International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2014): 5.611

Multiple Off-Grid Hybrid Block Simpson's Methods for Solution of Stiff Ordinary Differential Equations

Althemai, J. M.[1], Skwame, Y[2], Donald, J. Z.[3]

[1]Department of mathematics and statistics, Federal polytechnic, Mubi, Nigeria

[2,3]Department of mathematics, Adamawa State University, Mubi, Nigeria

Abstract: ***This paper is concerned with the construction of two-step hybrid block Simpson's methods with two and three off-grid points for the solutions of stiff systems of ordinary differential equations (ODEs). This is achieved by transforming a k-step multi-step method into continuous form and evaluating at various grid points to obtain the discrete schemes. The discrete schemes are applied each as a block for simultaneous integration. Each block matrix equation is A-stable and of order [5,5,5,6]^T and [6,6,6,6,6]^T respectively. These orders are achieved by the aid of Maple13 software program. The performance of the methods is demonstrated on some numerical experiments. The results revealed that the hybrid block Simpson's method with two-off grid points was more efficient than that hybrid block method with three off-grid points on mildly stiff problems.***

Keywords: hybrid method, off-step point, blocks method, first order system, multi-step method

1. Introduction

A considerable literature exists for the conventional *k-step* linear multi-step methods for the solution of ordinary differential equations (ODE's) of the form

$$y' = f(x,y), \quad y(a) = y_0, \quad x \in [a,b] \qquad (1)$$

Where y satisfies a given set of initial condition (Ibijola, *et al*, 2011), and we assume that the function f also satisfies the Lipschitz condition which guarantees existence, uniqueness and continuous differentiable solution, [6]. For the discrete solution of (1) linear multi-step methods has being studied by [7],[8], and continuous solutions of (1) [9]and[12],[13]. One important advantage of the continuous over the discrete approach is the ability to provide discrete schemes for simultaneous integration. These discrete schemes can as well be reformulated as general linear methods (GLM) [2]. The block methods are self-starting and can directly be applied to both initial and boundary value problems [3] and [17]. Block methods for solving ordinary differential equations have initially been proposed by[10] who advanced their use only as a means of obtaining starting values for predictor-corrector algorithms. Several authors [3],[4],[5],[11],[13,[15],[16],[17] among others, have modified it to be more efficient as a computational procedure for the integration of IVPs throughout the range of integration rather than just as a starting method for method for multistep methods [1].

In this paper we present a two-step hybrid block Simpson's method with two and three off-grid points. By using [12],[14] approach, the derived schemes will be applied in block form in other to achieve its order and error constants; the region of absolute stability, and the results of absolute errors.

2. Derivation of the Method

Consider the collocation methods defined for the step $|x_n, x_{n+1}|$ by

$$y(x) = \sum_{j=0}^{t-1} \alpha(x) y_{n+1} + h \sum_{j=0}^{m-1} \beta_j(x) f(x_j, y(\bar{x}_j)) \quad (2)$$

Where t denotes the number of interpolation points x_{n+j}, $j = 0, \cdots, t-1$, and m denotes the number of distinct collocation points $\bar{x}_j \in [x_n, x_{n+k}]$, $j = 0,1,\cdots,m-1$ the points $\bar{x}_j$ are chosen from the step x_{n+j} as well as one or more off-step points.

The following assumptions are made;
1) Although the step size can be variable, for simplicity in our presentation of the analysis in this paper, we assume it is constant $h = x_{n+1} - x_n$, $N = \frac{b-a}{h}$ with the steps given by $\{x_n / x_n = a + nh, n = 0,1,\cdots,N\}$,
2) That (1) has a unique solution and the coefficients $\alpha_j(x), \beta_j(x)$ in (2) can be represented by polynomial of the form

$$\alpha_j(x) = \sum_{j=0}^{t+m-1} \alpha_{j,i+1} x^i, \ j\varepsilon(0,1,2,\cdots,t-1) \qquad (3)$$

$$h\beta_j(x) = h \sum_{j=0}^{t+m-1} \beta_{j,i+1} x^i, \quad j\varepsilon(0,1,2,\cdots,m-1) \quad (4)$$

With constant coefficients $\alpha_{j,i+1}, h\beta_{j,i+1}$ and collocation conditions

$$\bar{y}(x_{n+j}) = y_{n+j}, \qquad j\varepsilon\{0,1,\cdots,t-1\} \qquad (5)$$

$$\bar{y}'(\bar{x}_j) = f(\bar{x}_j, \bar{y}(\bar{x}_j)), \qquad j\varepsilon\{0,1,\cdots m-1\} \qquad (6)$$

With these assumptions we obtained an MC polynomial in the form

$$y(x)=\sum_{j=0}^{t+m-1}\alpha_j x^i,\quad \alpha_j=\sum_{j=0}^{t-1}C_{i+1,j+1}+\sum_{j=0}^{m-1}C_{i+1,j+1}f_{n+j} \tag{7}$$

And also we get D Matrix as follows:

$$\underline{D}=\begin{bmatrix} 1 & x_n & x_n^2 & \cdots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+2}^2 & \cdots & x_n^{t+m-1} \\ \vdots & & & & \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & \cdots & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0 & \cdots & (t+m-1)\bar{x}_0^{t+M-2} \\ 0 & 1 & 2\bar{x}_0 & \cdots & (t+m-1)\bar{x}_1^{t+M-2} \\ \vdots & & & & \\ 0 & 1 & 2\bar{x}_{m-1} & \cdots & (t+m-1)\bar{x}_{m-1}^{t+M-2} \end{bmatrix} \tag{8}$$

The parameters required for equation (8) to obtain two step block hybrid Simpson's methods (BHSM2) are $k=2, t=1, m=k+3;$

$$\bar{x}_0=\left\{x_n,\bar{x}_{n+\frac{1}{2}},x_{n+1},x_{n+\frac{3}{2}},x_{n+2}\right\}$$

The matrix (8) becomes;

$$D=\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 \\ 0 & 1 & 2x_{n+\frac{3}{2}} & 3x_{n+\frac{3}{2}}^2 & 4x_{n+\frac{3}{2}}^3 & 5x_{n+\frac{3}{2}}^4 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 \end{pmatrix} \tag{9}$$

Using the maple software program and evaluating (9) at the grid-points, $x=\left\{x_{n+\frac{1}{2}},x_{n+1},x_{n+\frac{3}{2}},x_{n+2}\right\}$, We get four discrete schemes. Hence, the hybrid block methods are as follows

$$y_{n+\frac{1}{2}}=y_n+\tfrac{h}{1440}[251f_n+646f_{n+\frac{1}{2}}-264f_{n+1}+106f_{n+\frac{3}{2}}-19f_{n+2}]$$

$$y_{n+1}=y_n+\tfrac{h}{180}[29f_n+124f_{n+\frac{1}{2}}+24f_{n+1}+4f_{n+\frac{3}{2}}-f_{n+2}]$$

$$y_{n+\frac{3}{2}}=y_n+\tfrac{h}{160}[27f_n+102f_{n+\frac{1}{2}}+72f_{n+1}+42f_{n+\frac{3}{2}}-3f_{n+2}]$$

$$y_{n+2}=y_n+\tfrac{h}{45}[7f_n+32f_{n+\frac{1}{2}}+12f_{n+1}+32f_{n+\frac{3}{2}}+7f_{n+2}] \tag{10}$$

The parameters required for equation (8) to obtain two-step block hybrid Simpson's method with three off- grid points (BHSM3) are $k=2,\quad t=1,\quad m=k+4$ where,

$$\bar{x}_0=\left\{x_n,x_{n+\frac{1}{2}},x_{n+1},x_{n+\frac{5}{4}},x_{n+\frac{3}{4}},x_{n+2}\right\}$$

The matrix (8) becomes;

$$D=\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 & 6x_{n+\frac{1}{2}}^5 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 \\ 0 & 1 & 2x_{n+\frac{5}{4}} & 3x_{n+\frac{5}{4}}^2 & 4x_{n+\frac{5}{4}}^3 & 5x_{n+\frac{5}{4}}^4 & 6x_{n+\frac{5}{4}}^5 \\ 0 & 1 & 2x_{n+\frac{3}{2}} & 3x_{n+\frac{3}{2}}^2 & 4x_{n+\frac{3}{2}}^3 & 5x_{n+\frac{3}{2}}^4 & 6x_{n+\frac{3}{2}}^5 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 \end{pmatrix} \tag{11}$$

By using the maple software program and evaluating (11) at the grid- points $x=\left\{x_{n+\frac{1}{2}},x_{n+1},x_{n+\frac{5}{4}},x_{n+\frac{3}{2}},x_{n+2}\right\}$ we get the five discrete schemes as follows.

Therefore the hybrid block methods are

$$y_{n+\frac{1}{2}}=y_n+\tfrac{h}{720}[112f_n+413f_{n+\frac{1}{2}}-537f_{n+1}+576f_{n+\frac{5}{4}}-217f_{n+\frac{3}{2}}+13f_{n+2}]$$

$$y_{n+1}=y_n+\tfrac{h}{540}[81f_n+412f_{n+\frac{1}{2}}-108f_{n+1}+256f_{n+\frac{5}{4}}-108f_{n+\frac{3}{2}}+7f_{n+2}]$$

$$y_{n+\frac{5}{4}}=y_n+\tfrac{5h}{9216}[277f_n+1400f_{n+\frac{1}{2}}-150f_{n+1}+1152f_{n+\frac{5}{4}}-400f_{n+\frac{3}{2}}+25f_{n+2}]$$

$$y_{n+\frac{3}{2}}=y_n+\tfrac{h}{80}[12f_n+61f_{n+\frac{1}{2}}-9f_{n+1}+64f_{n+\frac{5}{4}}-9f_{n+\frac{3}{4}}+f_{n+2}]$$

$$y_{n+2}=y_n+\tfrac{h}{45}[7f_n+32f_{n+\frac{1}{2}}+12f_{n+1}+32f_{n+\frac{3}{2}}+7f_{n+2}] \tag{12}$$

3. Stability of Block method

The equations (10) when put together formed the block as

$$\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}\begin{pmatrix} y_{n+\frac{1}{2}} \\ y_{n+1} \\ y_{y+\frac{3}{2}} \\ y_{n+2} \end{pmatrix}=\begin{pmatrix} 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&1 \end{pmatrix}\begin{pmatrix} y_{n+2} \\ y_{n+\frac{3}{2}} \\ y_{y+1} \\ y_n \end{pmatrix}+h\begin{pmatrix} \frac{323}{720} & -\frac{11}{60} & \frac{53}{720} & -\frac{19}{1440} \\ \frac{31}{45} & \frac{2}{15} & \frac{1}{45} & -\frac{1}{180} \\ \frac{51}{80} & \frac{9}{20} & \frac{21}{80} & -\frac{3}{160} \\ \frac{32}{45} & \frac{4}{15} & \frac{32}{45} & \frac{7}{45} \end{pmatrix}\begin{pmatrix} f_{n+\frac{1}{2}} \\ f_{n+1} \\ f_{y+\frac{3}{2}} \\ f_{n+2} \end{pmatrix}+h\begin{pmatrix} 0&0&0&\frac{251}{1440} \\ 0&0&0&\frac{29}{180} \\ 0&0&0&\frac{27}{45} \\ 0&0&0&\frac{7}{45} \end{pmatrix}\begin{pmatrix} f_{n+2} \\ f_{n+\frac{3}{2}} \\ f_{y+1} \\ f_n \end{pmatrix} \tag{13}$$

The characteristic polynomial of the hybrid bock method (7) and (13) is given as

$$\rho(R)=\det[RA^0-A^1], \quad where\ A^0=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \quad and\ A^1=\begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}$$

$$\rho(R)=\det\left[R\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}-\begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}\right]=\det\begin{pmatrix}R&0&0&-1\\0&R&0&-1\\0&0&R&-1\\0&0&0&R-1\end{pmatrix}=0$$

$=R(R(R(1)))\Rightarrow R_1=0, R_2=0, R_3=0, R_4=1$

Since $|R_j|\le 1, \quad j\varepsilon\{1,2,3,4\}$ hence the method as a block is zero stable on its own, the hybrid block method is also consistent as its order $p>1$. The equations (4) when put together formed the block as

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_{n+1}\\y_{n+\frac{5}{4}}\\y_{n+\frac{3}{2}}\\y_{n+2}\end{bmatrix}=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+2}\\y_{n+\frac{3}{2}}\\y_{n+1}\\y_{n+\frac{1}{2}}\\y_n\end{bmatrix}+h\begin{bmatrix}\frac{413}{720}&-\frac{537}{720}&\frac{4}{5}&-\frac{217}{720}&\frac{13}{720}\\\frac{103}{135}&-\frac{1}{5}&\frac{64}{135}&-\frac{1}{5}&\frac{7}{540}\\\frac{175}{1154}&-\frac{25}{1536}&\frac{1}{8}&-\frac{25}{576}&\frac{25}{9216}\\\frac{61}{80}&-\frac{9}{80}&\frac{4}{5}&-\frac{9}{80}&\frac{1}{80}\\\frac{32}{45}&\frac{4}{15}&0&\frac{32}{45}&\frac{7}{45}\end{bmatrix}$$

$$\begin{bmatrix}f_{n+\frac{1}{2}}\\f_{n+1}\\f_{n+\frac{5}{4}}\\f_{n+\frac{3}{2}}\\f_{n+2}\end{bmatrix}+h\begin{bmatrix}0&0&0&0&\frac{7}{45}\\0&0&0&0&\frac{3}{20}\\0&0&0&0&\frac{277}{9216}\\0&0&0&0&\frac{3}{20}\\0&0&0&0&\frac{7}{45}\end{bmatrix}\begin{bmatrix}f_{n+2}\\f_{n+\frac{3}{2}}\\f_{n+\frac{5}{4}}\\f_{n+1}\\f_n\end{bmatrix} \tag{14}$$

The characteristic of polynomial of the hybrid block method (7) and (14) is given as

$$\rho(R)=\det[RA^0-A^1], where\ A^0=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix} and\ A^1=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}$$

Hence,

$$\det\begin{bmatrix}R&0&0&0&-1\\0&R&0&0&-1\\0&0&R&0&-1\\0&0&0&R&-1\\0&0&0&0&R-1\end{bmatrix}=0$$

$$=R(R(R(R(R-1))))=0$$

Which implies

$R_1=0, R_2=0, R_3=0, R_4=0, R_5=1. Since\,|R_j|\le 1, j\varepsilon\{1,...5\}$,

the method is zero stable and consistent as its order $P>1$.

3.1 Convergence Analysis

Order and Error constants of the Hybrid Block Simpson's Methods

The hybrid block methods which are obtained in a block form with the help of a maple software have the following order and error constants for each block hybrid method.

Evaluating point order Error constant

Table 1: HBSM2

$y(x = x_{n+\frac{1}{2}})$	5	$\frac{3}{10240}$
$y(x = x_{n+1})$	5	$\frac{1}{5760}$
$y(x = x_{n+\frac{3}{2}})$	5	$\frac{3}{10240}$
$y(x = x_{n+2})$	6	$-\frac{1}{15120}$

The method HBSM2 is of order 5, but for $y(x = x_{n+2})$ is order 6, and has error constants

$$C_6 = [\tfrac{3}{10240}, \tfrac{1}{5760}, \tfrac{3}{10240}, -\tfrac{1}{15120}]^T$$

Evaluating point order Error constant

Table 2: HBSM3

$y(x = x_{n+\frac{1}{2}})$	6	$-\frac{781}{15482880}$
$y(x = x_{n+1})$	6	$-\frac{13}{322560}$
$y(x = x_{n+\frac{5}{4}})$	6	$-\frac{4075}{99090432}$
$y(x = x_{n+\frac{3}{2}})$	6	$-\frac{23}{573440}$
$y(x = x_{n+2})$	6	$-\frac{1}{15120}$

The method HBSM3 is of order 6 and has error constants

$$C_7 = [-\tfrac{781}{15482880}, -\tfrac{13}{322560}, -\tfrac{4075}{99090432}, -\tfrac{23}{573440}, -\tfrac{1}{15120}]^T$$

3.2 Region of Absolute Stability

Using the MATLAB package, we were able to plot the stability regions of the block method (see fig1 and fig2). This is done by reformulating the block method as general linear method to obtain the values of the matrices A, B, U and V. These matrices are substituted into the stability matrix and using MATLAB software, the absolute stability regions of the new methods are plotted as shown in fig(1) for hybrid block method with two off-grid points and in fig (2) for hybrid block method with three off-grid points.

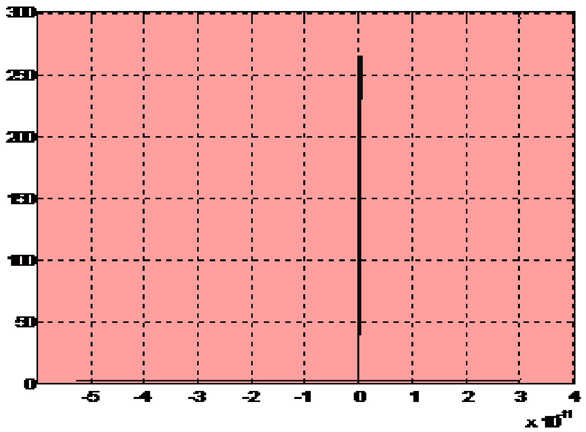

Figure 1: Stability region of the Block hybrid Simpson's method for *k=2* with two off-grid points (BHSM2)

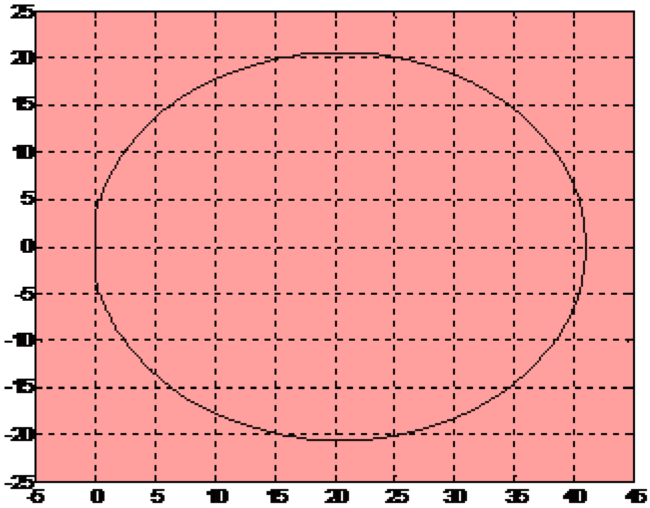

Figure 2: Stability region of the Block hybrid Simpson's method for *k=2* with three off-grid points (BHSM3)

4. Numerical Experiments

The newly constructed methods are demonstrated on some initial value problems and the results are displayed below

Example 1

$$y_1' = -8y_1 + 7y_2$$
$$y_2' = 42y_1 - 43y_2$$

where

$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 8$$

Exact Solution

$$y_1(x) = 2e^{-x} - e^{-50x}, \qquad y_2(x) = 2e^{-x} + 6e^{-50x}$$

with stiff ratio 5.0×10^1

Example 2

$$y_1' = 998y_1 + 1998y_2$$
$$y_2' = -999y_1 - 1999y_2$$

where

$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 1$$

and exact solution

$$y_1(x) = 4e^{-x} - 3e^{-1000x}, \qquad y_2(x) = -2e^{-x} + 3e^{-1000x}$$

with stiff ratio 1.0×10^3

Example 3

$$y_1' = -y_1 + 95y_2$$
$$y_2' = -y_1 - 97y_2$$

where

$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 1$$

and exact solution

$$y_1(x) = \tfrac{95}{47}e^{-2x} - \tfrac{48}{47}e^{-96x}, \quad y_2(x) = -\tfrac{48}{47}e^{-96x} - \tfrac{1}{47}e^{-2x}$$

with stiff ratio 4.8×10^1

Table 1: Comparing the Absolute Stability Errors for Example 1

	BHSM with two off-grid points		BHSM with three off-grid points	
X	Y_1	Y_2	Y_1	Y_2
0.1	1.284 E^{-2}	7.706 E^{-2}	1.358 E^{0}	3.201 E^{0}
0.2	4.574 E^{-2}	2.744 E^{-1}	9.020E^{-1}	7.357 E^{-1}
0.3	8.961 E^{-4}	5.377 E^{-3}	1.093 E^{0}	2.575 E^{0}
0.4	2.096 E^{-4}	1.258 E^{-2}	8.091 E^{-1}	5.315 E^{-1}
0.5	4.104 E^{-5}	2.462 E^{-4}	8.844 E^{-1}	2.097 E^{0}
0.6	9.595 E^{-5}	5.757 E^{-4}	7.222 E^{-1}	3.754 E^{0}
0.7	1.877 E^{-6}	1.127 E^{-5}	7.150 E^{-1}	1.708 E^{0}
0.8	4.393 E^{-6}	2.635 E^{-5}	6.418 E^{-1}	2.568 E^{0}
0.9	8.738 E^{-8}	5.146 E^{-7}	5.776 E^{-1}	1.391 E^{0}
1.0	2.792 E^{-5}	1.205 E^{-6}	5.682 E^{-1}	1.674 E^{-1}
1.1	4.596 E^{-10}	2.250 E^{-9}	4.664 E^{-1}	1.132 E^{0}
1.2	9.100 E^{-9}	5.460 E^{-8}	5.015 E^{-1}	1.009 E^{-1}

Table 2: Comparing the Absolute Stability Errors for Example 2

	BHSM with two off-grid points		BHSM with three off-grid points	
X	Y_1	Y_2	Y_1	Y_2
0.1	1.819 E^{0}	3.673 E^{0}	9.742 E^{-2}	3.522 E^{0}
0.2	1.623 E^{0}	3.190 E^{0}	1.460 E^{0}	4.735 E^{0}
0.3	1.482 E^{0}	2.963 E^{0}	4.743 E^{-2}	3.011 E^{0}
0.4	1.341 E^{0}	2.682 E^{0}	7.110 E^{-1}	1.970 E^{0}
0.5	1.213 E^{0}	2.426 E^{0}	2.309 E^{-2}	2.403 E^{0}
0.6	1.098 E^{0}	2.195 E^{0}	3.461 E^{-1}	2.541 E^{0}
0.7	9.932 E^{-1}	1.986 E^{0}	1.124 E^{-2}	1.998 E^{0}
0.8	8.987 E^{-1}	1.797 E^{0}	1.685 E^{-1}	1.629 E^{0}
0.9	8.131 E^{-1}	1.626 E^{0}	5.411 E^{-3}	1.621 E^{0}
1.0	7.358 E^{-1}	1.472 E^{0}	8.202 E^{-2}	1.554 E^{0}
1.1	6.657 E^{-1}	1.331 E^{0}	2.663 E^{-3}	1.334 E^{0}
1.2	6.024 E^{-1}	1.205 E^{0}	3.993 E^{-2}	1.165 E^{0}

Table 3: Comparing the Absolute Stability Errors for Example 3

	BHSM with two off-grid points		*BHSM with three off-grid points*	
X	Y_1	Y_2	Y_1	Y_2
0.1	1.475 E^{-1}	1.871 E^{0}	1.922 E^{0}	1.750 E^{0}
0.2	2.819 E^{-1}	1.656 E^{0}	1.729 E^{0}	1.461 E^{0}
0.3	3.723 E^{-1}	1.493 E^{0}	1.851 E^{0}	1.467 E^{0}
0.4	4.324 E^{-1}	1.350 E^{0}	1.761 E^{0}	1.319 E^{0}
0.5	4.695 E^{-1}	1.220 E^{0}	1.682 E^{0}	1.206 E^{0}
0.6	4.888 E^{-1}	1.104 E^{0}	1.586 E^{0}	1.090 E^{0}
0.7	4.947 E^{-1}	9.984 E^{-1}	1.488 E^{0}	9.879 E^{-1}
0.8	4.906 E^{-1}	9.030 E^{-1}	1.389 E^{0}	8.943 E^{-1}
0.9	4.790 E^{-1}	8.167 E^{-1}	1.292 E^{0}	8.096 E^{-1}
1.0	4.623 E^{-1}	7.386 E^{-1}	1.198 E^{0}	7.329 E^{-1}
1.1	4.418 E^{-1}	6.681 E^{-1}	1.108 E^{0}	6.634 E^{-1}
1.2	4.190 E^{-1}	6.043 E^{-1}	1.021 E^{0}	6.045 E^{-1}

5. Concluding Remarks

In this paper the newly constructed hybrid block methods were demonstrated on some stiff initial value problems (IVPs). From the absolute errors displayed on Tables (1-3) it can be seen that the BHSM2 has been shown to be more efficient and converges very well on example1 table (1) and performs fairly on the rest tables, while BHSM3 performs fair convergence throughout the tables. Therefore, the block hybrid Simpson's methods is efficient, accurate and convergent on mildly stiff problems.

6. Acknowledgements

We are grateful to the referees whose useful suggestions greatly improve the quality of this manuscript.

References

[1] Biala, T. A., Jator, S. N., Adeniyi, R. B., Ndukum, P.I (2015); Block Hybrid Simpson's Method with two Off-Grid Points for Stiff Systems. Intern. Journal of Science. Vol.20, No.1.pp.3-10.

[2] Butcher, J.C (1993); General Linear Methods for the Parallel Solution of Ordinary Differential Equations. Appl. Anal.2:97-111

[3] Donald, J.Z., Yusuf, S., Pius, T., and Paul, I.D., (2009); Construction of Two-Step Simpson's Multistep Methods as Parallel Integration for the Solution of Ordinary Differential Equations. Bagale J. Pure Appl. Sci.7:1-12

[4] Fatunla, S.O (1991); Block Methods for Second Order IVPs. Intern. J. Comput. Math. 41:55-63

[5] Ibijola, E.A., Skwame, Y., and Kumeleng, G (2011); Formation of Hybrid Block of Higher Step-Sizes, Through the Continuous Multi-Step Collocation. American Journal of Scientific and Industrial Research.

[6] Jain, M.K., Iyangar, S.R.K., Jain, R.K (2014); Numerical Methods for Scientific and Engineering Computation (Sixth Edition). New Age International Publishers, New Dethi.

[7] Lambert, J.D (1973): Hybrid Methods in Ordinary Differential Equations, John Wiley, Chi Chester, Network.

[8] Lambert, J.D (1996): Numerical Methods for Ordinary Differential Systems. John Willy and Sons, New York

[9] Lie, I. and Norsett (1989); Supper Convergence for the Multi-step Collocation Matt. Comp. 52.PP65-79

[10] Milne, W.E (1953); Numerical Solution of Differential Equations, John Wiley and Sons, New York, NY, USA,

[11] Ngwane, F.F., Jator, S.N (2012); Block Hybrid Second Derivatives Method for Stiff Systems, Intern. J. Pure Appl. Math. 4:543-559.

[12] Onumanyi, P., Awoyemi, D.O, Jator, S.N and Sirisena, U.W (1994), New Linear Multi-step Methods with Continuous Coefficients for First Order IVPs. Journal of the Nigerian Mathematical Society, 13, 37-51.

[13] Onumanyi, P., U.W. Sirisena and S.N Jator (1999); Continuous Finite Difference Approximations for Solving Differential Equations. Inter. J. Comp. Matt. 72, No. 1, Pp15-27

[14] Onumanyi, P., Sirisena, W.U and Chollom, J.P (2002); Some Theoretical Consideration of Continuous Linear Multi-Step Methods for U (V) =f(x,U), V=1,2,J.Pure and Appl. Sci.2 (2);1-5

[15] Roser, J.B (1967); A RungeKutta for all Seasons, SIAM Review. 9: 417-452

[16] Shampine, L.F., Watts, H.A (1969); Block Implicit One-Step Methods. Math. Comp. 23:731-740

[17] Skwame, Y. (2013); L-stable block hybrid Simpson's Methods for Numerical Solution of Initial Value

Problems in Stiff Ordinary Differential Equations (Ph.D. Thesis).

Author Profile

Althemai, J.M received the B.Sc. degree in Mathematics from the University of Maiduguri, Borno State Nigeria in 1983-1987, he is a lecturer in the Federal Polytechnic Mubi in the Department of mathematics and Statistics. He is a Postgraduate Student in the Department of Mathematics, Adamawa State University, Mubi-Nigeria.