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A Simulation Study on M/M/1 Queuing Model in a Medical Centre

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Abstract: ***In this paper, we discuss the application of Simulation in M/M/1 queueing model in a medical centre. The simulation table provides a systematic method of studying the future behaviour of the system over time. The main aim of this paper is to find the waiting time of a patient in the queue, the waiting time of a patient in the medical centre, the idle time of a doctor, the queue length and also to compare with the mathematical solution.***

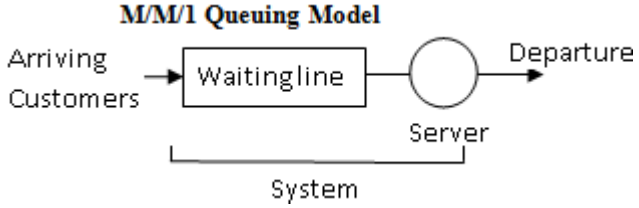
Keywords: Inter - arrival Time, Service time, Waiting time, M/M/1 queueing model, Simulation, Queue length

1. Introduction

Waiting lines or Queues are familiar phenomena, which we observe quite frequently in our daily life. The study of a queueing problem would take into account: 1.The arrival distribution 2.The service distribution 3.The queue discipline 4.The number of service points 5.System capacity 6.Calling source [2].

Units arrive, at regular or irregular intervals of time, at a given point called the Service Centre. For example, ships arriving at a loading station, customers entering a department store, persons arriving at a cinema hall, letters arriving at a typist's desk, etc. All these units are called entries or arrivals of customers [4]. If the service station is free, the arriving customer will be served immediately; if not, the arriving customer will wait in line until the service is provided. Once service has been completed, the customer leaves the system. Whenever we have customers coming to a service facility in such a way that the customers have to wait, we have a queueing problem [8].

Queueing theory was introduced by A.K.Erlang in 1909. He published various articles about the study of jamming in telephone traffic [1]. The main aim of all investigations in queueing theory is to get the main performance measures of the system, which are the probabilistic properties like distribution function, density function, mean, variance of random variables. Number of customers in the system, number of waiting customers, consumption of the servers, response time of a customer, waiting time of a customer, Idle time of the server, busy time of a server can also be computed [5].



Simulation is a representation of reality through the use of a model or other device which will meet in the same manner as reality under a given set of conditions. Simulation has also been defined as the utilization of a system model that has the designed features of reality in order to produce the essence of actual operation [6]. Before introducing several simulations of queueing systems, it is necessary to understand the concepts of the system state, events and simulation clock. The state of the system is the number of units in the system and the status of the server, busy or idle. In a single channel queueing system, there are only two possible events that can change the state of the system. They are the arrival event and the departure event [7]. The arrival event occurs when a unit enters the system. The unit will find the server either idle or busy, therefore, either the unit starts service immediately, or the queue forms for the service. If the server is busy, the unit enters the queue. If the server is at rest and the queue is free, the unit begins service. It is not possible for the server to be idle while the queue is busy. After the completion of a service, the server either will become idle or will remain busy with the next unit [3].

Simulation clock times for arrivals and departures are computed in a simulation table customized for each problem. In simulation, events usually take place at random times, the randomness imitating uncertainty in real life. Random number also can be generated in Simulation. In a single – channel queueing simulation, service times are generated from the distributions by using the random variables [10].

At a medical centre, there is one doctor who is to treat the patients. The patients arrival at the medical centre is a random phenomenon and the time between the arrivals varies from 6 a.m. to 12 p.m., and the service time varies from four minutes to thirty two minutes. The frequency distributions are given below.

Table 1: Arrival distribution

S. No	*Time*	*No of Patients*	*Probability*
1	6 - 8	4	0.008
2	8 - 10	37	0.076
3	10 - 12	76	0.157
4	12 - 14	91	0.188
5	14 - 16	48	0.099
6	16 - 18	63	0.130
7	18 - 20	107	0.221
8	20 - 22	54	0.111
9	22 - 24	5	0.010
Total	-	485	1

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Table 2: Service distribution

S.No	Time	No of Patients	Probability
1	0 - 4	5	0.077
2	4 - 8	7	0.108
3	8 - 12	15	0.231
4	12 - 16	13	0.202
5	16 - 20	12	0.185
6	20 - 24	7	0.117
7	24 - 28	3	0.050
8	28 - 32	2	0.030
Total	-	65	1

Table 3: Tag number table for arrival distribution

S. No	*Time*	*No of Patients*	*Probability*	*Cumulative Probability*	*Tag numbers*
1	6 - 8	4	0.008	0.008	0 - 7
2	8 - 10	37	0.076	0.084	8 - 83
3	10 - 12	76	0.157	0.241	84 – 240
4	12 - 14	91	0.188	0.429	241 – 428
5	14 - 16	48	0.099	0.528	429 – 527
6	16 - 18	63	0.130	0.658	528 – 657
7	18 - 20	107	0.221	0.879	658 – 878
8	20 - 22	54	0.111	0.990	879 – 989
9	22 - 24	5	0.010	1	990 - 1000
Total	-	485	1	-	-

Table 4: Tag number table for service distribution

S. No	*Time*	*No of Patients*	*Probability*	*Cumulative Probability*	*Tag numbers*
1	0 - 4	5	0.077	0.077	0 – 76
2	4 - 8	7	0.108	0.185	77 - 184
3	8 - 12	15	0.231	0.416	185 – 415
4	12 - 16	13	0.202	0.618	416 – 617
5	16 - 20	12	0.185	0.803	618 – 802
6	20 - 24	7	0.117	0.920	803 – 919
7	24 - 28	3	0.050	0.970	920– 969
8	28 - 32	2	0.030	1	970 – 1000
Total	-	65	1	-	-

Table 5: Simulation table

S.No	Random number	Inter arrival time (min)	Arrival time	Random number	Service begins	Service Time (min)	Service ends	Waiting time in Queue (min)	Waiting time in Medical Centre (min)	Idle Time of Doctor (min)	Queue length
1	483	-	6.00	837	6.00	6	6.06	0	6	0	-
2	517	5	6.05	705	6.06	5	6.11	1	6	0	1
3	063	2	6.07	068	6.11	1	6.12	4	5	0	1
4	229	4	6.11	129	6.12	2	6.14	1	3	0	1
5	797	7	6.18	597	6.18	4	6.22	0	4	4	-
6	562	6	6.24	467	6.24	4	6.28	0	4	2	-
7	066	2	6.26	543	6.28	4	6.32	2	6	0	1
8	914	8	6.34	041	6.34	1	6.35	0	1	2	-
9	511	5	6.39	513	6.39	4	6.43	0	4	4	-
10	134	3	6.42	999	6.43	8	6.51	1	9	0	1
11	657	6	6.48	840	6.51	6	6.57	3	9	0	1
12	602	6	6.54	812	6.57	6	7.03	3	9	0	1
13	511	5	6.59	153	7.03	2	7.05	4	6	0	1
14	504	5	7.04	364	7.05	3	7.08	1	4	0	1
15	132	3	7.07	126	7.08	2	7.10	1	3	0	1
16	946	8	7.15	540	7.15	4	7.19	0	4	5	-
17	579	6	7.21	978	7.21	8	7.29	0	8	2	-
18	262	4	7.25	001	7.29	1	7.30	4	5	0	1
19	787	7	7.32	494	7.32	4	7.36	0	4	2	-
20	331	4	7.36	442	7.36	4	7.40	0	4	0	-
21	600	6	7.42	135	7.42	2	7.44	0	2	2	-
22	318	4	7.46	232	7.46	3	7.49	0	3	2	-
23	151	3	7.49	457	7.49	4	7.53	0	4	0	-
24	647	6	7.55	546	7.55	4	7.59	0	4	2	-
25	890	8	8.03	248	8.03	3	8.06	0	3	4	-
26	746	7	8.10	504	8.10	4	8.14	0	4	4	-
27	995	9	8.19	292	8.19	3	8.22	0	3	5	-
28	636	6	8.25	618	8.25	5	8.30	0	5	3	-
29	589	6	8.31	220	8.31	3	8.34	0	3	1	-
30	830	7	8.38	471	8.38	4	8.42	0	4	4	-
31	444	5	8.43	627	8.43	5	8.48	0	5	1	-
32	642	6	8.49	823	8.49	6	8.55	0	6	1	-
33	590	6	8.55	527	8.55	4	8.59	0	4	0	-
34	031	2	8.57	182	8.59	3	9.02	2	5	0	1
35	590	6	9.03	638	9.03	5	9.08	0	5	1	-

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36	304	4	9.07	147	9.08	2	9.10	1	3	0	1
37	169	3	9.10	320	9.10	3	9.13	0	3	0	-
38	573	6	9.16	822	9.16	6	9.22	0	6	3	-
39	876	7	9.23	633	9.23	5	9.28	0	5	1	-
40	216	3	9.26	548	9.28	4	9.32	2	6	0	1
41	369	4	9.30	344	9.32	3	9.35	2	5	0	1
42	607	6	9.36	461	9.36	4	9.40	0	4	1	-
43	822	7	9.43	319	9.43	3	9.46	0	3	3	-
44	918	8	9.51	246	9.51	3	9.54	0	3	5	-
45	445	5	9.56	081	9.56	2	9.58	0	2	2	-
46	658	7	10.03	195	10.03	3	10.06	0	3	5	-
47	904	8	10.11	330	10.11	3	10.14	0	3	5	-
48	686	7	10.18	649	10.18	5	10.23	0	5	4	-
49	833	7	10.25	942	10.25	7	10.32	0	7	2	-
50	490	5	10.30	091	10.32	2	10.34	2	4	0	1
51	557	6	10.36	754	10.36	5	10.41	0	5	2	-
52	626	6	10.42	737	10.42	5	10.47	0	5	1	-
53	723	7	10.49	234	10.49	3	10.52	0	3	2	-
54	065	2	10.51	238	10.52	3	10.55	1	4	0	1
55	469	5	10.56	222	10.56	3	10.59	0	3	1	-
56	088	3	10.59	618	10.59	5	11.04	0	5	0	-
57	148	3	11.02	173	11.04	2	11.06	2	4	0	1
58	653	6	11.08	078	11.08	2	11.10	0	2	2	-
59	920	8	11.16	830	11.16	6	11.22	0	6	6	-
60	516	5	11.21	356	11.22	3	11.25	1	4	0	1
61	244	4	11.25	870	11.25	6	11.31	0	6	0	-
62	697	7	11.32	649	11.32	5	11.37	0	5	1	-
63	896	8	11.40	609	11.40	4	11.44	0	4	3	-
64	455	5	11.45	926	11.45	7	11.52	0	7	1	-
65	718	7	11.52	741	11.52	5	11.57	0	5	0	-
66	840	7	11.59	403	11.59	3	12.02	0	3	2	-
67	259	4	12.03	189	12.03	3	12.06	0	3	1	-
68	913	8	12.11	208	12.11	3	12.14	0	3	5	-
69	741	7	12.18	666	12.18	5	12.23	0	5	4	-
70	061	2	12.20	691	12.23	5	12.28	3	8	0	1
71	498	5	12.25	254	12.28	3	12.31	3	6	0	1
72	658	7	12.32	884	12.32	6	12.38	0	6	1	-
73	549	6	12.38	456	12.38	4	12.42	0	4	0	-
74	717	7	12.45	886	12.45	6	12.51	0	6	3	-
75	501	5	12.50	276	12.51	3	12.54	1	4	0	1
76	193	3	12.53	169	12.54	2	12.56	1	3	0	1
77	826	7	1.00	966	1.00	7	1.07	0	7	4	-
78	912	8	1.08	980	1.08	8	1.16	0	8	1	-
79	544	6	1.14	470	1.16	4	1.20	2	6	0	1
80	693	7	1.21	610	1.21	4	1.25	0	4	1	-
81	907	8	1.29	101	1.29	2	1.31	0	2	4	-
82	523	5	1.34	495	1.34	4	1.38	0	4	3	-
83	331	4	1.38	730	1.38	5	1.43	0	5	0	-
84	925	8	1.46	006	1.46	1	1.47	0	1	3	-
85	962	8	1.54	767	1.54	5	1.59	0	5	7	-
86	422	4	1.58	049	1.59	1	2.00	1	2	0	1
87	657	6	2.04	856	2.04	6	2.10	0	6	4	-
88	382	4	2.08	508	2.10	4	2.14	2	6	0	1
89	147	3	2.11	012	2.14	1	2.15	3	4	0	1
90	031	2	2.13	265	2.15	3	2.18	2	5	0	1
91	135	3	2.16	470	2.18	4	2.22	2	6	0	1
92	367	4	2.20	463	2.22	4	2.26	2	6	0	1
93	095	3	2.23	114	2.26	2	2.28	3	5	0	1
94	121	3	2.26	294	2.28	3	2.31	2	5	0	1
95	966	8	2.34	648	2.34	5	2.39	0	5	3	-
96	056	2	2.36	089	2.39	2	2.41	3	5	0	1
97	373	4	2.40	705	2.41	5	2.46	1	6	0	1
98	534	6	2.46	543	2.46	4	2.50	0	4	0	-
99	633	6	2.52	027	2.52	1	2.53	0	1	2	-
100	314	4	2.56	768	2.56	5	3.01	0	5	3	-
Total	-	*536*	-	-	-	*389*	-	*69*	*458*	*449*	*34*

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Simulation Calculation
Average Queue Length = 0.34
Average Waiting Time of a Patient in Queue = 0.69
Average Waiting Time of a Patient in a
Medical Centre = 4.58
Arrival rate = 0.186
Service rate = 0.257
Idle Time of a Doctor = 4.49

Analytical Calculation ($\lambda < \mu$)
Arrival rate = 0.01
Service rate = 0.25
Average Waiting Time of a patient in Queue= 0.132
Average Waiting Time of a Patient in a
Medical Centre = 4.13
Queue Length = 0.001

2. Numerical Study

Table 6: Calculation table

Categories	Simulation Calculation	Mathematical Calculation
Arrival rate	0.186	0.01
Service rate	0.257	0.25
Average Waiting Time of a Patient in Queue	0.69	0.132
Average Waiting Time of a Patient in a Medical Centre	4.58	4.13
Queue Length	0.34	0.001

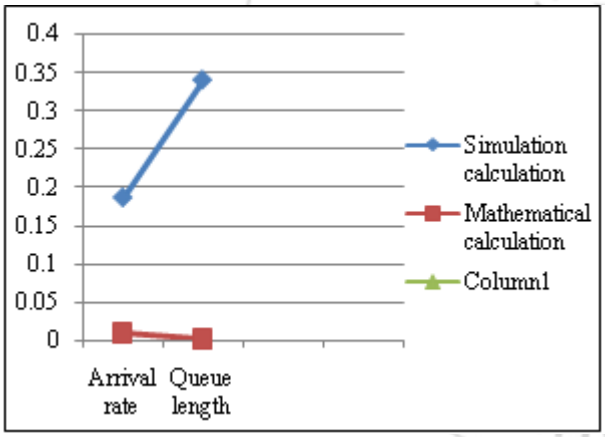

Figure 1: Comparison of Arrival rate with Queue length

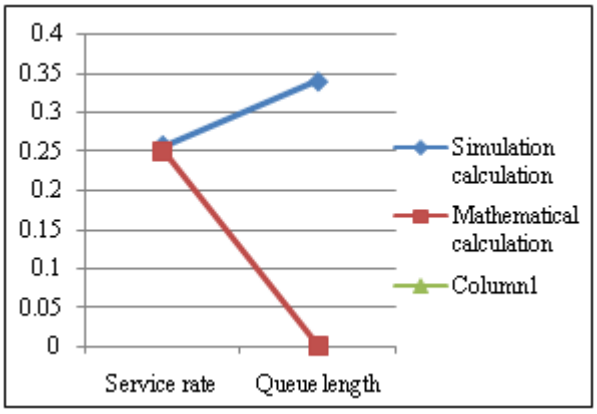

Figure 2: Comparison of Service rate with Queue length

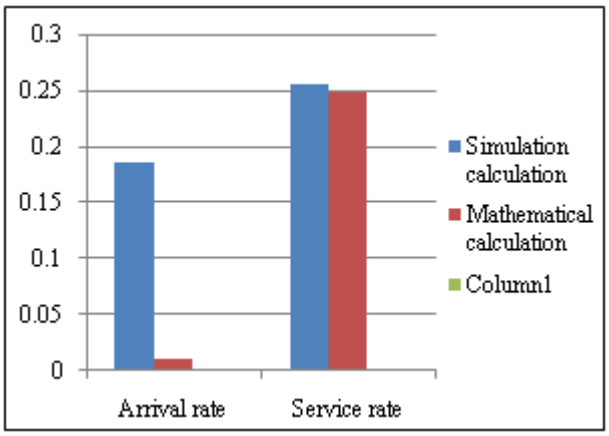

Figure 3: Comparison of Arrival rate with Service rate

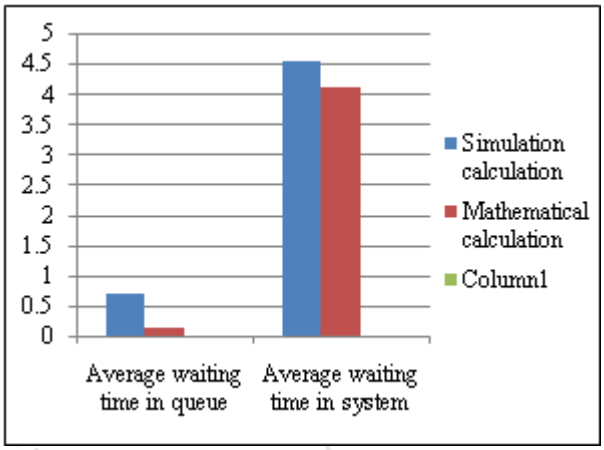

Figure 4: Comparison of waiting time of a patient in Queue and in medical centre

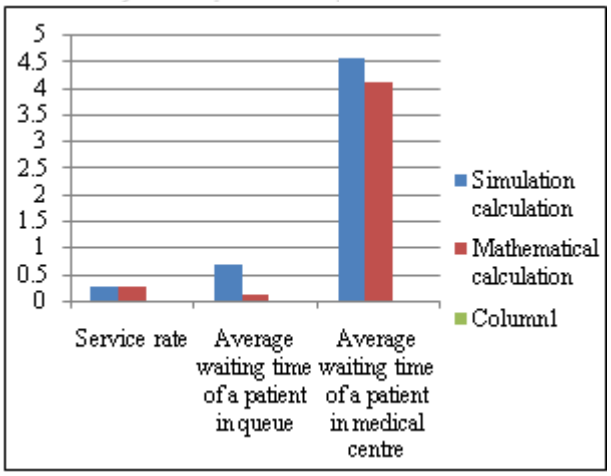

Figure 5: Comparison of Simulation Calculation with Mathematical Calculation

3. Conclusion

In this paper, we have studied the queueing system in simulation model of a medical centre assuming a single service station. In above discussion, we have calculated the queue length, average arrival time, average service time, idle time of a doctor, patient waiting time in the queue and in a medical centre. Also we have compared the calculation of simulation method and analytical method by using the bar diagram. This has given the feasibility of the system. The main purpose of this study is to develop an efficient

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procedure and to reduce the waiting time of the patients in a Medical Centre in future.

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