On Nano $(1,2)^*$ Generalized Pre Closed Sets in Nano Bitopological Spaces

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Abstract: The purpose of this paper is to define and study a new class of sets called Nano $(1, 2)^*$ generalized pre closed sets in Nano bitopological spaces. Basic properties are analyzed.

Keywords: Nano $(1, 2)^*$ generalized pre-closed sets, Nano $(1, 2)^*$ pre-closure, Nano $(1, 2)^*$ pre-interior

1. Introduction

In 1970, Levine \textsuperscript{9} introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In \textsuperscript{10} Maki et al introduced the concepts of gclosed sets in an analogous manner. The notion of Nano topology was introduced by Lellis-Thivagar \textsuperscript{5} which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. K. Bhuvaneswari et al.\textsuperscript{1} introduced the concept of Nano Generalized Pre Closed sets in Nano topological spaces. Kelly J.C., \textsuperscript{4} introduced the concept of Bitopological spaces. In this paper, we have introduced a new class of sets on Nano bitopological spaces called Nano$(1, 2)^*$ Generalized Pre closed sets and the relation of these set and investigate the some its relevant properties.

2. Preliminaries

Definition: 2.1\textsuperscript{11} A subset A of a topological space (X, $\tau$) is called a pre open set if $A \subseteq \text{Int}(\text{Cl}(A))$. The complement of a pre-open set of a space X is called pre closed set in X.

Definition: 2.2\textsuperscript{3} A pre-closure of a subset A of X is the intersection of all pre closed sets that contains A and it is denoted by pcl(A).

Definition: 2.3\textsuperscript{3} The union of all pre-open subsets of X contained in A is called pre-interior of A and it is denoted by pInt(A).

Definition: 2.4\textsuperscript{9} A subset A of $(X, \tau)$ is called a generalized closed set (briefly g-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.5\textsuperscript{10} A subset A of $(X, \tau)$ is called a generalized pre closed set (briefly gp-closed) if pcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.6\textsuperscript{6} Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X) = U \{R(x): R(x) \subseteq X, x \in U\}$ where $R(x)$ denotes the equivalence class determined by x$\in U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X) = U \{R(x) \subseteq X, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$

Definition: 2.7\textsuperscript{6} Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) $U$ and $\Phi$ $\in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $\tau_R(X)$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Remark: 2.8\textsuperscript{6} If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition: 2.9\textsuperscript{6} If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then
(i) The Nano interior of the set $A$ is defined as the union of all Nano open subsets contained in $A$ and is denoted by $N\text{Int}(A)$. $N\text{Int}(A)$ is the largest Nano open subset of $A$.

(ii) The Nano closure of the set $A$ is defined as the intersection of all Nano closed sets containing $A$ and is denoted by $N\text{Cl}(A)$. $N\text{Cl}(A)$ is the smallest Nano closed set containing $A$.

**Definition: 2.10** [6] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then $A$ is said to be

(i) Nano semi open if $A \subseteq N\text{Int}(N\text{Int}(A))$

(ii) Nano semi closed if $N\text{Int} \subseteq N\text{Cl}(A) \subseteq A$

(iii) Nano pre open if $A \subseteq N\text{Int}(N\text{Cl}(A))$

(iv) Nano open if $A \subseteq N\text{Int}(N\text{Cl}(A))$.

$NSO(U, X)$, $NSF(U, X)$, $NPO(U, X)$ and $\tau_R(X)$ respectively denote the families of all Nano semi open, Nano semi closed, Nano pre-open and Nano open subsets of $U$.

**Definition: 2.11** [1] A subset $A$ of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng-closed) if $N\text{Cl}(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is Nano open in $(U, \tau_R(X))$.

**Definition: 2.12** [1] A subset $A$ of $(U, \tau_R(X))$ is called Nano generalized pre closed set (briefly Ngp-closed) if $N\text{Cl}(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is Nano open in $(U, \tau_R(X))$.

**Definition: 2.13** [7] A subset $A$ of a bitopological space $(X, \tau_{1,2})$ is called a $(1, 2)^*_{\text{pre}}$ open set if $A \subseteq \tau_{1,2}\text{Int}(\tau_{1,2}\text{Cl}(A))$. The complement of a $(1, 2)^*_{\text{pre}}$ open set of a space $X$ is called a $(1, 2)^*_{\text{pre}}$ closed set in $X$.

**Definition: 2.14** [8] A subset $A$ of $(X, \tau_{1,2})$ is called a $(1, 2)^*_{\text{gp}}$ closed set (briefly $\tau_{1,2}\text{gp}$-closed) if $\tau_{1,2}\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(1, 2)^*_{\text{open}}$ in $X$.

**Definition: 2.15** [2] Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_{R_{1,2}}(X) = \bigcup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_{R_k}(X) = \{U, \Phi, L_{\text{R}(X)}, U_{\text{R}(X)}, B_{\text{R}(X)}\}$ and $X \subseteq U$. Then $\tau_{R_k}(X)$ satisfies the following axioms:

(i) $U$ and $\Phi$ are in $\tau_{R_k}(X)$

(ii) The union of the elements of any sub-collection of $\tau_{R_k}(X)$ is in $\tau_{R_k}(X)$

(iii) The intersection of the elements of any finite sub-collection of $\tau_{R_k}(X)$ is in $\tau_{R_k}(X)$

Then $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano $(1, 2)^*_{\text{pre}}$ open sets in $U$. Elements of $[\tau_{R_{1,2}}(X)]^C$ are called Nano$(1, 2)^*_{\text{closed}}$ sets in $\tau_{R_{1,2}}(X)$.

**Definition: 2.16** [2] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to $X$ where $X \subseteq U$ and $A \subseteq U$, then

(i) The Nano $(1, 2)^*_{\text{pre}}$ closure of $A$ is defined as the intersection of all Nano $(1, 2)^*_{\text{pre}}$ closed sets containing $A$ and it is denoted by $N\tau_{1,2}\text{Cl}(A)$. $N\tau_{1,2}\text{Cl}(A)$ is the smallest Nano $(1, 2)^*_{\text{pre}}$ closed set containing $A$.

(ii) The Nano $(1, 2)^*_{\text{pre}}$ interior of $A$ is defined as the union of all Nano $(1, 2)^*_{\text{pre}}$ open subsets of $A$ contained in $A$ and it is denoted by $N\tau_{1,2}\text{Int}(A)$. $N\tau_{1,2}\text{Int}(A)$ is the largest Nano $(1, 2)^*_{\text{pre}}$ open subset of $A$.

3. Nano $(1, 2)^*_{\text{Generalized Pre Closed Sets}}$

Throughout this paper, $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to $X$ where $X \subseteq U$, $R$ is an equivalence relation on $U$. Then $U/R$ denotes the family of equivalence classes of $U$ by $R$. In this section, we define and study the forms of Nano $(1, 2)^*_{\text{Generalized Pre closed sets}}$.

**Definition: 3.1** If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to $X$ where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano $(1, 2)^*_{\text{pre}}$ closure of $A$ is defined as the intersection of all Nano $(1, 2)^*_{\text{pre}}$ closed sets containing $A$ and it is denoted by $N\tau_{1,2}\text{Cl}(A)$. $N\tau_{1,2}\text{Cl}(A)$ is the smallest Nano $(1, 2)^*_{\text{pre}}$ closed set containing $A$.

(ii) The Nano $(1, 2)^*_{\text{pre}}$ interior of $A$ is defined as the union of all Nano $(1, 2)^*_{\text{pre}}$ open subsets of $A$ contained in $A$ and it is denoted by $N\tau_{1,2}\text{Int}(A)$. $N\tau_{1,2}\text{Int}(A)$ is the largest Nano $(1, 2)^*_{\text{pre}}$ open subset of $A$.

**Definition: 3.2** Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then $A$ is said to be Nano$(1, 2)^*_{\text{pre}}$ open if $A \subseteq N\tau_{1,2}\text{Int}[N\tau_{1,2}\text{Cl}(A)]$. The complement of a Nano$(1, 2)^*_{\text{pre}}$ open set in $U$ is called Nano$(1, 2)^*_{\text{pre}}$ closed in $U$.

**Definition: 3.3** A subset $A$ of $(U, \tau_{R_{1,2}}(X))$ is called Nano$(1, 2)^*_{\text{generalized pre}}$ closed set (briefly $N\tau_{1,2}\text{gp}$-closed) if $N\tau_{1,2}\text{pcl}(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is Nano $(1, 2)^*_{\text{pre}}$ open in $(U, \tau_{R_{1,2}}(X))$.
Example: 3.4

Let U={a, b, c, d} with U/R1= \{\{a\}, \{d\}, \{b, c\}\} and X1= \{a, c\}

Then \( \mathcal{T}_{R_1} = \{U, \emptyset, \{a\}, \{d\}, \{b, c\}\} \), U/R2= \{\{a\}, \{c\}\}, \{b, d\}\} and X2= \{a, d\}. Then \( \mathcal{T}_{R_2} = \{U, \emptyset, \{a\}, \{c\}, \{d\}, \{b, d\}\} \).

Then \( \mathcal{T}_{R_{1,2}} (X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\} \) are open sets.

The Nano(1, 2)* closed sets = \{U, \emptyset, \{a\}, \{c\}, \{d\}, \{b, c\}, \{a, d\}, \{a, c\} \}. The Nano (1, 2)* pre closed sets = \{\Phi, U, \{a\}, \{c\}, \{d\}, \{b, c\}, \{a, d\}, \{a, c\}\} which are open sets.

\( \mathcal{T}_{R_{1,2}} (X) = \{U, \emptyset, \{a\}, \{c\}, \{d\}, \{b, c\}, \{a, d\}, \{a, c\}\} \), \( N_{\tau_1,2} (N_{\tau_1,2}(A)) = N_{\tau_1,2} (A) \).

Theorem: 3.5 Let A be a Nano (1, 2)* closed set in \( (U, \mathcal{T}_{R_{1,2}} (X)) \), then Nano (1, 2)* pre closed set.

Proof: Let A be a Nano (1, 2)* closed set. Then we have \( N_{\tau_1,2}(A) = A \). To prove that

\[ N_{\tau_1,2} (A) \subseteq A \] \implies \( A \) is a Nano (1, 2)* pre closed set.

Remark: 3.6 The converse of the above theorem (3.5) is not true. In the example 3.4, the sets \{b, c\}, \{a, c\} \} are Nano (1, 2)* pre closed sets but not Nano (1, 2)* closed sets.

Theorem: 3.7 If A is a Nano (1, 2)* closed set in \( (U, \mathcal{T}_{R_{1,2}} (X)) \), then Nano (1, 2)* pre generalized closed set.

Proof: Let A be a Nano (1, 2)* closed set of U and A \subseteq V is Nano (1, 2)* open in U. Since A is Nano (1, 2)* closed, \( N_{\tau_1,2}(A) = A \subseteq V \). That is \( N_{\tau_1,2}(A) \subseteq V \).

Remark: 3.8 The converse of the theorems (3.7) need not be true which is shown in the example 3.4.

Theorem: 3.9 If a subset A is a Nano (1, 2)* pre closed set, then Nano (1, 2)* general pre closed set. This is shown in the example 3.4.

Theorem: 3.10 Arbitrary union of two Nano (1, 2)* generalized pre closed sets in \( (U, \mathcal{T}_{R_{1,2}} (X)) \) is also a Nano (1, 2)* generalized pre closed set.
Hence B is also a Nano \((1, 2)^*\) generalized pre closed subset of \(\mathcal{T}_{K,12}(X)\).

4. Conclusion

In this paper, some of the properties of Nano \((1, 2)^*\) Generalized Pre closed sets are discussed. This shall be extended in the future Research with some applications.

References


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