On Nano (1,2)* Generalized Pre Closed Sets in Nano Bitopological Spaces

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Abstract: ***The purpose of this paper is to define and study a new class of sets called Nano (1, 2)* generalized pre closed sets in Nano bitopological spaces. Basic properties are analyzed.***

Keywords: Nano (1, 2)* generalized pre-closed sets, Nano (1, 2)* pre-closure, Nano (1, 2)* pre-interior

1. Introduction

In 1970, Levine [9] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In [10] Maki et al introduced the concepts of gp-closed sets in an analogous manner. The notion of Nano topology was introduced by LellisThivagar [5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. K.Bhuvaneswari et al[1] introduced the concept of Nano Generalized Pre Closed sets in Nano topological spaces. Kelly J.C., [4] introduced the concept of Bitopological spaces,. In this paper, we have introduced a new class of sets on Nano bitopological spaces called Nano(1, 2)* Generalized Pre closed sets and the relation of these set and investigate the some its relevant properties.

2. Preliminaries

Definition: 2.1[11] A subset A of a topological space (X, τ) is called a pre open set if A ⊆Int(Cl(A)). The complement of a pre-open set of a space X is called pre closed set in X.

Definition: 2.2 [3] A pre-closure of a subset A of X is the intersection of all pre closed sets that contains A and it is denoted by pcl(A).

Definition: 2.3[3] The union of all pre-open subsets of X contained in A is called pre-interior of A and it is denoted by pInt(A).

Definition: 2.4[9] A subset A of (X, τ) is called a generalized closed set (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition: 2.5[10] A subset A of (X, τ) is called a generalized pre closed set (briefly gp-closed) if pcl(A) ⊆ U whenever A⊆ U and U is open in X.

Definition: 2.6[6]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆U.

Then,

(i)The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.$L_R(X)=U\{R(x): R(x) \subseteq X, x \in U\}$ where R(x) denotes the equivalence class determined by xϵU.

(ii)The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi, x \in U\}$

(iii)The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not –X with respect to R and it is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$

Definition: 2.7 [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆U. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and Φϵ $\tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Remark: 2.8[6] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B = { U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition: 2.9 [6] If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆ U and if A⊆U, then (i)The Nano interior of the set A is defined as the union of all Nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest Nano open subset of A.

(ii)The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest Nano closed set containing A.

Definition: 2.10 [6] Let (U, τ_R(X)) be a Nano topological space and A ⊆ U. Then A is said to be

(i) Nano semi open if A⊆NCl[NInt(A)]
(ii) Nano semi closed if NInt [NCl (A)] ⊆A
(iii)Nano pre open if A⊆NInt [NCl (A)]
(iv)Nano**α** open if A⊆NInt[NCl(NInt(A))].

NSO(U, X), NSF(U, X), NPO(U, X) and τ_R^{α} (X) respectively denote the families of all Nano semi open, Nano semi closed, Nano pre-open and Nano**α** open subsets of U.

Definition: 2.11 [1] A subset A of (U, τ_R (X)) is called Nano generalized closed set (briefly Ng-closed) if Ncl(A)⊆ V whenever A⊆ V and V is Nano open in (U, τ_R(X)).

Definition: 2.12 [1] A subset A of (U, τ_R (X)) is called Nano generalized pre closed set (briefly Ngp-closed) if Npcl(A) ⊆V whenever A⊆ V and V is Nano open in (U, τ_R(X)).

Definition: 2.13[7] A subset A of a bitopological space (X, $\tau_{1,2}$) is called a (1, 2)*pre open set if A ⊆ $\tau_{1,2}$ Int ($\tau_{1,2}$ Cl(A)). The complement of a (1, 2)*pre open set of a space X is called (1, 2)* pre closed set in X.

Definition: 2.14[8] A subset A of (X, $\tau_{1,2}$) is called a (1, 2)*generalized pre closed set (briefly $\tau_{1,2}$ gp-closed) if $\tau_{1,2}$ pcl (A) ⊆ U whenever A⊆ U and U is (1, 2)* open in X.

Definition: 2.15[2] Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where τ_R(X) = {U, Φ, L_R(X), U_R(X), B_R(X)} and $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and Φ ϵ $\tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

Then $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano (1, 2)* open sets in U. Elements of $[\tau_{R1,2}(X)]^C$ are called Nano(1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Definition: 2.16[2] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano (1, 2)* closure of A is defined as the intersection of all Nano (1, 2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest Nano (1, 2)* closed set containing A.

(ii) The Nano (1, 2)* interior of A is defined as the union of all Nano (1, 2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ is the largest Nano (1, 2)* open subset of A.

3. Nano (1, 2)*Generalized Pre Closed Sets

Throughout this paper, (U, $\tau_{R_{1,2}}(X)$) is a Nano bitopological space with respect to X where X ⊆ U, R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R. In this section, we define and study the forms of Nano (1, 2)*Generalized Pre closed sets.

Definition: 3.1 If (U, $\tau_{R_{1,2}}(X)$) is a Nano bitopological space with respect to X where X ⊆ U and if A ⊆U, then

(i)The Nano (1, 2)*pre closure of A is defined as the intersection of all Nano (1, 2)* pre closed sets containing A and it is denoted by N $\tau_{1,2}$ pcl(A). N $\tau_{1,2}$pcl (A) is the smallest Nano (1, 2)*pre closed set containing A.

(ii)The Nano (1, 2)* pre interior of A is defined as the union of all Nano (1, 2)*pre-open subsets of A contained in A and it is denoted by N $\tau_{1,2}$ pInt(A). N $\tau_{1,2}$ pInt(A) is the largest Nano(1, 2)*pre open subset of A.

Definition: 3.2 Let (U, $\tau_{R_{1,2}}(X)$) be a Nano bitopological space and A ⊆ U. Then A is said to be Nano(1, 2)*pre open if A⊆N $\tau_{1,2}$Int [N $\tau_{1,2}$Cl (A)]. The complement of a Nano(1, 2)*pre open set in U is called Nano(1, 2)*pre closed in U.

Definition: 3.3 A subset A of (U, $\tau_{R_{1,2}}(X)$) is called Nano(1, 2)*generalized pre-closed set (briefly N $\tau_{1,2}$gp-closed) if N $\tau_{1,2}$pcl(A) ⊆V whenever A⊆ V and V is Nano (1, 2)*open in (U, $\tau_{R_{1,2}}(X)$).

Example: 3.4

Let U={a, b, c, d} with U/R_1={{a}, {d}, {b, c}} and X_1= {a, c}

Then τ_{R_1}={U, Φ, {a}{b, c}{a, b, c}}, U/R_2={{a}, {c}, {b, d}}, and X_2={a, d}.Then τ_{R_2}={U, Φ, {a}, {a, b, d}{b, d}}.

Then $\tau_{R_{1,2}}(X)$={ U, Φ, {a}, {b, c}, {b, d}, {a, b, c}, {a, b, d}}which are open sets.

The Nano(1, 2)* closed sets = {U, Φ, {b, c, d}, {c}, {d}, {a, d}, {a, c}}. The Nano (1, 2)* pre closed sets = {Φ, U, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {a, c}, {c, d}, { {b, c, d}, {a, c, d}}. The Nano (1, 2)*generalized pre closed sets are {U, Φ, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {c, d}, {a, c, d}, {b, c,.d}}

Theorem: 3.5 Let A be a Nano (1, 2)* closed set in (U, $\tau_{R_{1,2}}(X)$), then Nano (1, 2)*pre closed set

Proof: Let A be a Nano (1, 2)* closed set. Then we have $N\tau_{1,2}cl$ (A) = A. To prove that

$N\tau_{1,2}$ Cl ($N\tau_{1,2}$ Int (A)) ⊆A which implies that A is a Nano (1, 2)*pre closed set.
$N\tau_{1,2}$Cl ($N\tau_{1,2}$ Int(A)) = $N\tau_{1,2}$Int (A)⊆A. Hence A is a Nano (1, 2)* pre closed set.

Remark: 3.6 The converse of the above theorem (3.5) is not true. In the example 3.4, the sets {a, b}, {a, c, d} are Nano (1, 2)* pre closed sets but not Nano (1, 2)* closed sets.

Theorem: 3.7 If A is a Nano (1, 2)* closed set in (U, $\tau_{R_{1,2}}(X)$), then Nano (1, 2)* generalised pre closed set.

Proof: Let A be a Nano (1, 2)* closed set of U and A ⊆ V, V is Nano (1, 2)* open in U. Since A is Nano (1, 2)* closed, $N\tau_{1,2}cl$ (A) =A ⊆ V. That is $N\tau_{1,2}cl$(A) ⊆ V. Also, $N\tau_{1,2}pcl$(A) ⊆
$N\tau_{1,2}cl$(A) ⊆ V, where V is Nano (1, 2)* open in U. Therefore, A is a Nano (1, 2)* generalised pre closed set.

Remark: 3.8 The converse of the theorems (3.7) need not be true which is shown in the example 3.4

Theorem: 3.9 If a subset A is a Nano (1, 2)* pre closed set, then Nano (1, 2)* generalized pre closed set.This is shown in the example 3.4

Theorem: 3.10 Arbitrary union of two Nano (1, 2)* generalized pre closed sets in (U, $\tau_{R_{1,2}}(X)$) is also a Nano (1, 2)* generalized pre closed set in (U, $\tau_{R_{1,2}}(X)$).

Proof: Let A and B be two Nano (1, 2)* generalized pre closed sets in (U, $\tau_{R_{1,2}}(X)$). Let V be a Nano (1, 2)* open set in U such that A⊆V and B⊆V. Then we have AUB⊆V. A and B are Nano (1, 2) * generalized pre closed sets in (U, $\tau_{R_{1,2}}(X)$), $N\tau_{1,2}$ pcl(A)⊆V and $N\tau_{1,2}$ pcl (B)⊆V. Now $N\tau_{1,2}$pcl (AUB) = $N\tau_{1,2}$pcl (A) U$N\tau_{1,2}$pcl (B) ⊆V. Thus we have $N\tau_{1,2}$pcl (AUB)⊆V whenever AUB⊆V, V is Nano (1, 2)* open set in (U, $\tau_{R_{1,2}}(X)$).This implies AUB is a Nano(1, 2)* generalized pre closed set in (U, $\tau_{R_{1,2}}(X)$).

Remark: **3.11** The Intersection of two Nano (1, 2)* generalized pre closed sets in (U, $\tau_{R_{1,2}}(X)$) is also a Nano (1, 2)* generalized pre closed set in (U, $\tau_{R_{1,2}}(X)$) as seen from the following example.

Example: 3.12 Let U={a, b, c, d} with U/R_1={{a}, {d}, {b, c}} and X_1= {a, c}

Then τ_{R_1}={U, Φ, {a}{b, c}{a, b, c}}, U/R_2={{a}, {c}, {b, d}}, andX_2={a, d}.Then τ_{R_2}={U, Φ, {a}, {a, b, d}{b, d}}. Then $\tau_{R_{1,2}}(X)$={ U, Φ, {a}, {b, c}, {b, d}, {a, b, c}, {a, b, d}} which are open sets.

The Nano (1, 2)* generalized pre closed sets are {U, Φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, c, d}, {b, c, d}}. Here {a, d} ∩ {c, d} ={d} which is again a Nano (1, 2)* generalized pre closed set.

Theorem: 3.13 If A is Nano (1, 2)*generalized pre closed sets in (U, $\tau_{R_{1,2}}(X)$), then it is Nano (1, 2)* pre generalized closed set.

Proof: Let A⊆V and V is Nano (1, 2)* open in $\tau_{R_{1,2}}(X)$.

Then $N\tau_{1,2}$pcl(A) ⊆V as V is Nano (1, 2)* generalized pre closed sets. Since every Nano (1, 2)* open set is Nano (1, 2)* pre open, $N\tau_{1,2}$pcl(A) ⊆V where V is Nano (1, 2)* pre open set. This implies A is Nano (1, 2)* pre generalized closed set. The converse of the above theorem (3.13) is not true as seen in the example 3.3.

Theorem: 3.14 Let A be a Nano (1, 2)* generalized pre closed subset of (U, $\tau_{R_{1,2}}(X)$). If A ⊆B ⊆$N\tau_{1,2}$pcl(A), then B is also a Nano (1, 2)* generalized pre closed subset of (U, $\tau_{R_{1,2}}(X)$).

Proof: Let V be a Nano (1, 2)* open set of a Nano (1, 2)* generalized pre closed subset of $\tau_{R_{1,2}}(X)$ such that B ⊆ V. As A ⊆B, we have A ⊆ V. As A is a Nano (1, 2)* generalized pre closed set, $N\tau_{1,2}$pcl(A) ⊆ V. Given B⊆$N\tau_{1,2}$pcl(A), we have $N\tau_{1,2}$pcl(B)⊆$N\tau_{1,2}$pcl(A). As $N\tau_{1,2}$ pcl(B) ⊆$N\tau_{1,2}$pcl(A) and $N\tau_{1,2}$pcl(A)⊆ V, we have $N\tau_{1,2}$ pcl (B) ⊆ V whenever B⊆V and V is Nano (1, 2)* open.

Hence B is also a Nano (1, 2)* generalized pre closed subset of $\tau_{R_{1,2}}(X)$.

4. Conclusion

In this paper, some of the properties of Nano (1, 2)* Generalized Pre closed sets are discussed. This shall be extended in the future Research with some applications.

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