N- Fourier Series Equations Involving Heat Polynomials

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Abstract: ***In this paper, we have considered the N-Fourier series equations involving heat polynomials of the first and second kind and solved the two sets of series equations.***

Keywords and Phrases: Integral equation, Series equation, Fourier series, Integral theorems, Heatpolynomials.

1. Introduction

If we review the literature then we observe that the existing solutions on series equations are derived only from dual to six Fourier series equations. No further generalizations are available till date. This tempted us to find the solution of n-Fourier series equations involving some special functions and in this paper we have obtained certain results. By considering the special values of n = 2,3,4,5,6 we shall be able to derive solutions of dual, triple,quadruple,5-tuple and 6-tuple Fourier series equations involving respective special functions.

2. N- Series Equations of the first kind

(i) N-series equations of the first kind

$$\sum_{m=0}^{\infty}\frac{A_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,t)=f_i(x),$$
$$a_{i-1}<x<a_i \qquad (1)$$

where, $i = 1,3,5,\ldots,n-1$ and a_0 =0.

$$\sum_{m=0}^{\infty}\frac{t^{-m}l^m A_m}{\left(\boldsymbol{\nu}+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\nu}(x,-t)=f_j(x),$$
$$a_{i-1}<x<a_i \qquad (2)$$

where, $j = 2,4,6,\ldots,n$.

Here n is taken as an even number. If n is odd then the equations will be

$$\sum_{m=0}^{\infty}\frac{B_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,t)=f_i(x),$$
$$a_{i-1}<x<a_i \qquad (3)$$

Where, $i = 1,3,5,\ldots,n$ and a_0 =0.

$$\sum_{m=0}^{\infty}\frac{t^{-m}l^m B_m}{\left(\boldsymbol{\nu}+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\nu}(x,-t)=f_j(x),$$
$$a_{j-1}<x<a_j \qquad (4)$$

Where, $j = 2,4,6,\ldots,n-1$.

(ii) N-series equations of the second kind

$$\sum_{m=0}^{\infty}\frac{t^{-m}l^m D_m}{\left(\boldsymbol{\nu}+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,t)=g_i(x),$$
$$a_{i-1}<x<a_i \qquad (5)$$

where, $i = 1,3,5,\ldots,n-1$ and $a_0 = o$

$$\sum_{m=0}^{\infty}\frac{D_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\nu}(x,-t)=g_j(x),$$
$$a_{j-1}<x<a_j \qquad (6)$$

where, $j = 2,4,6,\ldots,n$.

Here also n is taken as an even number. If n is odd then the equations will be

$$\sum_{m=0}^{\infty}\frac{t^{-m}l^m E_m}{\left(\boldsymbol{\nu}+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,t)=g_i(x),$$
$$a_{i-1}<x<a_i \qquad (7)$$

where, $i = 1,3,5,\ldots,n$ and a_0 =0.

$$\sum_{m=0}^{\infty}\frac{D_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\nu}(x,-t)=g_j(x),$$
$$a_{j-1}<x<a_j \qquad (8)$$

where, $j = 2,4,6,\ldots,n-1$.

Also $c>0$,l is an arbitrary non- negative integer.$f_i(x), g_i(x)$, where i = 1,3,5,....,n-1 and $f_j(x), g_j(x)$ where $j = 2,4,6,\ldots,n$ are prescribed functions.A_m, B_m, D_m andE_m areunknown co- efficients.In generalmin$(\alpha,\beta,\upsilon,\delta,\lambda,\mu,\sigma,\rho)>1$.Here we solve only equations (1),(2)of first kind and equations (5),(6) of the first kind and equations (7),(8) of the second kind will follow easily.

3. Preliminary Results

In the course of analysis, we shall use the following results:

(i) The orthogonality relation for the heat polynomials

$$\int_0^{\infty}W_{m,\boldsymbol{\nu}}(x,t)P_{n,\boldsymbol{\nu}}(x,-t)d\boldsymbol{\Omega}(\boldsymbol{x})=\frac{\boldsymbol{\delta_{mn}}}{\boldsymbol{K_n}} \qquad (9)$$

whereδ_{mn} is the Kronecker delta,

$$d\Omega\ (x)=2^{\frac{1}{2}-\boldsymbol{\nu}}\left[\Gamma\left(\upsilon+\frac{1}{2}\right)\right]^{-1}x^{2\upsilon}dx \qquad (10)$$

$$\text{and}K_n=\frac{\Gamma\left[\upsilon+\frac{1}{2}\right]}{2^{4n}n!\Gamma\left[\upsilon+\frac{1}{2}+n\right]} \qquad (11)$$

(ii)The series ,

$$S(x,\xi,t) = 2^{\frac{1}{2}-\sigma}\sum_{n=0}^{\infty}\frac{\left(\frac{l}{t}\right)^n \Gamma\left(\mu+\frac{1}{2}+n+\rho\right)P_{n+\rho,v}(x,-t)W_{n+\rho,\sigma}(\xi,t)}{2^{4(n+\rho)}(n+\rho)!\,\Gamma\left(\sigma+\frac{1}{2}+n+\rho\right)\Gamma\left(v+\frac{1}{2}+n+\rho\right)} \quad (12)$$

$$S(x,\xi,t) = \frac{x^{1-2v}\xi^{1-2\sigma}e^{-\xi^2/4t}}{\Gamma m\Gamma(v-\sigma+m)}a_n{}^* \int_0^{\omega}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy \quad (13)$$

where, $a_n{}^* = \frac{l^n t^{-(m+n)}2^{2(1-m)}\Gamma\left(\mu+\frac{1}{2}+n+\rho\right)}{\Gamma\left(\sigma-m+\frac{1}{2}+n+\rho\right)}$, $(v-\sigma+m>0)$ **(14)**

$\eta(y) = y^{2(\sigma-m)}e^{y^2/4t}$ and, $\omega = \min(\xi,x)$

If h(y) is strictly monotonically increasing and differentiable function in (a,b) and h(y) ≠0 in this interval, then the solutions to the Abel integral equations.

$$f(x) = \int_a^x \frac{\Phi(y)}{\{h(x)-h(y)\}^{\alpha}}dy, 0<\alpha<1 \quad (15)$$

and

$$f(x) = \int_x^b \frac{\Phi(y)}{\{h(y)-h(x)\}^{\alpha}}dy, 0<\alpha<1 \quad (16)$$

are given by ,

$$\Phi(y) = \frac{\sin(\alpha\pi)}{\pi}\frac{d}{dy}\int_a^y \frac{h'(x)F(x)}{\{h(y)-h(x)\}^{1-\alpha}}dx \quad (17)$$

and

$$\Phi(y) = -\frac{\sin(\alpha\pi)}{\pi}\frac{d}{dy}\int_y^b \frac{h'(x)F(x)}{\{h(x)-h(y)\}^{1-\alpha}}dx \quad (18)$$

respectively.

4. The Solution

(i) Equations of the first kind:

Let us assume

$$\sum_{m=0}^{\infty}\frac{A_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,t) = \phi_i(x), \quad a_{i-1}<x<a_i \quad (19)$$

where, i = 2,4,6,…..,n.

and where $\phi_i(x)$ are unspecified functions. Using orthogonality relation it follows from equations (1) and (19)

$$A_m = \frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+m+\rho\right)}{2^{4(m+\rho)}(m+\rho)!} . \sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(x,t) + \int_{a_{2i+1}}^{a_{2i+2}}\phi_{2i+2}(x,t)\right\} .W_{m+\rho,\sigma}(x,t)d\Omega(x) \quad (20)$$

Substituting this value of A_m in equation (2)and interchanging the order of integration and summation, we get

$$\sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(\xi,t) + \int_{a_{2i+1}}^{a_{2i+2}}\phi_{2i+2}(\xi,t)\right\} .S(x,\xi,t)d\Omega(\xi) = \frac{2^{\frac{1}{2}-\sigma}}{\Gamma\left(\sigma+\frac{1}{2}\right)}f_j(x,t) \quad (21)$$

$a_{j-1}<x<a_j$, j = 2,4,6,…..,n

$$\sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}}\phi_{2i+2}(\xi,t)S(x,\xi,t)d\xi = M_j(x,t), \quad a_{j-1}<x<a_j, \quad j = 2,4,6,\ldots,n. \quad (22)$$

where

$$M_j(x,t) = \frac{2^{\frac{1}{2}-\sigma}}{\Gamma\left(\sigma+\frac{1}{2}\right)}f_j(x,t) - \sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(\xi,t) .S(x,\xi,t)d\xi$$

for all j = 2,4,6,…..,n.

Taking j = k in equation (20), where k is an even integer and $2 \le k \le n$ and n is the total number of considered equations, we get

$$\int_{a_{k-1}}^{x}\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi \int_0^{\xi}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy + \int_x^{a_k}\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi \int_0^{x}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy$$
$$= \frac{\Gamma m\Gamma(v-\sigma+m)}{a_n * x^{1-2v}}M_k(x,t) - \sum_{i=0}^{\frac{k-4}{2}}\int_{a_{2i+1}}^{a_{2i+2}}\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi . \int_0^{x}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy$$
$$- \sum_{i=\frac{k}{2}}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}}\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi \int_0^{x}\eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy \quad (23)$$

Inverting the order of integration and assuming

$$\int_y^{a_k}\frac{\phi_k(\xi,t)\,\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-y^2)^{1-m}} = \bar{\phi}_k(y), \quad a_{k-1}<x<a_k$$

For all k = 2,4,6,…..,n **(24)**

we get

$$\int_{a_{k-1}}^{x}\frac{\eta(y)\bar{\phi}_k(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} = \frac{\Gamma m\Gamma(v-\sigma+m)}{a_n * x^{1-2v}}M_k(x,t) - \int_0^{a_{k-1}}\frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_{a_{k-1}}^{a_k}\frac{\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-y^2)^{1-m}}$$
$$- \sum_{i=0}^{(k-4)/2}\left\{\int_0^{a_{2i+1}}\frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_{a_{2i+1}}^{a_{2i+2}}\frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-y^2)^{1-m}} + \int_{a_{2i+1}}^{a_{2i+2}}\frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}}\right.$$

$$\cdot \int_{t}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} \Bigg\}$$

$$- \sum_{i=k/2}^{(n-2)/2} \int_0^x \frac{\eta(y)dy}{(x^2-y^2)^{1-\upsilon+\sigma-m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} \quad \mathbf{(25)}$$

This equation is an Abel type integralequation and its solution is given by

$$\eta(t)\bar{\phi}_k(y) = F_k(y,t) - \frac{\sin(1-\upsilon+\sigma-m)\pi}{\pi}$$

$$\Bigg[\int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{t} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \int_{a_{k-1}}^{a_k} \frac{\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} +$$

$$\sum_{i=0}^{(k-4)/2} \Bigg\{ \int_0^{a_{2i+1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} + \int_{a_{2i+1}}^{a_{2i+2}} \eta(z)dz$$

$$\frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{\xi}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg\}$$

$$+ \sum_{i=k/2}^{(n-2)/2} \frac{d}{dy} \int_{a_{k-1}}^{y} \eta(z)dz \int_0^x \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg] \quad \mathbf{(26)}$$

where

$$F_k(y,t) = \frac{\sin(1-\upsilon+\sigma-m)\pi\ \Gamma m\ \Gamma(\upsilon-\sigma+m)}{\pi\ a_n^*} \frac{d}{dy} \cdot \int_{a_{k-1}}^{y} \frac{2x^{2\upsilon} M_k(x)\,dx}{(y^2-x^2)^{\upsilon+\sigma-m}} \quad \mathbf{(27)}$$

Changingthe order of integration of the last integral of equation (26), we get

$$\eta(y)\bar{\phi}_k(y) = F_k(y,t) - \frac{\sin(1-\upsilon+\sigma-m)\pi}{\pi}$$

$$\Bigg[\int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{a_{k-1}}^{a_k} \frac{\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} +$$

$$\sum_{i=0}^{(k-4)/2} \Bigg\{ \int_0^{a_{2i+1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} + \int_{a_{2i+1}}^{a_{2i+2}} \eta(z)dz$$

$$\frac{d}{dy} \cdot \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{z}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg\} + \sum_{i=k/2}^{(n-2)/2} \Bigg\{ \int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} \cdot$$

$$\int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}} \xi e^{-\frac{\xi^2}{4t}} \frac{\phi_{2i+2}(\xi,t)d\xi}{(\xi^2-z^2)^{1-m}} +$$

$$\frac{d}{dy} \int_{a_{k-1}}^{y} \eta(\xi)d\xi \int_{z}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg\} \Bigg] \quad \mathbf{(28)}$$

Using orthogonality,

$$\int_{z}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\upsilon+\sigma-m}(y^2-x^2)^{\upsilon+\sigma-m}} = \frac{\pi}{\sin(1-\upsilon+\sigma-m)\pi} \quad \mathbf{(29)}$$

We get,

$$\eta(y)\bar{\phi}_k(y) = F_k(y,t) - \frac{\sin(1-\upsilon+\sigma-m)\pi}{\pi}$$

$$\Bigg[\int_0^{a_{k-1}} \frac{\eta(z)(a^2-z^2)^{\upsilon-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\upsilon-\sigma+m}} \cdot \int_{a_{k-1}}^{a_k} \frac{\phi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}}$$

$$+ \sum_{i=0}^{(k-4)/2} \Bigg\{ \int_0^{a_{2i+1}} \frac{\eta(z)(a^2-z^2)^{\upsilon-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\upsilon-\sigma+m}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}}$$

$$+ \int_{a_{2i+1}}^{a_{2i+2}} \frac{\eta(z)(a^2-z^2)^{\upsilon-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\upsilon-\sigma+m}} \int_{z}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg\}$$

$$+ \sum_{i=k/2}^{(n-2)/2} \Bigg[\int_0^{a_{2i+1}} \frac{\eta(z)(a^2-z^2)^{\upsilon-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\upsilon-\sigma+m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \Bigg]$$

$$- \sum_{i=k/2}^{(n-2)/2} \frac{d}{dy} \int_{a_{k-1}}^{y} \eta(z)dz \int_{a_{2i+1}}^{a_{2i+2}} \frac{\phi_{2i+2}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \quad \mathbf{(30)}$$

Equation (30) is also Abel type integral equation.Therefore its solution is given by

$$\emptyset_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} = \frac{-\sin(1-m)\pi}{\pi}\frac{d}{d\xi}\int_{\xi}^{a_k}\frac{2y\,\bar{\emptyset}_k(y)\,dy}{(y^2-\xi^2)^m}$$

for all k = 2,4,6,....,n. **(31)**

Therefore,

$$\int_{a_{k-1}}^{a_k}\frac{\emptyset_k(\xi,t)\xi e^{\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}} = \frac{\sin(1-m)\pi}{\pi(a^2-z^2)^{-m}} \cdot \int_{a_{k-1}}^{a_k}\frac{2x\bar{\emptyset}_k(x)\,dx}{(x^2-z^2)(x^2-a^2)^m}$$

for all k = 2,4,6,....,n. **(32)**

Applying the above result in equation (30) and also applying the Leibnitz theorem we get

$$\eta(y)\bar{\emptyset}_k(y) = F_k(t) - \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2}\Bigg[\int_0^{a_{k-1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-a_{k-1}{}^2)^{\nu-\sigma+m}}dz \int_{a_{k-1}}^{a_k}\frac{2x\,\bar{\emptyset}_k(x)\,dx}{(x^2-z^2)(x^2-a_{k-1}{}^2)^m}$$

$$\sum_{i=0}^{(k-4)/2}\Bigg\{\int_0^{a_{2i+1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-a_{2i+1}{}^2)^{\nu-\sigma+m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}}\frac{2x\,\bar{\emptyset}_{2i+2}(x)\,dx}{(x^2-z^2)(x^2-a_{2i+1}{}^2)^m}$$

$$-m\int_{a_{2i+1}}^{a_{2i+2}}\frac{2x\bar{\emptyset}_{2i+2}(x)}{(x^2-z^2)(x^2-a_{2i+1}{}^2)^m}dx\int_{a_{2i+1}}^{\xi}\frac{2\xi d\xi}{(\xi^2-z^2)^{1+m}} \cdot \int_{a_{2i+1}}^{y}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-a_{2i+1}{}^2)^{\nu-\sigma+m}}\Bigg\}$$

$$+\sum_{i=k/2}^{\frac{n-2}{2}}\int_0^{a_{k-1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-a_{2i+1}{}^2)^{\nu-\sigma+m}} \cdot \int_{a_{2i+1}}^{a_{2i+2}}\frac{2x\bar{\emptyset}_{2i+2}(x)}{(x^2-z^2)(x^2-a_{2i+1}{}^2)^m}\Bigg]$$

$$-\sum_{i=k/2}^{(n-2)/2}\frac{d}{dy}\int_{a_{k-1}}^{y}\frac{\eta(z)dz}{(a_{2i+1}-z^2)^{-m}}\int_{a_{2i+1}}^{a_{2i+2}}\frac{2x\,\bar{\emptyset}_{2i+2}(x)\,dx}{(x^2-z^2)(x^2-a_{2i+1}{}^2)^m}$$

or

$$\eta(y)\bar{\emptyset}_k(y) = F_k(y,t) - \int_{a_{k-1}}^{a_k}\bar{\emptyset}_k(x)P_k(x,y)\,dx - \sum_{i=0}^{(k-4)/2}\int_{a_{2i+1}}^{a_{2i+2}}\bar{\emptyset}_{2i+2}(x)Q_k(x,y)dx - \sum_{i=k/2}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}}\bar{\emptyset}_{2i+2}(x)R_k(x,y)dx,$$

$$a_{k-1} < r < a_k \qquad \textbf{(33)}$$

where

$$P_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-a_{k-1}{}^2)^{\nu-\sigma+m}}\frac{2x}{(x^2-a_{k-1}{}^2)^m} \cdot \int_0^{a_{k-1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)} \qquad \textbf{(34)}$$

$$Q_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-a_{k-1}{}^2)^{\nu-\sigma+m}}\frac{2x}{(x^2-a_{2i+1}{}^2)^m} \cdot \int_0^{a_{2i+1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)} - m\int_{a_{2i+1}}^{\xi}\frac{2\xi d\xi}{(\xi^2-z^2)^{1+m}} \cdot \int_{a_{2i+1}}^{y}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)}\Bigg\} \qquad \textbf{(35)}$$

$$R_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-a_{2i+1}{}^2)^{\nu-\sigma+m}}\frac{2x}{(x^2-a_{2i+1}{}^2)^m}\Bigg\{\int_0^{a_{k-1}}\frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)} - \frac{d}{dy}\cdot\int_{a_{k-1}}^{y}\frac{\eta(z)dz}{(a_{2i+1}-z^2)^{-m}(x^2-z^2)}\Bigg\} \qquad \textbf{(36)}$$

Substituting k = 2,4,6,....,n. in equation (33) we will get n/2simultaneous Fredholm Integral equations of the second kind. With the help of these n/2 simultaneous equations we can calculate $\bar{\emptyset}_2(t), \bar{\emptyset}_4(t), \ldots\ldots\bar{\emptyset}_n(t)$ and Then the values of $\emptyset_2(t), \emptyset_4(t), \ldots.\emptyset_n(t)$ can be determined .After all these calculations we can compute the coefficient A_m with the help of equation (20).

(ii) Equations of the second kind:

Let us assume

$$\sum_{m=0}^{\infty}\frac{D_m}{\left(\mu+m+\frac{1}{2}+\rho\right)}P_{m+\rho,\sigma}(x,-t) = \Psi_i(x),$$

$$a_{i-1} < x < a_i \qquad \textbf{(37)}$$

where, i = 1,3,5,....,n − 1.

and where $\Psi_i(x)$ are unspecified functions. Using orthogonality relation it follows from equations (6) and (37)

$$D_m = \frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\mu+\frac{1}{2}+m+\rho\right)}{2^{4(m+\rho)}(m+\rho)!} \cdot \sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i+1}}^{a_{2i+2}}f_{2i+2}(x,t) + \int_{a_{2i}}^{a_{2i+1}}\Psi_{2i+1}(x,t)\right\} .W_{m+\rho,\sigma}(x,t)d\Omega(x) \qquad \textbf{(38)}$$

Substituting this value of D_m in equation (6) and interchanging the order of integration and summation, we get

$$\sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i+1}}^{a_{2i+2}}f_{2i+2}(\xi,t) + \int_{a_{2i}}^{a_{2i+1}}\Psi_{2i+1}(\xi,t)\right\} .\,S(x,\xi,t)d\Omega(\xi) = \frac{2^{\frac{1}{2}-\sigma}}{\Gamma\left(\sigma+\frac{1}{2}\right)}g_j(x,t) \qquad \textbf{(39)}$$

$$a_{j-1} < x < a_j \quad , \; j = 1,3,5,\ldots.,n-1$$

$$\sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}}\Psi_{2i+1}(\xi,t)S(x,\xi,t)d\xi = N_j(x,t),$$

$$a_{j-1} < x < a_j, \quad j = 1,3,5,\ldots.,n-1. \qquad \textbf{(40)}$$

where

$$N_j(x,t) = \frac{2^{\frac{1}{2}-\sigma}}{\Gamma\left(\sigma+\frac{1}{2}\right)}g_j(x,t) - \sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}}f_{2i+2}(\xi,t)$$

$. S(x,\xi,t)d\xi$
for all $j = 1,3,5,\ldots,n-1$.

Taking $j = k$ in equation (38), where k is an odd integer and $1 \le k \le n-1$ and n is the total number of considered equations, we get

$$\int_{a_{k-1}}^{x} \Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi \int_0^{\xi} \eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy + \int_x^{a_k} \Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi \int_0^{x} \eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy$$
$$= \frac{\Gamma m \Gamma(v-\sigma+m)}{a_n * x^{1-2v}} N_k(x,t) - \sum_{i=0}^{\frac{k-3}{2}} \int_{a_{2i}}^{a_{2i+1}} \Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi . \int_0^{x} \eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy$$
$$- \sum_{i=\frac{k+1}{2}}^{\frac{n-2}{2}} \int_{a_{2i}}^{a_{2i+1}} \Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi \int_0^{x} \eta(y)(\xi^2-y^2)^{m-1}(x^2-y^2)^{v-\sigma+m-1}dy \quad \textbf{(41)}$$

Inverting the order of integration and assuming

$$\int_y^{a_k} \frac{\Psi_k(\xi,t)\,\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} = \bar{\Psi}_k(y), \quad a_{k-1} < x < a_k$$

For all $k = 1,3,5,\ldots,n-1$ **(42)**

we get

$$\int_{a_{k-1}}^{x} \frac{\eta(y)\bar{\Psi}_k(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} = \frac{\Gamma m \Gamma(v-\sigma+m)}{a_n * x^{1-2v}} N_k(x,t) - \int_0^{a_{k-1}} \frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_{a_{k-1}}^{a_k} \frac{\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}}$$
$$- \sum_{i=1}^{(k-3)/2} \left\{ \int_0^{a_{2i}} \frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} + \int_{a_{2i}}^{a_{2i+1}} \frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_t^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} \right\}$$
$$- \sum_{i=k+1/2}^{(n-2)/2} \int_0^{x} \frac{\eta(y)dy}{(x^2-y^2)^{1-v+\sigma-m}} . \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-y^2)^{1-m}} \quad \textbf{(43)}$$

This equation is an Abel type integral equation and its solution is given by

$$\eta(t)\bar{\Psi}_k(y) = G_k(y,t) - \frac{\sin(1-v+\sigma-m)\pi}{\pi} \left[\int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{t} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} \int_{a_{k-1}}^{a_k} \frac{\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \right.$$
$$+ \sum_{i=1}^{(k-3)/2} \left\{ \int_0^{a_{2i}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} + \int_{a_{2i}}^{a_{2i+1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} . \int_{\xi}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \right\}$$
$$\left. + \sum_{i=(k+1)/2}^{(n-2)/2} \frac{d}{dy} \int_{a_{k-1}}^{y} \eta(z)dz \int_0^{x} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} . \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \right] \quad \textbf{(44)}$$

where

$$G_k(y,t) = \frac{\sin(1-v+\sigma-m)\pi \; \Gamma m \, \Gamma(v-\sigma+m)}{\pi \; a_n^*} \frac{d}{dy} . \int_{a_{k-1}}^{y} \frac{2x^{2v} N_k(x)\,dx}{(y^2-x^2)^{v+\sigma-m}} \quad \textbf{(45)}$$

Changing the order of integration of the last integral of equation (44), we get

$$\eta(y)\bar{\Psi}_k(y) = G_k(y,t) - \frac{\sin(1-v+\sigma-m)\pi}{\pi} \left[\int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} . \int_{a_{k-1}}^{a_k} \frac{\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} + \right.$$
$$\sum_{i=1}^{(k-3)/2} \left\{ \int_0^{a_{2i}} \eta(z)dz \frac{d}{dy} \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} + \int_{a_{2i}}^{a_{2i+1}} \eta(z)dz \frac{d}{dy} . \int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-v+\sigma-m}(y^2-x^2)^{v+\sigma-m}} . \int_z^{a_{2i+1}} \frac{\bar{\Psi}_{2i+1}(\xi,t)\,\xi e^{-\frac{\xi^2}{4t}} d\xi}{(\xi^2-z^2)^{1-m}} \right\} + \sum_{i=(k+1)/2}^{(n-2)/2} \left\{ \int_0^{a_{k-1}} \eta(z)dz \frac{d}{dy} . \right.$$

$$\int_{a_{k-1}}^{y} \frac{2x\,dx}{(x^2-z^2)^{1-\nu+\sigma-m}(y^2-x^2)^{\nu+\sigma-m}} \cdot \int_{a_{2i}}^{a_{2i+1}} \frac{\xi e^{-\frac{\xi^2}{4t}}\Psi_{2i+1}(\xi,t)d\xi}{(\xi^2-z^2)^{1-m}} + \frac{d}{dy}\int_{a_{k-1}}^{y}\eta(\xi)d\xi \int_{z}^{y}\frac{2x\,dx}{(x^2-z^2)^{1-\nu+\sigma-m}(y^2-x^2)^{\nu+\sigma-m}} \cdot \int_{a_{2i}}^{a_{2i+1}}\frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}}\Bigg\}\Bigg] \quad (46)$$

Using orthogonality,

$$\int_{z}^{y}\frac{2x\,dx}{(x^2-z^2)^{1-\nu+\sigma-m}(y^2-x^2)^{\nu+\sigma-m}} = \frac{\pi}{\sin(1-\nu+\sigma-m)\pi}$$

We get,

$$\eta(y)\overline{\Psi}_k(y) = G_k(y,t) - \frac{\sin(1-\nu+\sigma-m)\pi}{\pi}\Bigg[\int_0^{a_{k-1}}\frac{\eta(z)(a^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\nu-\sigma+m}} \cdot \int_{a_{k-1}}^{a_k}\frac{\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}} + \sum_{i=1}^{(k-3)/2}\Bigg\{\int_0^{a_{2i+1}}\frac{\eta(z)(a^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\nu-\sigma+m}}\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}} + \int_{a_{2i}}^{a_{2i+1}}\frac{\eta(z)(a^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\nu-\sigma+m}}\int_{z}^{a_{2i+1}}\frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}}\Bigg\}$$

$$+\sum_{i=(k+1)/2}^{(n-2)/2}\left[\int_0^{a_{2i}}\frac{\eta(z)(a^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-a^2)^{\nu-\sigma+m}} \cdot \int_{a_{2i}}^{a_{2i+1}}\frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}}\right] - \sum_{i=(k+1)/2}^{(n-2)/2}\frac{d}{dy}\int_{a_{k-1}}^{y}\eta(z)dz\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi_{2i+1}(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}} \quad (47)$$

Equation (47) is also Abel type integral equation. Therefore its solution is given by

$$\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}} = \frac{-\sin(1-m)\pi}{\pi}\frac{d}{d\xi}\int_{\xi}^{a_k}\frac{2y\overline{\Psi}_k(y)\,dy}{(y^2-\xi^2)^m}$$

for all $k = 1,3,5,\ldots,n-1$. **(48)**

Therefore,

$$\int_{a_{k-1}}^{a_k}\frac{\Psi_k(\xi,t)\xi e^{-\frac{\xi^2}{4t}}d\xi}{(\xi^2-z^2)^{1-m}} = \frac{\sin(1-m)\pi}{\pi(a^2-z^2)^{-m}} \cdot \int_{a_{k-1}}^{a_k}\frac{2x\overline{\Psi}_k(x)\,dx}{(x^2-z^2)(x^2-a^2)^m}$$

for all $k = 1,3,5,\ldots,n-1$. **(49)**

Applying the above result in equation (47) and also applying the Leibnitz theorem we get

$$\eta(y)\overline{\Psi}_k(y) = G_k(t) - \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2}\Bigg[\int_0^{a_{k-1}}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+m}dz}{(y^2-z^2)(y^2-{a_{k-1}}^2)^{\nu-\sigma+m}}dz\int_{a_{k-1}}^{a_k}\frac{2x\overline{\Psi}_k(x)\,dx}{(x^2-z^2)(x^2-{a_{k-1}}^2)^m}$$

$$\sum_{i=1}^{(k-3)/2}\Bigg\{\int_0^{a_{2i}}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-{a_{2i}}^2)^{\nu-\sigma+m}} \cdot \int_{a_{2i}}^{a_{2i+1}}\frac{2x\overline{\emptyset}_{2i+1}(x)\,dx}{(x^2-z^2)(x^2-{a_{2i}}^2)^m} - m\int_{a_{2i}}^{a_{2i+1}}\frac{2x\overline{\Psi}_{2i+1}(x)}{(x^2-z^2)(x^2-{a_{2i+1}}^2)^m}dx\int_{a_{2i}}^{\xi}\frac{2\xi d\xi}{(\xi^2-z^2)^{1+m}} \cdot \int_{a_{2i}}^{y}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-{a_{2i}}^2)^{\nu-\sigma+m}}\Bigg\}$$

$$+\sum_{i=(k+1)/2}^{\frac{n-2}{2}}\int_0^{a_{k-1}}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(y^2-{a_{2i}}^2)^{\nu-\sigma+m}} \cdot \int_{a_{2i}}^{a_{2i+1}}\frac{2x\overline{\Psi}_{2i+1}(x)}{(x^2-z^2)(x^2-{a_{2i}}^2)^m}\Bigg] - \sum_{i=(k+1)/2}^{(n-2)/2}\frac{d}{dy}\int_{a_{k-1}}^{y}\frac{\eta(z)dz}{(a_{2i}-z^2)^{-m}}\int_{a_{2i}}^{a_{2i+1}}\frac{2x\overline{\Psi}_{2i+1}(x)\,dx}{(x^2-z^2)(x^2-{a_{2i}}^2)^m}$$

or

$$\eta(y)\overline{\Psi}_k(y) = G_k(y,t) - \int_{a_{k-1}}^{a_k}\overline{\Psi}_k(x)L_k(x,y)\,dx - \sum_{i=1}^{(k-3)/2}\int_{a_{2i+1}}^{a_{2i+2}}\overline{\Psi}_{2i+1}(x)M_k(x,y)dx - \sum_{i=(k+1)/2}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}}\overline{\Psi}_{2i+1}(x)N_k(x,y)dx,$$

$$a_{k-1} < r < a_k \quad (50)$$

where

$$L_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-{a_{k-1}}^2)^{\nu-\sigma+m}}\frac{2x}{(x^2-{a_{k-1}}^2)^m} \cdot \int_0^{a_{k-1}}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)} \quad (51)$$

$$M_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-{a_{k-1}}^2)^{\nu-\sigma+m}}\frac{2x}{(x^2-{a_{2i}}^2)^m}\Bigg\{\int_0^{a_{2i}}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)} - m\int_{a_{2i}}^{\xi}\frac{2\xi d\xi}{(\xi^2-z^2)^{1+m}} \cdot \int_{a_{2i}}^{y}\frac{\eta(z)({a_{k-1}}^2-z^2)^{\nu-\sigma+2m}dz}{(y^2-z^2)(x^2-z^2)}\Bigg\} \quad (52)$$

$$N_k(x,y) = \frac{\sin(1-\nu+\sigma-m)\pi\sin(1-m)\pi}{\pi^2(y^2-a_{2i+1}{}^2)^{\nu-\sigma+m}} \frac{2x}{(x^2-a_{2i}{}^2)^m} \left\{ \int_0^{a_{k-1}} \frac{\eta(z)(a_{k-1}{}^2-z^2)^{\nu-\sigma+2m} dz}{(y^2-z^2)(x^2-z^2)} - \frac{d}{dy} \cdot \int_{a_{k-1}}^{y} \frac{\eta(z)dz}{(a_{2i}-z^2)^{-m}(x^2-z^2)} \right\} \quad (53)$$

Substituting $k = 1,3,5,\ldots,n-1.$ in equation (50) we will get n/2simultaneous Fredholm Integral equations of the second kind. With the help of these n/2 simultaneous equations we can calculate $\overline{\Psi}_2(t),\overline{\Psi}_4(t),\ldots\ldots\overline{\Psi}_{n-1}(t)$ and then the values of $\Psi_2(t),\Psi_4(t),\ldots\Psi_{n-1}(t)$ can be determined .After all these calculations we can compute the coefficient D_m with the help of equation (38).

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References

[1] Pathak,R.S, Dual series equations involving heat polynomials ,*Indag.Math,41,1979 , 456-463.*
[2] Cooke , J.C., The solution of triple and quadruple integral equations and Fourier, series equations, *Quart . J . Mech. Appl. Math., 45, 1992, 247-263.*
[3] Shukla Gunjan,Tripathi K.C., Triple simultaneous Fourier series equations involving heat polynomials ,*Int.J.Sc.Res. 3(12), 2014, 965-970.*
[4] Shukla Gunjan ,Tripathi K.C. , Quadruple simultaneous Fourier series equations involving heat polynomials , *Int.J.Sc.Res. 4 (1),2015 ,147-155.*
[5] Dwivedi A.P.,Chandel J.& Siddiqi Tarannum , Family of triple and quadruple series equations involving heat polynomials,*Ganita, 52 ,2001, 43-53.*
[6] Shukla Indu ,Dwivedi A.P. , Upadhayay Virendra ,Six Fourier series equations involving heat polynomials , *Int.J.Sc.Res. 5 (12),2016 ,465-470.*