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New Iterative Method for Solution of System of linear Differential Equations

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Abstract: ***In this study new iterative method is used for the solution of system of linear differential equations. The solution is obtained in series form and then obtained the exact solution. This shows the efficiency of the new iterative method. Because using new iterative method we can find its solution easily and more accurately.***

Keywords: Linear differential equations, New Iterative Method, scientific work place

1. Introduction

A differential equation is an equation involving dependent variable and independent variables and derivatives of one or more dependent variables with respect to one or more independent variables. An equation involving one or more derivatives of dependent variable with respect to single independent variable is called an ordinary differential equation.

An ODE is known as linear if:

1) The derivative of the dependent variable is one and also the power of the dependent variable is one.
2) The coefficient of the derivative and the coefficient of the dependent variable are constants or independent variables.

In this paper we have used new iterative method to solve linear differential equations which are already solved with differential transform method. N.Patil and A. khambayat [1] used differential transformation method to solve the equations.

2. The New Iterative Method

Consider the following general functional equation

$$y(\bar{x}) = f(\bar{x}) + N\big(y(\bar{x})\big), \tag{1}$$

Where N is nonlinear from a Banach space B→B, f is a known function and $\bar{x} = (x_1, x_2, \dots\dots, x_n)$. We are looking for a solution y of eq. (1) having the series form

$$y(\bar{x}) = \sum_{n=0}^{\infty} y_i(\bar{x}) \tag{2}$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{n=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\} \tag{3}$$

From Eqs. (2) And eq (3), Eq. (1) is equivalent to

$$\sum_{i=0}^{\infty} y_i = f + N(y_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{4}$$

We define the recurrence relation as

$$y_0 = f,$$
$$y_1 = N(y_0), \tag{5}$$
$$y_{m+1} = N(y_0 + \dots + y_m) - N(y_0 + \dots + y_{m-1})$$

Then

$$(y_1 + \dots + y_{m+1}) = N(y_0 + \dots + y_m),$$

And

$$\sum_{i=0}^{\infty} y_i = f + N\left(\sum_{i=0}^{\infty} y_i\right). \tag{6}$$

The L-term approximation solution of Eq. (1) is given by $y = y_0 + y_1 + \dots + y_{k-1}$

If N contracts i.e. $\| N(x) - N(y) \| \le L \| x - y \|, 0 < L < 1$, then

$$\| y_{m+1} \| = \| N(y_0 + \dots + ys_m \| - N \| y_0 + \dots + y_{m-1} \| \le L \| y_m \| \le L^m \| y_0 \|, m = 0, 1, 2 \dots\dots$$

And series $\sum_{i=0}^{\infty} y_i$ uniformly and absolutely converges to solution of equation (1). A unique solution, with respect to Banach fixed point theorem [12].

3. Numerical Examples

Example 1; Consider this system of simultaneous linear differential equations

$$\frac{dx}{dt} - 2x + 3y = 0,$$
$$\frac{dy}{dt} + 2x - y = 0,$$

The conditions are as $x(0) = 8,\ y(0) = 3$

The corresponding integral equations are as follows

$$x(t) = 8 + \int_0^t (2x - 3y)dt$$
$$y(t) = 3 + \int_0^t (-2x + y)dt$$

Setting $x_{10} = 8, y_{20} = 3$ and

$$N_1(x_{10}, y_{20}) = \int_0^t (2x - 3y)dt,\quad N_2(x_{10}, y_{20}) = \int_0^t (-2x + y)dt$$

Following the algorithm of NIM we obtain following approximations:

$$x_{11} = N_1(x_{10}, y_{20}) = 7t$$

$y_{21} = N_2(x_{10}, y_{20}) = -13t$

$x_{12} = \frac{53}{2}t^2$

$y_{22} = \frac{-27}{2}t^2$

$x_{13} = \frac{187}{6}t^3$

$y_{23} = \frac{187}{6}t^3$

The solution in the series form is

$x(t) = 8 + 7t + \frac{53}{2}t^2 + \frac{187}{6}t^3$

$y(t) = 3 - 13t - \frac{27}{2}t^2 - \frac{133}{6}t^3$

Example 2; Consider this system of simultaneous linear differential equations

$\frac{dx}{dt} - y = e^t$

$\frac{dy}{dt} + x = sint$

And the initial conditions are $x(0) = 1, y(0) = 0$

The corresponding integral equations are as follows

$x(t) = 8 + \int_0^t (2x - 3y)dt$

$y(t) = 3 + \int_0^t (-2x + y)dt$

Setting $x_{10} = e^t, y_{20} = 1 - cost$, and

$N_1(x_{10}, y_{20}) = \int_0^t y(t)dt, \quad N_2(x_{10}, y_{20}) = -\int_0^t x(t)dt$

Following the algorithm of NIM we obtain following approximations:

$x_{11} = N_1(x_{10}, y_{20}) = t - sint$

$y_{21} = N_2(x_{10}, y_{20}) = 1 - e^t$

$x_{12} = t - e^t + 1$

$y_{22} = 1 - \frac{1}{2}t^2 - cost$

$x_{13} = t - sint - \frac{1}{6}t^3$

$y_{23} = e^t - t - \frac{1}{2}t^2 - 1$

The solution in the series form is

$x(t) = 1 + t + \frac{1}{6}t^3 - \frac{1}{60}t^5$

$y(t) = -t - \frac{1}{12}t^4$

Example 3; Consider the system of nonhomogeneous differential equations as

$\frac{dx}{dt} = z(t) - cost,$

$\frac{dy}{dt} = z(t) - e^t,$

$\frac{dz}{dt} = x(t) - y(t),$

The conditions are as $x(0) = 1, \ y(0) = 0, z(0) = 2,$

The corresponding integral equations are as follows

$x(t) = 1 - sint + \int_0^t z(t)dt$

$y(t) = 1 - e^t + \int_0^t z(t)dt$

$z(t) = 2 + \int_0^t (x - y)dt$

Setting $x_{10} = 1 - sint, y_{20} = 1 - e^t, z_{30} = 2$, and

$N_1(x_{10}, y_{20}, z_{30}) = \int_0^t z(t)\,dt,$

$N_2(x_{10}, y_{20}, z_{30}) = \int_0^t z(t)\,dt,$

$N_3(x_{10}, y_{20}, z_{30}) = \int_0^t (x - y)dt$

Following the algorithm of NIM we obtain following approximations:

$x_{11} = N_1(x_{10}, y_{20}, z_{30}) = 2t$

$y_{21} = N_2(x_{10}, y_{20}, z_{30}) = 2t$

$z_{31} = N_3(x_{10}, y_{20}, z_{30}) = \cos t + e^t - 2$

$x_{12} = e^t - 2t + sin\ t - 1$

$y_{22} = e^t - 2t + sin\ t - 1$

$z_{32} = 0$

$x_{13} = 0$

$y_{23} = 0$

$z_{33} = 0$

The solution in the series form is

$x(t) = e^t$

$y(t) = sint$

$z(t) = cos\ t + e^t$

Which is the exact solution.

4. Conclusion

In this study, we have solved the linear differential equations using new iterative method. The solution is obtained in series form and exact solution of some of them. The obtained results show the efficiency of the method. Using NIM the calculation size is also reduced, which shows that the method is efficient and convenient.

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