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On e-Chromatic Number of Adjacency Matrix of a Dutchwindmill Graph D_3^m and Central Graphs

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Abstract: ***In this paper we have found out the end vertex adjacency matrix and end vertex incidence matrix on Dutchwind mill graphs. We have also found out the rank of the end vertex adjacency matrix and finalized the bounds for the e-chromatic number of some Dutchwind mill graph.***

Keywords: end vertex adjacency matrix, end vertex incidence matrix, dutchwind mill graph, e-spectrum, e-chromatic number, e-eigen value

AMS Subject classification Code: 05C69

1. Introduction

Vivin J. Vernold , M. Venkatachalam and Ali M. M. Akbar gave a note on achromatic coloring of central graphs. Let G be a finite undirected graph with no loops and multiple edges. Mostly the concept of matrices is very easy and convenient and it is widely used in various fields.

The basic definitions are given in section 2 . The end vertex adjacency matrices and end vertex incidency matrices are given in section 3.We find the e-chromatic number For the Dutchwind mill graph D_3^m , we find the duplication and triplication and finally we find the end vertex adjacency matrix and thenWe find the e-chromatic number in section 4**.**

2. Basic Definitions

The basic definitions which are used in section 4 are

Definition 2.1 A Dutchwindmill graph G= (V,X) on the non empty set of vertices and the set of edges X consists of n edges satisfying the conditions.

1) Two edges have atmost one vertex in common
2) Two edges $(u_1, u_2, u_3, \ldots, u_n)$ is the same as $(u_n, u_{n-1}, u_{n-2}, \ldots, u_1)$

Definition 2.2 For the edge E= $(u_1, u_2, u_3, \ldots, u_n)$, u_1 andu_2 are said to be the end vertices of the edge E while $u_2, u_3, \ldots, u_{n-1}$ are said to be middle vertices of E.

Definition 2.3 Two vertices in a dutchwind mill graph are said to be adjacent if they belong to the same edge

Definition 2.4 Two edges are adjacent if they have a common vertex

Definition 2.5 Let G= (V,X) be a dutchwind mill graph. Two vertices are said to be the e-adjacent if they are the end vertices of an edge in G**.**

Definition 2.6 For the end vertex we define the various types of degrees as follows.

Degree: deg(v) is the number of edges having a as an end vertex

Edge degree: $deg_e(v)$ is the number of edges containing v

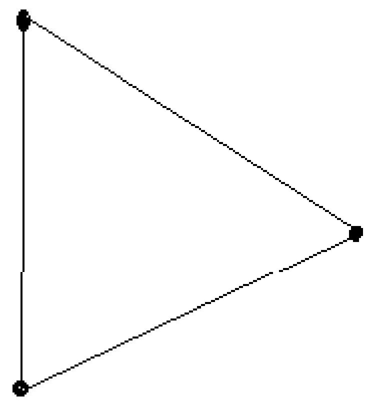
Adjacency degree:$deg_{ca}(v)$ is the number of vertices which are consecutive adjacent to v

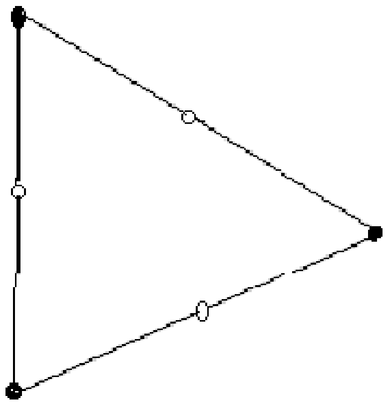
Definition 2.7 A Dutchwind mill Graph G is complete if any two vertices in G are adjacent.

3. End Vertex Adjacency Matrix and End Vertex Incidence Matrix

Definition 3.1 Let G = (V,X) be a Dutchwind mill graph with V={ v_1, v_2, v_3 } and X = { e_1, e_2, e_3 }. The end vertex adjacency matrix $A_e = (a_{ij})$ of Dutchwind mill graph with p points is the p x p matrix in which

(a_{ij}) = 1, if v_i is e-adjacent with v_j
(a_{ij}) = 0,otherwise.





$D_3^1\, C(D_3^1)$

The end vertex adjacency matrix of the Dutchwind mill graph D_3^2 is

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition 3.2

Let G bea graph with nvertices, eedges and noself-loops. Define an n by e matrix B= [bij],whose nrows correspond to the nvertices and the ecolumns correspond tothe eedges, asfollows :

(b_{ij})= 1, end vertex v_i is incident with e_j and

(b_{ij})= 0,otherwise.

Such a matrix B iscalled vertex-edge incidence matrix (or) simply incidence matrix.

The end vertex incidence matrix of the Dutchwind mill graph D_3^2 is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition 3.3

Let A_e (G) denote the end vertex adjacency matrix of a Dutchwind mill graph D_3^m. The characteristic polynomial of G is defined to be det(I - A_e (G)) where I stands for the identity matrix of the same order of A_e (G). The roots of the characteristic polynomial must be real and they are called eigen values of A_e (G) or e-eigen values of G. The sequence of e-eigen values of G is called the e-spectrum of G.

We make the following observations about the end vertex adjacency and end vertex incidence matrix about the dutchwind mill graph

1)The end vertex adjacency matrix A_e is symmetric.
2)The sum of the i^{th} row of A_e is equal to the i^{th} coloumn of A_e is equal to the degree of v_i
3)The entries along the leading diagonal of $A_e$are zero.
4)If one row or one coloumn of the end vertex adjacency matrix A_e of the dutchwind mill graph has all its entries zero then the corresponding vertex is a middle vertex.
5)If sum of i^{th} row or i^{th} coloumn of A_e is one then v_i is a middle end vertex
6)Since each edge has exactly two end vertices each coloumn of the end vertex incidence matrix B_e contains exactly two ones
7)The sum of i^{th} row of B_e is equal to the degree of v_i
8)A_e has both positive negative real eigen values

Harray proved that $A(L(G)) = B^TB-2I_q$ where A is the adjacency matrix of the line graph G and B is the incidence matrix if G.

Here we prove that the same result for Dutchwind mill graph with end vertex adjacency and end vertex incidence matrix.

Theorem: For any dutchwind mill graph G=(V,X) with the end vertex incidence matrix B_e , the end vertex adjacency matrix A_e can be obtained from the following relation A_e (G) = $B_eB_e^T$ – D. Where B_e^T is the transpose of the end vertex incidence matrix and D is the diagonal matrix whose diagonal entries are the degree of the vertices.

4. Bounds on e – Chromatic Number

Sampathkumar introduced colouring numbers of a semigraph as follows Definition 4.1: An e-colouring of semigraph G is a colouring of vertices so that no two end vertices of an edge are coloured the same. The e-chroamtic number χ_e is the minimum number of colours needed to colour n-vertices of edges of semigraph G. The following theorem presents bound involoving the e-chromatic number and rank of A_e of semigraphs.

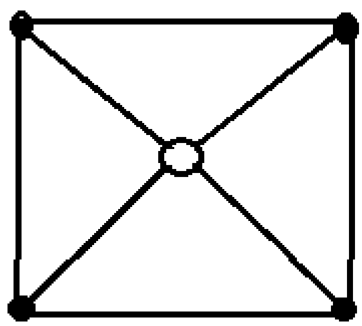
Theorem:

If A_e be the end vertex adjacency matrix of a semigraph G and χ_e is the e-chromatic number of G then $\chi_e \leq$ Rank of the matrix A_e.

Lemma 4.3:

If A_e be the end vertex adjacency matrix of a complete semigraph G with no middle end vertex and χ_e is e-chromatic number of G then χ_e = Rank of the matrix A_e.

Example 4.4: Consider a complete semigraph H with no middle end vertex in fig:3



D_3^2

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

χ_e= Rank of A_e (H)=4

The following theorem gives bounds for the e-chromatic number in terms of e - eigen values**.**

Theorem:
If G is a dutchwind mill graph with largerst e-eigen value λ and smallest e-eigen value μ, then $(1-\lambda)\mu \leq \chi_e(G) \leq 1+\lambda$.

The eigen values of A_e are 3,-1,-1 and -1.
$\lambda=3, \mu=-1$ and $\chi_e=4$
$(1-\lambda)\mu=2$ and $1+\lambda.=4$
$(1-\lambda)\mu \leq \chi_e \leq 1+\lambda$.
Thus the inequalities in theorem is true for H.

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