3 + D_3\alpha_4 = 0 \quad (1.31)$$

$$E_1 + E_2\alpha_3 + E_3\alpha_4 = 0 \quad (1.32)$$

Then, acceleration equations are obtained as follows:

$$\alpha_3 = \frac{-D_1E_3 + D_3E_1}{D_2E_3 - D_3E_2} \quad (1.33)$$

$$\alpha_4 = \frac{-D_2E_1 + D_1E_2}{D_2E_3 - D_3E_2} \quad (1.34)$$

By carrying out the substitution and simplification, angular acceleration expressions of link 3 and link 4 are found as (Söylemez 1999):

$$\alpha_3 = \frac{r_2\omega_2^2 \cos(\theta_2 - \theta_4) + r_2\alpha_2 \sin(\theta_2 - \theta_4) + r_3\omega_3^2 \cos(\theta_4 - \theta_3) - r_4\omega_4^2}{r_3 \sin(\theta_4 - \theta_3)} \quad (1.35)$$

$$\alpha_4 = \frac{r_2\omega_2^2 \cos(\theta_2 - \theta_3) + r_2\alpha_2 \sin(\theta_2 - \theta_4) - r_4\omega_4^2 \cos(\theta_4 - \theta_3) + r_3\omega_3^2}{r_4 \sin(\theta_4 - \theta_3)} \quad (1.36)$$

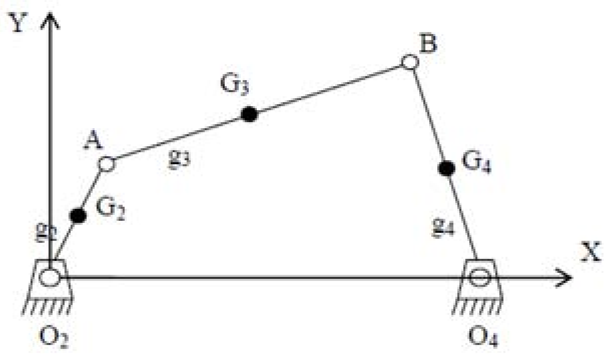

Figure 1.2: Four-bar mechanism showing the centers of gravity of links

The centers of gravity of the links are shown in Figure 1.2. Accelerations of the centers of gravity can be found using the standard kinematic relationships as follows:

$$a_{G2x} = -g_2\omega_2^2 \cos\theta_2 - g_2\alpha_2 \sin\theta_2 \quad (1.37)$$

$$a_{G2y} = -g_2\omega_2^2 \sin\theta_2 + g_2\alpha_2 \cos\theta_2 \quad (1.38)$$

$$a_{G3x} = -r_2\omega_2^2 \cos\theta_2 - r_2\alpha_2 \sin\theta_2 - g_3\omega_3^2 \cos\theta_3 - g_3\alpha_3 \sin\theta_3 \quad (1.39)$$

$$a_{G3y} = -r_2\omega_2^2 \sin\theta_2 + r_2\alpha_2 \cos\theta_2 - g_3\omega_3^2 \sin\theta_3 + g_3\alpha_3 \cos\theta_3 \quad (1.40)$$

$$a_{G4x} = -g_4\omega_4^2 \cos\theta_4 - g_4\alpha_4 \sin\theta_4 \quad (1.41)$$

$$a_{G4y} = -g_4\omega_4^2 \sin\theta_4 + g_4\alpha_4 \cos\theta_4 \qquad (1.42)$$

Kinetics of Rigid Four-Bar Mechanism

Kinetic analysis of rigid four-bar mechanism is based on the accelerations of the centers of gravity given by Equations 1.37 to 1.42. Inertial force and inertial moment of the link *i*are given by

$$\vec{F}_i^{\,inertia} = -m_i \vec{a}_i \qquad (1.43)$$

$$\vec{M}_i^{\,inertia} = -I_{Gi}\vec{\alpha}_i \qquad (1.44)$$

Finite Element Model for Flexible Four-Bar Mechanism

The finite element model shown in Figure 1.3 is used to model any link of theflexible four-bar mechanism (Turcic and Midha 1984).

$$d = Nu^e \qquad (1.45)$$

where d is the local displacement vector of any point on element and u^e is the nodal displacements vector including nodal displacements shown in Figure 1.3

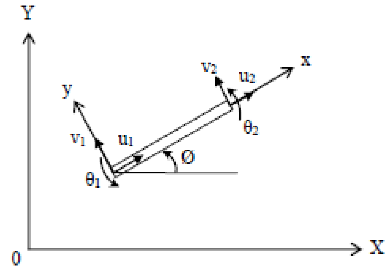

Fig.1.3. A finite element for flexible link

$$u^e = [u_1\ v_1\ \theta_1\ u_2\ v_2\ \theta_2]^T \qquad (1.46)$$

The equation of motion for a single finite element of the mechanism is derived by using Lagrangian equation:

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{u}^e}\right) - \frac{\partial KE}{\partial u^e} + \frac{\partial PE}{\partial u^e} = Q \qquad (1.47)$$

Where$Q$ are the generalized forces acting on elements.

The position of any point in the finite element Rshown in figure1.4 can be written as:

$$R = R_0 + T_m d \qquad (1.48)$$

Where$R_0$is the position of the origin of the local (x, y) coordinate system, $T_m$is the transformation matrix between the local (x,y) coordinate system and the reference (X,Y) coordinate system which is given by:

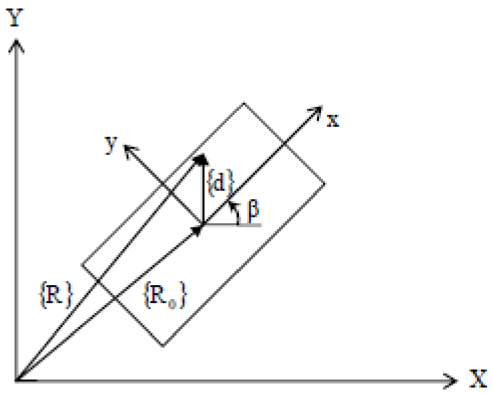

Figure 1.4: Positions of any point in terms of d

$$T_m = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad (1.49)$$

The kinetic energy of the element is given below:

$$KE = \frac{1}{2}\int \rho \dot{R}^T \dot{R}\, dv \qquad (1.50)$$

The equation of motion of a single finite element is expressed as:

$$m^e\ddot{u}^e + k^e u^e = Q_{e1}^e + Q_1^e + Q_{ex}^e - m^e\ddot{U}_0^e - 2m_{vel}^e\dot{u}^e - m_{acc}^e u^e$$

(1.51)

Where

$$m^e = \rho AL\begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 12/35 & 11L/210 & 0 & 9/70 & -13L/420 \\ 0 & 11L/210 & L^2/105 & 0 & 13L/420 & -L^2/140 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 9/70 & 13L/420 & 0 & 13/35 & -11L/210 \\ 0 & -13L/420 & -L^2/140 & 0 & -11L/210 & L^2/105 \end{bmatrix} \qquad (1.52)$$

$$k^e = \begin{bmatrix} EA_x/L & 0 & 0 & -EA_x/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & -2EI/L \\ -EA_x/L & 0 & 0 & EA_x/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \qquad (1.53)$$

$$m_{vel}^e = \int \rho (N^T T_m^T \dot{T}_m N)\, dv \qquad (1.54)$$

$$m_{acc}^e = \int \rho (N^T T_m^T \ddot{T}_m N)\, dv \qquad (1.55)$$

Q_{e1}^e is the force vector having the forces acting on the elements from adjacent elements, Q_1^e is the force vector due to the adjacent links, Q_{ex}^e is the external force vector acting on the element. $2m_{vel}^e\dot{u}^e$ has the forces resulting from Coriolis acceleration and $m_{acc}^e u^e$ has the forces resulting from tangential and normal accelerations.

The equation of motion of links is expressed as (Turcic and Midha 1984):

$$m^1\ddot{u}^1 + k^1u^1 = Q_1^1 + Q_{ex}^1 - m^1\ddot{U}_0^1 - 2m_{vel}^1\dot{u}^1 - m_{acc}^1 u^1$$

(1.56)

The equation of motion of entire mechanism is given by (Turcic and Midha 1984):

$$M\ddot{u} + C\dot{u} + K\ddot{u} = Q_{ex} - M\ddot{U}_0 - 2(Md + M_{vel})\dot{u} - (Mdd + 2Md_{vel} + M_{acc})u$$

(1.57)

Where$C$is the viscous damping matrix, $u$is the displacement vector, $\dot{u}$ is the velocity vector, $\ddot{u}$ is the acceleration vector, and $\ddot{U}_0$ is the rigid body acceleration vector of the mechanism.

The derivation of equation of motion is based on small strain theory. However axial force is effective of the stiffness properties of the beam. Using the large strain theory, the geometric stiffness matrix is found as follows (Turcic and Midha 1984):

$$k_G^e = \frac{F}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6/5 & L/10 & 0 & -6/5 & L/10 \\ 0 & L/10 & 2L^2/15 & 0 & -L/10 & -L^2/30 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6/5 & -L/10 & 0 & 6/5 & -L/10 \\ 0 & L/10 & -L^2/30 & 0 & -L/10 & 2L^2/15 \end{bmatrix} \quad (1.58)$$

Where F is axial force acting on element.

The equation of motion given by Equation 1.57 has been modified as (Yang and Sadler 2000):

$$M\ddot{U} + C_0 + 2M_\omega \dot{U} + K_0 + M_\alpha U = F - M\ddot{U}_0 \quad (1.59)$$

where

$$M_\omega = \tilde{T}^T (\int \rho_i A_i N^T T_2 \dot{T}_2^T N dx)\tilde{T} \quad (1.60)$$

and

$$M_\alpha = \tilde{T}^T (\int \rho_i A_i N^T T_2 \ddot{T}_2^T N dx)\tilde{T} \quad (1.61)$$

Eigenanalysis is applied to the system having mass and structural stiffness matrices; due to lack of generalized force vector in free vibration analysis as follows (Yu and Xi 2003):

$$M_r(\theta_2)\ddot{U}_r + K_r(\theta_2)U_r = 0 \quad (1.62)$$

From the following equation the natural frequencies and modal vectors can be obtained for flexible four-bar mechanism:

$$K_r(\theta_{2,k})X = \omega_n^2 (M_r \theta_{2,k})X \quad (1.63)$$

3. Result and Discussion

Figure 1.5 shows an example for four-bar mechanism modeled by using lumped parameter approach. The inertia forces given by Equation 1.40 for the lumped masses can be found by using the acceleration expressions given by Equations 1.34 to 1.39.Considering these forces, kinetic analysis of rigid four-bar mechanism can be carriedout by using the standard procedure based on the Equation 1.42 to 1.44.

The inertia forces acting on these lumped masses are summarized below:

$$F_{ix} = -m_i a_{ix} \quad i=1,\ldots\ldots, 12 \quad (1.64)$$

$$F_{iy} = -m_i a_{iy} \quad i=1,\ldots\ldots, 12 \quad (1.65)$$

Where a_i is the acceleration of mass m_i and can be calculated by using the formulation
given in Equations 1.34 to 1.39.

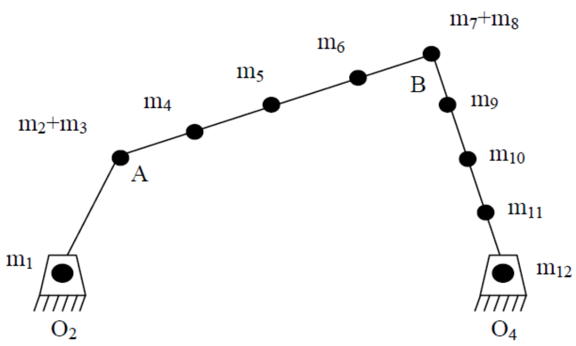

Figure 1.5: A four-bar mechanism showing lumped masses.

The numerical values of the first mechanism shown in Figure 1.5 are listedbelow:
r_1=254mm (length of the link1)
r_2=100mm (length of the link2)
r_3=279.4mm (length of the link3)
r_4=266.7mm (length of the link4)
d_4=69.85mm (distance between A and m_4)
d_5=139.7mm (distance between A and m_5)
d_6=209.55mm (distance between A and m_6)
d_9=200.025mm (distance between O_4 and m_9)
d_{10}=133.35mm (distance between O_4 and m_{10})
d_{11}=66.675mm (distance between O_4 and m_{11})
In the second mechanism, the numerical values are taken as the same listed above except r_2=102 *mm*.

The cross-sectional dimensions and material properties of the first flexible fourbarmechanism are listed below (Yu and Xi 2003):
b =6.1mm (width of the links)
h=4.25mm (height of the links)
E=200000 MPa (modulus of elasticity)
$\rho=7.8\times10^{-9}$ tonnes/mm^3 (density)
v=0.3 (Poisson's ratio)

For the second mechanism, the cross-sectional dimensions and material properties are given as (Yu and Xi 2003):
b_2 =4.24 mm (width of the link2)
h_2=25.4 mm (height of the link2)
b_3=1.6 mm (width of the link3)
h_3=25.4 mm (height of the link3)
b_4=1.6 mm (width of the link4)
h_4=25.4 mm (height of the link4)
E=68900 MPa (modulus of elasticity)
$\rho_2=2.698\times10^{-9}$ tonnes/mm^3 (density of link2)
$\rho_3=2.923\times10^{-9}$ tonnes/mm^3 (density of link3)
$\rho_2=2.923\times10^{-9}$ tonnes/mm^3 (density of link4)
v=0.3 (Poisson's ratio)

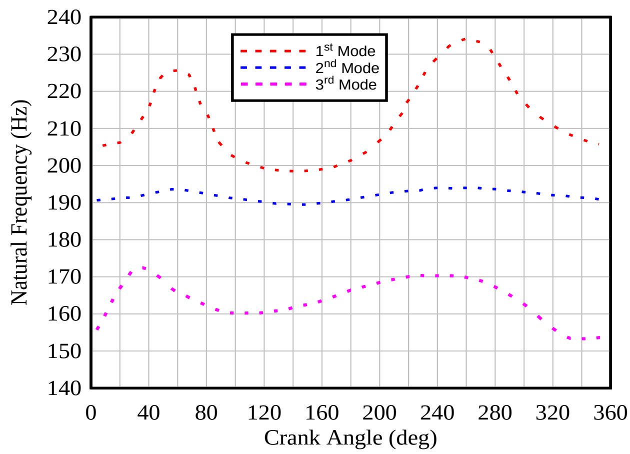

Figure 1.6: Natural frequencies of flexible four-bar mechanism.

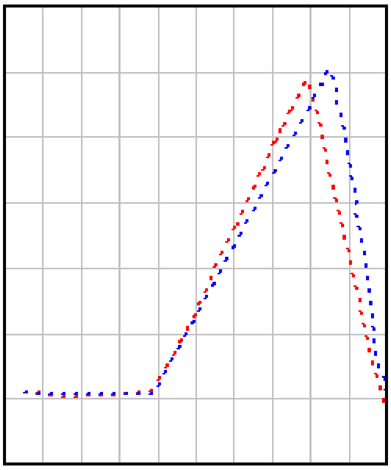

Figure 1.7: First mode shape of flexible four-bar mechanism for $\theta_2=0^o$.

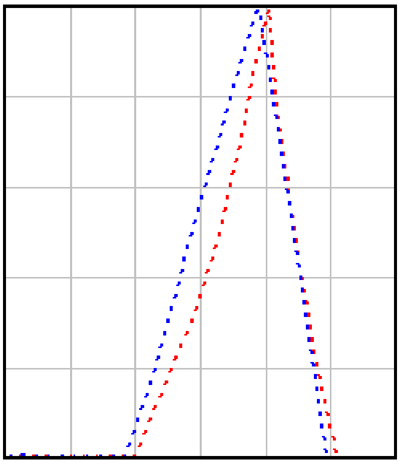

Figure 1.8: Second mode shape of flexible four-bar mechanism for $\theta_2=0^o$.

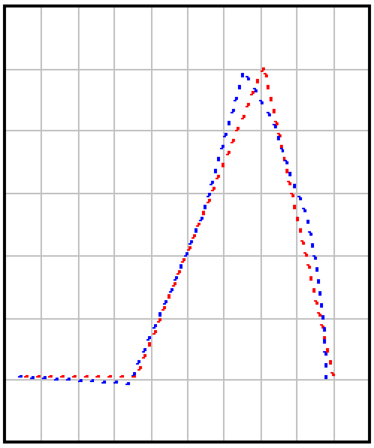

Figure 1.9: Third mode shape of flexible four-bar mechanism for $\theta_2=0^o$.

Natural Frequencies of Mechanism with Internal Force

The internal force due to the inertia force is taken into account in finding thenatural frequencies of mechanism for different angular velocities. Inertia forcesacting on the lumped masses of the mechanism are considered in finite element modelcreated in ADAMS. The results are shown in Figures 2.0 to 2.8.

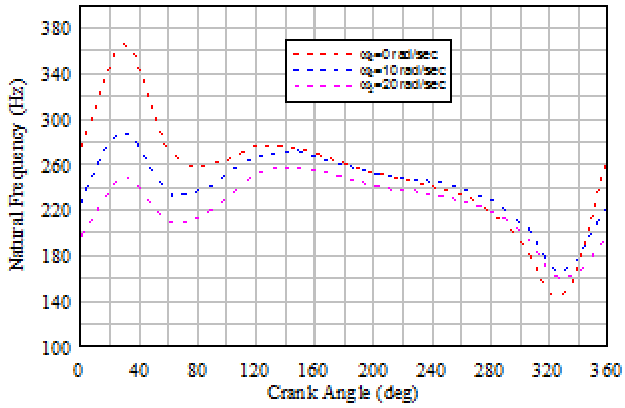

Figure 2.0: First natural frequencies of flexible four-bar mechanism.

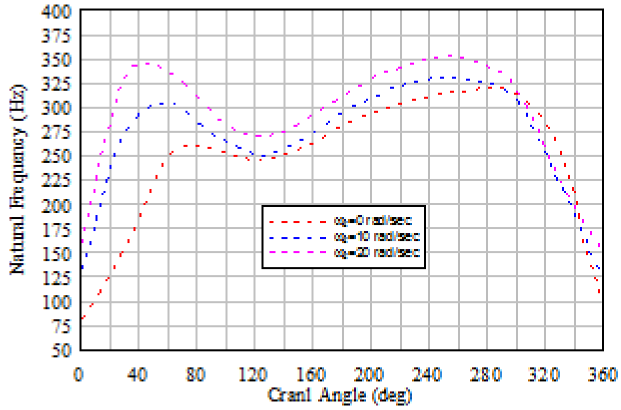

Figure 2.1: Second natural frequencies of flexible mechanism.

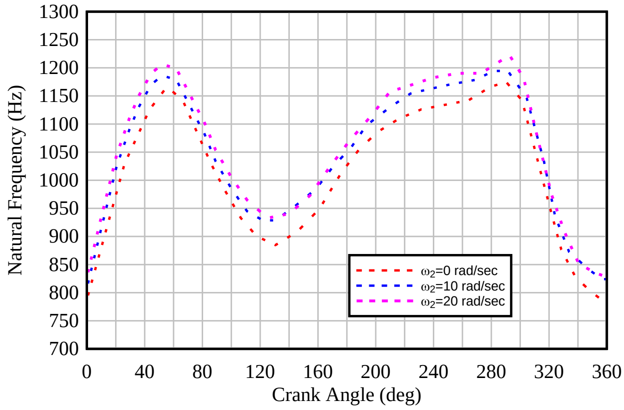

Figure 2.2: Third natural frequencies of flexible four-bar mechanism.

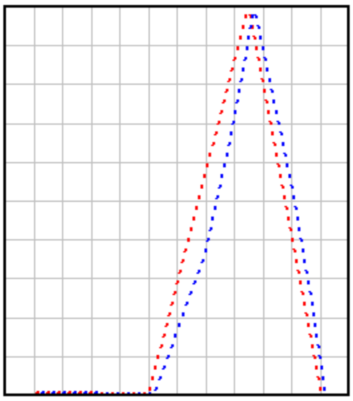

Figure 2.3: First mode shape of flexible four-bar mechanism for ω_2=10*rad/sec.*

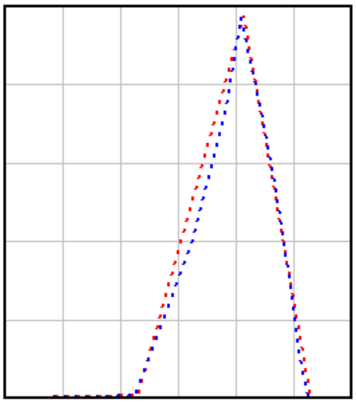

Figure 2.4: First mode shape of flexible four-bar mechanism for ω_2=20*rad/sec.*

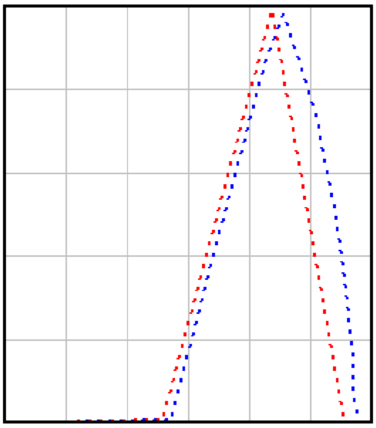

Figure 2.5: Second mode shape of flexible four-bar mechanism for ω_2=10*rad/sec.*

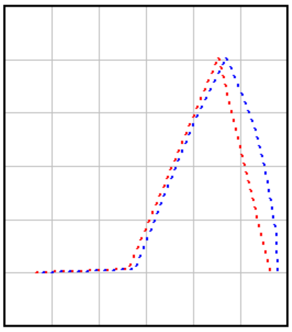

Figure 2.6: Second mode shapes of flexible four-bar mechanism for ω_2=20*rad/sec.*

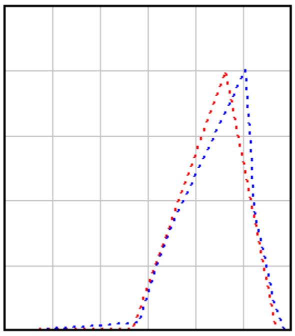

Figure 2.7: Third mode shape of flexible four-bar mechanism for ω_2=10*rad/sec.*

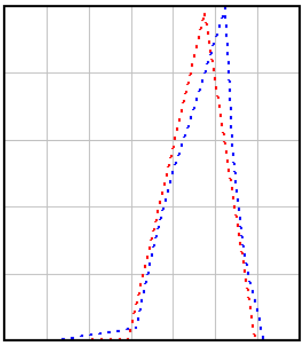

Figure 2.8: Third mode shapes of flexible four-bar mechanism for ω_2=20*rad/sec.*

The upper and lower limits of the static natural frequencies of the flexible four bar mechanism based on the crank angular position can be seen from the Figure 1.6.The first mode shapes of the mechanism plotted in Figure1.7-1.9. Similarly second and third mode shapes are plotted. They are consistent in each other. It can be seen from Figure 2.0-2.8 that the crank angular velocity is effective on the first and second dynamic natural frequencies but not on the third one. The first and second mode shapes of the mechanism plotted in Figures 2.3-2.8 are very consistent in each other, namely the links have the similar displacements for these modes.

This study presents an Eigen analysis of flexible four-bar mechanism by using finite element model conjunction with kinematic and kinetic relationships. The solution procedure based on the discrete crank positions and the discrete inertia forces applied to the nodes of the finite element model has been developed in ADAMS to accomplish this analysis.

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