

Stochastic Model for Magazine Retailer Problem

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Abstract: In this paper magazine retailer model extensions are developed. These two extensions allow modelling of the situations where there are certain percentage of customers who are willing to wait for the next regular order. The first model assumes that it is possible to identify customers who are willing to wait for the next regular order at the beginning of planning period, while the second model assumes back order option at time when stock out occurs.

Keywords: Inventory, Magazine Retailer Model, Stochastic Model.

1. Introduction

There are a number of logistics systems and cases in which orders may be placed only at certain times. Ordering dates of these systems are known in advance, because they are imposed by some limitations of the logistics system itself, or are due to the specific characteristics of the products kept on stock. Objective of inventory control, in these systems, is to determine the number of orders that will be sufficient to cover demand between two consecutive orders. Ordering too little, in these systems and cases, means that a demand cannot be satisfied, and additional costs will occur, such as additional order costs, lost sales costs, reputation and business credibility loss, future contracts and sales loss, etc. Ordering too much, means the occurrence of excessive inventory levels, that generate costs whose values depends on the nature of the product. Modeling of described inventory control problems can be successfully done using magazine retailer model and its extensions.

2. Literature Review

Magazine retailer problem has been presenting the literature for over 100 years [1], and magazine retailer model that solves magazine retailer problem is one of the most famous models in the operating management and operational research, in general [2]. This model, even if introduced in the middle of the last century [3], still attracts the attention of a large number of authors in recent years [4]. Widespread of magazine retailer problems and magazine retailer model popularity have resulted in numerous articles dealing exclusively with taxonomy of magazine retailer model extensions, or articles exclusively dealing only with literature reviews in this filed [4,5,6,7]. Applicability of magazine retailer model is manifold. Inventories in the food and the clothing industry are often modeled using magazine retailer model [8]. Magazine retailer model is also used in modelling and solving problems in the production capacity management, and in service industries, such as airline and hotel reservations [9,10]. As the lifetime of the product continues to shorten, the importance of the magazine retailer models grows, so many magazine retailer model extensions are developed in the last few years [4,11,12].

3. The Magazine Retailer Model

In this chapter we discuss the problem of controlling the inventory of a single item with stochastic demands over a

single period. This problem is also known as the Magazine retailer Problem because the prototype is the problem faced by a magazine retailer trying to decide how many newspapers to stock on a newsstand before observing demand. The magazine retailer faced both overage and underage costs if he orders too much or if he orders too little. The Magazine retailer Problems is therefore the problem of deciding the size of a single order that must be placed before observing demand when there are overage and underage costs. The problem is particularly important for items with significant demand uncertainty and large overage and underage costs.

Let D denote the one period random demand, with mean $\mu = E[D]$ and variance $\sigma^2 = V[D]$. Let c be the unit cost, $p > c$ the selling price and $s < c$ the salvage value. If Q units are ordered, then $\min(Q;D)$ units are sold and $(Q-D)^+ = \max(Q-D; 0)$ units are salvaged. The profit is given by $p \min(Q;D) + s(Q - D)^+ - cQ$. The expected profit is well defined and given by:

$$\pi(Q) = pE \min(Q;D) + sE(Q - D)^+ - cQ$$

Using the fact that $\min(Q;D) = D - (D - Q)^+$ we can write the expected profit as

$$\pi(Q) = (p-c)\mu - G(Q) \quad (1)$$

where

$$G(Q) = (c - s)E(Q - D)^+ + (p - c)E(D - Q)^+ \geq 0$$

Let $h = c - s$ and $b = p - c$. It is convenient to think of h as the per unit overage cost and of b as the per unit underage cost. Sometimes the underage cost is inflated to take into account the *ill-will cost* associated with unsatisfied demand.

Equation (1) allow us to view the problem of maximizing $\pi(Q)$ as that of minimizing the expected overage and underage cost $G(Q)$.

Let $G^{det}(Q) = h(\mu - Q)^+ + b(Q - \mu)^+$. This represents the cost when D is deterministic, i.e., $Pr(D = \mu) = 1$. Clearly $Q = \mu$ minimizes $G^{det}(Q)$ and $G^{det}(\mu) = 0$, so $\sigma^{det}(\mu) = (p - c)\mu$. Thus, the Magazine retailer Problem is only interesting when demand is random. Notice that the problem also becomes trivial when $s = c$ for in this case we can order an infinite amount, satisfy all demand, and then return all unsold items.

Let $g(x) = hx^+ + bx^-$, then $G(Q)$ can be written as $G(Q) = E[g(Q-D)]$. Since g is convex and convexity is preserved by

linear transformations and by the expectation operator it follows that G is also convex. By Jensen's inequality $G(Q) \geq G^{det}(Q)$. As a result, $\pi(Q) \leq \pi^{det}(Q) \leq \pi^{det}(\mu) = (p-c)\mu$. Thus, the expected profit is lower than it would be in the case of deterministic demand. If the distribution of D is continuous, we can find an optimal solution by taking the derivative of G and setting it to zero. Since we can interchange the derivative and the expectation operators, it follows that $G'(Q) = hE\delta(Q-D) - bE\delta(D-Q)$ where $\delta(x) = 1$ if $x > 0$ and zero otherwise. Since $E\delta(Q-D) = Pr(Q-D > 0)$ and $E\delta(D-Q) = Pr(D-Q > 0)$, it follows that

$$G'(Q) = hPr(Q-D > 0) - bPr(D-Q > 0).$$

Setting the derivative to zero reveals that

$$F(Q) \equiv Pr(D \leq Q) = \frac{b}{b+h} = \frac{p-c}{p-s} \equiv \beta. \quad (2)$$

If F is continuous then there is at least one Q satisfying Equation (2). We can select the smallest such solution by letting

$$Q^* = \inf\{Q \geq 0 : F(Q) \geq \beta\}. \quad (3)$$

It is clear that Q^* , selected this way, is increasing in β and therefore it is increasing in b and decreasing in h .

If F is strictly increasing then F has an inverse and there is a unique optimal solution given by

$$Q^* = F^{-1}(\beta). \quad (4)$$

In practice, D often takes values in the set of natural numbers $N = \{0, 1, \dots\}$. In this case it is useful to work with the forward difference $\Delta G(Q) = G(Q+1) - G(Q)$, $Q \in N$. By writing

$$E(D-Q)^+ = \sum_{j=Q}^{\infty} Pr(D > j),$$

It is easy to see that

$$\Delta G(Q) = h - (h+b)Pr(D > Q)$$

is non-decreasing in Q , and that $\lim_{Q \rightarrow \infty} \Delta G(Q) = h > 0$, so an optimal solution is given by $Q = \min\{Q \in N : \Delta G(Q) \geq 0\}$, or equivalently,

$$Q^* = \min\{Q \in N : F(Q) \geq \beta\}, \quad (5)$$

The origin of the Magazine retailer model appears to date back to the 1888 paper by Edgeworth [2] who used the Central Limit Theorem to determine the amount of cash to keep at a bank to satisfy random cash withdrawals from depositors with high probability. The fractile solution (2) appeared in 1951 in the classical paper by Arrow, Harris and Marchak [1].

The magazine retailer solution can be interpreted as providing the smallest supply quantity that guarantees that all demand will be satisfied with probability at least $100\beta\%$. Thus, the profit maximizing solution results in a service level $100\beta\%$. In practice, managers often specify β and then Q accordingly. This service level should not be confused with the fraction of demand served from stock, or fill-rate, which is defined as $\alpha = E \min(D; Q) / ED$.

4. Normal Demand Distribution

An important special case arises when the distribution D is normal. The normal assumption is justified by the Central Limit Theorem when the demand comes from many different independent or weakly dependent customers. If D is normal, then we can write $D = \mu + \sigma Z$ where Z is a

standard normal random variable. Let $\phi(z) = Pr(Z \leq z)$ be the cumulative distribution function of the standard normal random variable. Although the function Φ is not available in closed form, it is available in tables and also in electronic spreadsheets. Let $z_\beta = \Phi^{-1}(\beta)$. In Microsoft Excel, for example, the command `NORMSINV(0.75)` returns 0.6745 so $z_{.75} = 0.6745$. Since $Pr(D \leq \mu + z\beta\sigma) = \Phi(z_\beta) = \beta$, it follows that

$$Q^* = \mu + z_\beta\sigma \quad (6)$$

Satisfies Equation (4), so Equation (6) gives the optimal solution for the case of normal demand. The quantity z_β is known as the safety factor and $Q^* - \mu = z_\beta\sigma$ is known as the safety stock. It can be shown that

$$E(D - Q^*)^+ = \sigma E(Z - z_\beta)^+ = \sigma[\phi(z_\beta) - (1 - \beta)z_\beta]$$

where ϕ is the density of the standard normal random variable. As a consequence,

$$\begin{aligned} G(Q^*) &= hE(Q^* - D)^+ + bE(D - Q^*)^+ \\ &= h(Q^* - \mu) + (h+b)E(D - Q^*)^+ \\ &= hz_\beta\sigma + (h+b)\sigma E(Z - z_\beta)^+ \\ &= hz_\beta\sigma + (h+b)\sigma[\phi(z_\beta) - (1 - \beta)z_\beta] \\ &= (h+b)\sigma\phi(z_\beta), \end{aligned}$$

So

$$\begin{aligned} \pi(Q^*) &= (p-c)\mu - (h+b)\sigma\phi(z_\beta) \\ &= (p-c)\mu - (p-s)\sigma\phi(z_\beta). \end{aligned}$$

In addition, since $E \min(D, Q^*) = ED - E(D - Q^*)^+$, we can divide by ED and write the fill-rate as

$$\alpha = 1 - cv[\phi(z_\beta) - (1 - \beta)z_\beta]$$

where $cv = \sigma/\mu$ is the coefficient of variation of demand. Since $\phi(z_\beta) - (1 - \beta)z_\beta \geq 0$ is decreasing in β , it follows that the α is increasing in β and decreasing in cv . Numerical results show that $\alpha \geq \beta$ for all reasonable values of cv , including $cv \leq 1/3$, which is about the highest cv value for which the normal model is appropriate. Notice, for example, that $\alpha = 97\%$ when $\beta = 75\%$ and $cv = 0.2$, while $\alpha = 99.1\%$ when $\beta = 90\%$ and $cv = 0.2$.

Example Normal Demand: Suppose that D is normal with mean $\mu = 100$ and standard deviation $\sigma = 20$. If $c = 5$, $h = 1$ and $b = 3$, then $\beta = 0.75$ and $Q^* = 100 + 0.6745 * 20 = 113.49$. Notice that the order is for 13.49 units (safety stock) more than the mean. Typing `NORMDIST(.6574, 0, 1, 0)` in Microsoft Excel, returns $\phi(.6745) = 0.3178$ so $G(113.49) = 4 * 20 * .3178 = 25.42$, and $\pi(113.49) = 274.58$, with $\alpha = 97\%$.

5. Illustrative examples

Magazine Retailer Problem

A Bookstore must decide how many magazines it should order. Each magazine costs \$3 and is sold for \$5. Any unsold magazines at the end of the day incur a wastage cost of \$1 per unit. The daily demand for magazines is assumed to follow a Binomial distribution with 40 successes and a probability of .5.

The object is to determine how many magazines to order to maximize total profit, while keeping the shortage and wastage costs at a minimum. Specifically, the wastage and shortage costs must not exceed \$5 and \$27, respectively, at least 90% of the time.

Problem Data	
Selling Price	\$5.00
Wastage Cost	\$1.00
Order Cost	\$3.00
Demand	17
Max Shortage Cost	\$27.00
Max Surplus Cost	\$5.00
Number ordered	40
Wastage Cost Constraint	23.00 #N/A
Shortage Cost Constraint	115.00 #N/A
Total Profit	\$80.00

Uncertain Variab
The uncertain variable in cell simulates the magazine dem
Binomial distribution is a c
distribution which means that
distribution returns only integ

Decis
The decision variable
of magazine to order.
integer.

Objective Function
The objective in cell D24 is maximizing the total
profit function.

Model Building Tip: Chance Constraints
We could express the chance constraint in cell
D20 by placing =PsiPercentile(D12*(D15-
D14),0.9) in cell D20, and using the Ribbon to
define a normal constraint D20 <= D16.
However, while PsiPercentile and other PSI
Statistics functions are acceptable for
simulation optimization, the transformation for
robust optimization requires that we express
the chance property directly in the constraint
itself, not in a formula.

Chance Constraints
This example includes two chance
constraints. These constraints must be
met at least 90% of the time.
 $VaR_{.9}(D20) \leq D16$ Ensures that the
surplus cost does not exceed \$5.
 $VaR_{.9}(D22) \leq D16$ Ensures that the
shortage cost does not exceed \$27.

6. Conclusion

Derived extensions of magazine retailer model with backorder option are models that allow achieving the same service level with lower inventory levels in the case when there is a percentage of customers willing to wait for the next regular order. In situations where there are no buyers who are willing to wait for the next order, than these situations can be successfully modelled with developed extensions, where developed extensions become a classic magazine retailer model. In situations where there are buyers who are willing to wait for the next regular order, magazine retailer model with backorder option, that is triggered after the stock out, is more efficient than the classic magazine retailer model, and the magazine retailer model with backorder option, that is triggered at the beginning of the planning period, is more efficient than both models. In situations where all customers are willing to wait for the next regular order, both extensions of the magazine retailer model developed in this paper, will give the same result. Developed extensions could be even more efficient if they could enable modelling the percentage of customers who are not willing to wait for the next regular order, but are willing to wait for emergency order.

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