

# On e-Chromatic Number of Adjacency Matrix of a Dutchwindmill Graph $D_3^m$ and Central Graphs

S. Uma Maheswari<sup>1</sup>, Dr. K. Indirani<sup>2</sup>

<sup>1</sup>Assistant Professor, Department of Mathematics, CMS College of Science & Commerce, Coimbatore

<sup>2</sup>Associate Professor, Head, Department of mathematics, Nirmala College for Women, Coimbatore

**Abstract:** In this paper we have found out the end vertex adjacency matrix and end vertex incidence matrix on Dutchwind mill graphs. We have also found out the rank of the end vertex adjacency matrix and finalized the bounds for the e-chromatic number of some Dutchwind mill graph.

**Keywords:** end vertex adjacency matrix, end vertex incidence matrix, dutchwind mill graph, e-spectrum, e-chromatic number, e-eigen value

AMS Subject classification Code: 05C69

## 1. Introduction

Vivin J. Vernold, M. Venkatachalam and Ali M. M. Akbar gave a note on achromatic coloring of central graphs. Let  $G$  be a finite undirected graph with no loops and multiple edges. Mostly the concept of matrices is very easy and convenient and it is widely used in various fields.

The basic definitions are given in section 2. The end vertex adjacency matrices and end vertex incidence matrices are given in section 3. We find the e-chromatic number for the Dutchwind mill graph  $D_3^m$ , we find the duplication and triplication and finally we find the end vertex adjacency matrix and then we find the e-chromatic number in section 4.

## 2. Basic Definitions

The basic definitions which are used in section 4 are

**Definition 2.1** A Dutchwindmill graph  $G=(V,X)$  on the non empty set of vertices and the set of edges  $X$  consists of  $n$  edges satisfying the conditions.

- 1) Two edges have at most one vertex in common
- 2) Two edges  $(u_1, u_2, u_3, \dots, u_n)$  is the same as  $(u_n, u_{n-1}, u_{n-2}, \dots, u_1)$

**Definition 2.2** For the edge  $E=(u_1, u_2, u_3, \dots, u_n)$ ,  $u_1$  and  $u_2$  are said to be the end vertices of the edge  $E$  while  $u_2, u_3, \dots, u_{n-1}$  are said to be middle vertices of  $E$ .

**Definition 2.3** Two vertices in a dutchwind mill graph are said to be adjacent if they belong to the same edge

**Definition 2.4** Two edges are adjacent if they have a common vertex

**Definition 2.5** Let  $G=(V,X)$  be a dutchwind mill graph. Two vertices are said to be the e-adjacent if they are the end vertices of an edge in  $G$ .

**Definition 2.6** For the end vertex we define the various types of degrees as follows.

**Degree:**  $\deg(v)$  is the number of edges having  $v$  as an end vertex

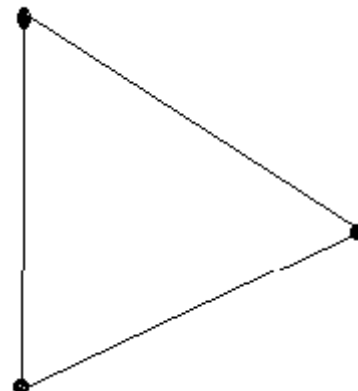
**Edge degree:**  $\deg_e(v)$  is the number of edges containing  $v$

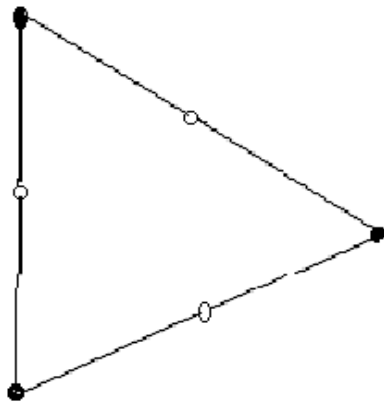
**Adjacency degree:**  $\deg_{ca}(v)$  is the number of vertices which are consecutive adjacent to  $v$

**Definition 2.7** A Dutchwind mill Graph  $G$  is complete if any two vertices in  $G$  are adjacent.

## 3. End Vertex Adjacency Matrix and End Vertex Incidence Matrix

**Definition 3.1** Let  $G=(V,X)$  be a Dutchwind mill graph with  $V=\{v_1, v_2, v_3\}$  and  $X=\{e_1, e_2, e_3\}$ . The end vertex adjacency matrix  $A_e=(a_{ij})$  of Dutchwind mill graph with  $p$  points is the  $p \times p$  matrix in which  $(a_{ij})=1$ , if  $v_i$  is e-adjacent with  $v_j$   
 $(a_{ij})=0$ , otherwise.





$D_3^1 C(D_3^1)$

The end vertex adjacency matrix of the Dutchwind mill graph  $D_3^2$  is

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Definition 3.2**

Let G be a graph with n vertices, e edges and no self-loops. Define an n by e matrix  $B = [b_{ij}]$ , whose rows correspond to the n vertices and the e columns correspond to the e edges, as follows :

- $(b_{ij}) = 1$ , end vertex  $v_i$  is incident with  $e_j$  and
- $(b_{ij}) = 0$ , otherwise.

Such a matrix B is called vertex-edge incidence matrix (or simply incidence matrix).

The end vertex incidence matrix of the Dutchwind mill graph  $D_3^2$  is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Definition 3.3**

Let  $A_e(G)$  denote the end vertex adjacency matrix of a Dutchwind mill graph  $D_3^m$ . The characteristic polynomial of G is defined to be  $\det(I - A_e(G))$  where I stands for the identity matrix of the same order of  $A_e(G)$ . The roots of the characteristic polynomial must be real and they are called eigen values of  $A_e(G)$  or e-eigen values of G. The sequence of e-eigen values of G is called the e-spectrum of G.

We make the following observations about the end vertex adjacency and end vertex incidence matrix about the dutchwind mill graph

- 1) The end vertex adjacency matrix  $A_e$  is symmetric.
- 2) The sum of the  $i^{th}$  row of  $A_e$  is equal to the  $i^{th}$  column of  $A_e$  is equal to the degree of  $v_i$
- 3) The entries along the leading diagonal of  $A_e$  are zero.

- 4) If one row or one column of the end vertex adjacency matrix  $A_e$  of the dutchwind mill graph has all its entries zero then the corresponding vertex is a middle vertex.
- 5) If sum of  $i^{th}$  row or  $i^{th}$  column of  $A_e$  is one then  $v_i$  is a middle end vertex
- 6) Since each edge has exactly two end vertices each column of the end vertex incidence matrix  $B_e$  contains exactly two ones
- 7) The sum of  $i^{th}$  row of  $B_e$  is equal to the degree of  $v_i$
- 8)  $A_e$  has both positive negative real eigen values

Harray proved that  $A(L(G)) = B^T B - 2I_q$  where A is the adjacency matrix of the line graph G and B is the incidence matrix if G.

Here we prove that the same result for Dutchwind mill graph with end vertex adjacency and end vertex incidence matrix.

**Theorem:** For any dutchwind mill graph  $G=(V,X)$  with the end vertex incidence matrix  $B_e$ , the end vertex adjacency matrix  $A_e$  can be obtained from the following relation  $A_e(G) = B_e B_e^T - D$ . Where  $B_e^T$  is the transpose of the end vertex incidence matrix and D is the diagonal matrix whose diagonal entries are the degree of the vertices.

**4. Bounds on e – Chromatic Number**

Sampathkumar introduced colouring numbers of a semigraph as follows Definition 4.1: An e-colouring of semigraph G is a colouring of vertices so that no two vertices of an edge are coloured the same. The e-chromatic number  $\chi_e$  is the minimum number of colours needed to colour n-vertices of edges of semigraph G. The following theorem presents bound involving the e-chromatic number and rank of  $A_e$  of semigraphs.

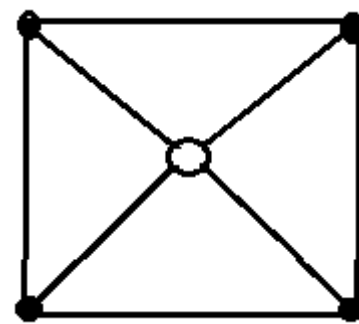
**Theorem:**

If  $A_e$  be the end vertex adjacency matrix of a semigraph G and  $\chi_e$  is the e-chromatic number of G then  $\chi_e \leq$  Rank of the matrix  $A_e$ .

**Lemma 4.3:**

If  $A_e$  be the end vertex adjacency matrix of a complete semigraph G with no middle end vertex and  $\chi_e$  is e-chromatic number of G then  $\chi_e =$  Rank of the matrix  $A_e$ .

**Example 4.4:** Consider a complete semigraph H with no middle end vertex in fig:3



$D_3^2$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\chi_e = \text{Rank of } A_e(H) = 4$$

The following theorem gives bounds for the e-chromatic number in terms of e - eigen values.

**Theorem:**

If G is a dutchwind mill graph with largerst e-eigen value  $\lambda$  and smallest e-eigen value  $\mu$ , then  $(1 - \lambda)\mu \leq \chi_e(G) \leq 1 + \lambda$ .

The eigen values of  $A_e$  are 3,-1,-1 and -1.

$$\lambda=3, \mu=-1 \text{ and } \chi_e=4$$

$$(1-\lambda)\mu=2 \text{ and } 1+\lambda=4$$

$$(1-\lambda)\mu \leq \chi_e \leq 1+\lambda.$$

Thus the inequalities in theorem is true for H.

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