N- Fourier Series Equations Involving Jacobi H_m Polynomials

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Abstract: ***In this paper , we have considered the N-Fourier series equations involving Jacobi H_m polynomials of the first and second kind and solved the two sets of series equations.***

Keywords: Integral equation, Series equation, Fourier series, integral theorems, Jacobi polynomials

1. Introduction

If we review the literature then we observe that the existing solutions on series equations are derived only from dual to six Fourier series equations. No further generalizations are available till date. This tempted us to find the solution of n-Fourier series equations involving some special functions and in this paper we have obtained certain results. By considering the special values of n = 2,3,4,5,6 we shall be able to derive solutions of dual, triple,quadruple,5-tuple and 6-tuple Fourier series equations involving respective special functions.

2. N- Series Equations of the first kind

(i) N-series equations of the first kind

$$\sum_{m=0}^{\infty} A_m j_m(\alpha,\gamma;\sigma) = f_i(\sigma), a_{i-1} < \sigma < a_i \quad (1)$$

where, i = 1,3,5,...., n − 1 and a_0 =0.

$$\sum_{m=0}^{\infty} A_m P_m(\lambda-\rho,\gamma)(1+H_m) j_m(\alpha,\lambda;\sigma) = f_j(\sigma), a_{j-1} < \sigma < a_j \quad (2)$$

where, j = 2,4,6,...., n and a_n = 1.

Here n is taken as an even number. If n is odd then the equations will be

$$\sum_{m=0}^{\infty} B_m j_m(\alpha,\gamma;\sigma) = f_i(\sigma), a_{i-1} < \sigma < a_i \quad (3)$$

where, i = 1,3,5,...., n and a_0 =0.

$$\sum_{m=0}^{\infty} B_m P_m(\lambda-\rho,\gamma)(1+H_m) j_m(\alpha,\lambda;\sigma) = f_j(\sigma), a_{j-1} < \sigma < a_j \quad (4)$$

where, j = 2,4,6,...., n − 1.

(ii) N-series equations of the second kind

$$\sum_{m=0}^{\infty} D_m P_m(\lambda-\rho,\gamma)(1+H_m) j_m(\alpha,\lambda;\sigma) = g_i(\sigma), a_{j-1} < \sigma < a_j \quad (5)$$

where, i = 1,3,5,...., n − 1 and a_0 = 0.

$$\sum_{m=0}^{\infty} D_m j_m(\alpha,\gamma;\sigma) = g_j(\sigma), a_{i-1} < \sigma < a_i \quad (6)$$

where, j = 2,4,6,...., n and a_n = 1.

Here also n is taken as an even number. If n is odd then the equations will be

$$\sum_{m=0}^{\infty} E_m P_m(\lambda-\rho,\gamma)(1+H_m) j_m(\alpha,\lambda;\sigma) = g_i(\sigma), a_{j-1} < \sigma < a_j \quad (7)$$

where, i = 1,3,5,...., n and a_0 =0

$$\sum_{m=0}^{\infty} E_m j_m(\alpha,\gamma;\sigma) = g_j(\sigma), a_{i-1} < \sigma < a_i \quad (8)$$

where, j = 2,4,6,...., n − 1 and a_n = 1.

$$P_m(\lambda-\rho,\gamma) = \frac{\Gamma(\lambda-\rho+m)\Gamma(1+\alpha+\gamma+m)}{\Gamma(\gamma+m)\Gamma(1+\alpha-\lambda+\rho+m)} \quad (9)$$

J_m is the Jacobi polynomial. H_m is a known coefficient of m. $f_i(\sigma), g_i(\sigma)$ where (i= 1,2,3,…n) all are known functions and A_m, B_m, D_m, E_m are known coefficients.

Here we solve only equations (1),(2)of first kind and equations (5),(6) of the second kind and the solution of equation (3),(4) of first kind and equations (7),(8) of the second kind can be obtained easily by following similar procedure.

3. Preliminary Results

In the analysis, we shall use the following results:

(i)The orthogonality relation for the Jacobi polynomial is,

$$\int_0^1 t^{\gamma-1}(1-t)^{\alpha-\gamma} j_m(\alpha,\gamma;t) j_n(\alpha,\gamma;t)\,dt = \frac{\boldsymbol{\delta_{mn}}}{\boldsymbol{\Delta_n^2}} \quad (10)$$

Where $\alpha+1>\gamma>0$, δ_{mn} is the kronecker delta.

$$\Delta_n^2 = \frac{(\alpha+2n)\Gamma(\alpha+n)\Gamma(\gamma+n)}{\Gamma(n+1)\gamma^2\Gamma(1+\alpha-\gamma+n)} \quad (11)$$

(ii) When $\alpha+1+\rho>\lambda>$,$\gamma+\rho>\lambda>\rho>0$.

$$K(\sigma,t) = \frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\ \sigma^{1-\lambda} t^{1-\gamma}} \sum_{n=0}^{\infty} \Delta_n^2 P_n(\lambda-\rho,\gamma).\ . j_n(\alpha,\lambda;\sigma) j_n(\alpha,\gamma;t) \quad (12)$$

$$= \int_0^s m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho+1}\,dx \quad (13)$$

$= K_s(\sigma,t)$

Where, m(x) = $x^{\lambda-\rho-1}(1-x)^{\lambda-\rho-\alpha}$ and δ= min(σ,t).

(iii) Iff(x) and $f'(x)$ are continuous in a a<x<b and if $0<\rho<1$ .then the solutions to the Abel integral equations.

$$f(x) = \int_a^x \frac{f(y)}{\{x-y\}^\rho}\,dy \quad (14)$$

$$\text{and} f(x) = \int_x^b \frac{f(y)}{\{y-x\}^\rho}\,dy \quad (15)$$

are given by ,

$$f(y) = \frac{\sin(\rho\pi)}{\pi}\frac{d}{dy}\int_a^y \frac{F(x)}{\{y-x\}^{1-\rho}}\,dx \quad (16)$$

$$\text{and} f(y) = -\frac{\sin(\rho\pi)}{\pi}\frac{d}{dy}\int_y^b \frac{F(x)}{\{x-y\}^{1-\rho}}\,dx \quad (17)$$

respectively.

4. The Solution

(i) Equations of the first kind:

Let us assume

$\sum_{m=0}^{\infty} A_m\, j_m(\alpha,\gamma;\sigma) = \phi_i(\sigma), a_{i-1} < x < a_i$ **(18)**

where, i $=$ 2,4,6, ..., n. and $\phi_i(x)$ are unknown functions.

Using orthogonality relation it follows from equations (1) and (18).

$$A_m = \Delta_n^2 \sum_{i=0}^{\frac{n-2}{2}} \left\{ \int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(\sigma) + \int_{a_{2i+1}}^{a_{2i+2}} \phi_{2i+2}(\sigma) \right\} .\sigma^{\gamma-1}(1-\sigma)^{\alpha-\gamma} j_m(\alpha,\gamma;\sigma).\, D\sigma \quad \textbf{(19)}$$

$$f'_{2i+1}(\sigma) = (1-\sigma)^{\alpha-\gamma} f_{2i+1}(\sigma) \quad \textbf{(20)}$$

i = 0,1,2,.........(n-2)/2

$$\emptyset'_{2i+1}(\sigma) = (1-\sigma)^{\alpha-\gamma}\emptyset_{2i+1}(\sigma \quad \textbf{(21)}$$

i = 0,1,2,.........(n-2)/2

equation(19) reduces to,

$$A_m = \Delta_n^2 \sum_{i=0}^{\frac{n-2}{2}} \left\{ \int_{a_{2i}}^{a_{2i+1}} f'_{2i+1}(\sigma) + \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+1}(\sigma) \right\} .\sigma^{\gamma-1} j_m(\alpha,\gamma;\sigma).\, d\sigma \quad \textbf{(22)}$$

Substituting this expression for A_m in equation (2) we obtain

$$\sum_{i=0}^{\frac{n-2}{2}} \left[\int_{a_{2i}}^{a_{2i+1}} f'_{2i+1}(\sigma)\{K(\sigma,t)+S(\sigma,t)\}dt + \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+1}(\sigma)\{K(\sigma,t)+S(\sigma,t)\}dt \right] = \frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\ \ \sigma^{1-\lambda}} f_j(\sigma); a_{j-1} < \sigma < a_j \quad \textbf{(23)}$$

j $=$ 2,4,6, ..., n

where K (σ ,t)is defined by equation (12) and

$$S(\sigma,t) = \frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\ \ \sigma^{1-\lambda}\, t^{1-\gamma}} \sum_{m=0}^{\infty} \Delta_m^2\, P_m(\lambda-\rho,\gamma).\, H_m.j_m(\alpha,\lambda;\sigma) j_m(\alpha,\gamma;t) \quad \textbf{(24)}$$

It is assumed that H_m is in the form such that S(σ,t) converges. Now starting with equation (23) if j=k where k is an even number and 2 ≤k ≤ m we have ,

$$\sum_{i=0}^{\frac{k-4}{2}} \left[\int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t)\, K_t(\sigma,t)dt + \int_{a_{k-1}}^{\sigma} \emptyset'_k(t) K_t(\sigma,t)dt + \int_{\sigma}^{a_k} \emptyset'_k(t) K_\sigma(\sigma,t)dt \right] + \sum_{i=0}^{\frac{n-2}{2}} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t)\, S(\sigma,t)dt + \sum_{i=k/2}^{\frac{n-2}{2}} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t)\, K_\sigma(\sigma,t)dt = M_k(\sigma), a_{k-1} < \sigma < a_k \quad \textbf{(25)}$$

Where,

$$M_k(\sigma) = \frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\ \ \sigma^{1-\lambda}} f_k(\sigma) - \sum_{i=0}^{\frac{n-2}{2}} \int_{a_{2i}}^{a_{2i+1}} f'_{2i+1}(t)\{K(\sigma,t)+S(\sigma,t)\}dt \quad \textbf{(26)}$$

Using equation (13) in equation (25) , we get

$$\int_{a_{k-1}}^{\sigma} \emptyset'_k(t) \int_0^t m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\, dx\, dt + \int_{\sigma}^{a_k} \emptyset'_k(t) \int_0^{\sigma} m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\, dx\, dt$$

$$= M_K(\sigma) - \sum_{i=0}^{\frac{k-4}{2}} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t) . \int_0^t m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\, dx\, dt - \sum_{i=k/2}^{\frac{n-2}{2}} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t) . \int_0^{\sigma} m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\, dxdt$$

$$-\sum_{i=0}^{\frac{n-2}{2}} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t) S(\sigma,t)dt$$

Inverting the order of integration, we obtain

$$\int_{a_{k-1}}^{\sigma} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} \int_x^{a_k} \frac{\emptyset'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \int_0^{a_{k-1}} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} \int_{a_{k-1}}^{a_k} \frac{\emptyset'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} = M_K(\sigma)$$

$$-\sum_{i=0}^{\frac{k-4}{2}} \left\{ \int_{a_{2i+1}}^{a_{2i+2}} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} \int_x^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \right.$$

0a2i+1mxdxσ−x1−ρa2i+1a2i+2∅2i+2′tdtt−x1−γ+λ−ρ−

$$\sum_{i=k/2}^{(n-2)/2} \left\{ \int_0^{\sigma} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} \right\} +$$

$$\sum_{i=0}^{(n-2)/2} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t) S(\sigma,t)dt \quad \textbf{(27)}$$

Assuming,

$$\int_x^{a_k} \frac{\emptyset'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} = \overline{\emptyset}_k(x) \quad \textbf{(28)}$$

for allk $=$ 2,4,6, ..., n.

with the help of equation (28) the equation (27) takes the form

$$\int_{a_{k-1}}^{\sigma} \frac{m(x)\overline{\emptyset}_k(x)dx}{(\sigma-x)^{1-\rho}} = M_K(\sigma) - \int_0^{a_{k-1}} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} . \int_{a_{k-1}}^{a_k} \frac{\emptyset'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} - \sum_{i=0}^{\frac{k-4}{2}} \left\{ \int_{a_{2i+1}}^{a_{2i+2}} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} . \int_x^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \int_0^{a_{2i+1}} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} . \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} \right\}$$

$$\sum_{i=k/2}^{(n-2)/2} \int_0^{\sigma} \frac{m(x)dx}{(\sigma-x)^{1-\rho}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \sum_{i=0}^{(n-2)/2} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t)S(\sigma,t)dt$$

This is an Abel type integral equation and when $0 < \rho < 1$.the we can solve it as,

$$m(x)\overline{\emptyset}_k(x) = A_k(x) - \frac{\sin(1-\rho)\pi}{\pi} \left[\int_0^{a_{k-1}} m(\xi)d\xi . \frac{d}{dx} \int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{a_{k-1}}^{a_k} \frac{\emptyset'_k(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \right.$$

$$\sum_{i=0}^{(k-4)/2} \left\{ \int_{a_{2i+1}}^{a_{2i+2}} m(\xi)d\xi . \frac{d}{dx} \int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \frac{d}{dx} \int_{a_{k-1}}^{x} m(\xi)d\xi . \int_{\xi}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset'_{2i+2}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} \right\}$$

$$\left. + \sum_{i=0}^{(n-2)/2} \frac{d}{dx} \int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}} . \int_{a_{2i+1}}^{a_{2i+2}} \emptyset'_{2i+2}(t)s(\sigma,t)dt \right] \quad \textbf{(31)}$$

Using the results,

$$\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}=\frac{(a_{k-1}-\xi)^{\rho}}{(x-a_{k-1})^{\rho}(x-\xi)} \quad (32)$$

And, $\int_{\xi}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}=\frac{\pi}{\sin(1-\rho)\pi}$ (33)

Now equation (31) will be

$$m(x)\bar{\emptyset}_k(x)=A_k(x)-\frac{\sin(1-\rho)\pi}{\pi}.$$

$$\left[\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)d\xi}{(x-a_{k-1})^{\rho}(x-\xi)}\int_{a_{k-1}}^{a_k}\frac{\emptyset_k'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}+\right.$$

$$\sum_{i=0}^{(k-4)/2}\left\{\int_{a_{2i+1}}^{a_{2i+2}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-a_{k-1})^{\rho}(x-\xi)}d\xi\int_{\xi}^{a_{2i+2}}\frac{\emptyset_{2i+2}'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}+\right.$$

$$\left.\int_0^{a_{2i+1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-a_{k-1})^{\rho}(x-\xi)}d\xi\int_{a_{2i+1}}^{a_{2i+2}}\frac{\emptyset_{2i+2}'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\right\}+$$

$$\sum_{i=k/2}^{(n-2)/2}\left\{\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)d\xi}{(x-a_{k-1})^{\rho}(x-\xi)}.\int_{a_{2i+1}}^{a_{2i+2}}\frac{\emptyset_{2i+2}'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}+\right.$$

$$\left.\frac{\pi}{\sin(1-\rho)}\frac{d}{dx}\int_{a_{k-1}}^{x}m(\xi)d\xi.\int_{a_{2i+1}}^{a_{2i+2}}\frac{\emptyset_{2i+2}'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\right\}+$$

$$\left.+\sum_{i=0}^{(n-2)/2}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}}.\int_{a_{2i+1}}^{a_{2i+2}}\emptyset_{2i+2}'(t)s(\sigma,t)dt\right] \quad (34)$$

Equation (28) is also an Abel type integral equation and its solution is given by

$$\emptyset_k'(t)=\frac{-\sin(1-\gamma+\lambda-\rho)\pi}{\pi}\frac{d}{dt}\int_t^{a_k}\frac{\bar{\emptyset}_k(x)dx}{(x-t)^{\gamma-\lambda+\rho}} \quad (35)$$

For all k= 2,4,6,……n.

Therefore,

$$\int_{a_{k-1}}^{a_k}\frac{\emptyset_k'(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}$$

$$=\frac{-\sin(1-\gamma+\lambda-\rho)\pi}{\pi(a_{k-1}-\xi)^{-\gamma+\lambda-\rho}}\int_{a_{k-1}}^{a_k}\frac{\bar{\emptyset}_k(x)dx}{(x-\xi)(x-a_{k-1})^{\gamma-\lambda+\rho}}$$

For all k= 2,4,6,……n. (36)

Applying the equation (36) in equation (34) and also applying the Leibnitz theorem, we get

$$m(x)\bar{\emptyset}_k(x)=A_k(x)-\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}.$$

$$\left[\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{2\rho+\gamma-\lambda}m(\xi)d\xi}{(x-\xi)}.\int_{a_{k-1}}^{a_k}\frac{\bar{\emptyset}_k(y)dy}{(y-\xi)(y-a_{k-1})^{\gamma-\lambda+\rho}}+\right.$$

$$\sum_{i=0}^{(k-4)/2}\left\{-\frac{1}{(\gamma-\lambda+\rho)^{-1}}\int_{a_{2i+1}}^{a_{2i+2}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-\xi)}d\xi\right..$$

$$\int_{\xi}^{a_{2i+2}}\frac{dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}.\int_t^{a_{2i+2}}\frac{\bar{\emptyset}_{2i+2}(y)dy}{(y-t)^{1+\gamma-\lambda+\rho}}+$$

$$\left.\int_0^{a_{2i+1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)d\xi}{(x-\xi)(a_{2i+1}-\xi)^{-\gamma+\lambda-\rho}}.\int_{a_{2i+1}}^{a_{2i+2}}\frac{\bar{\emptyset}_{2i+2}(y)dy}{(y-\xi)(y-a_{2i+1})^{\gamma-\lambda+\rho}}\right\}$$

$+$

$$\sum_{i=k/2}^{(n-2)/2}\left\{\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)d\xi}{(a_{2i+1}-\xi)^{-\gamma+\lambda-\rho}(x-\xi)}.\right.$$

$$\left.\left.\int_{a_{2i+1}}^{a_{2i+2}}\frac{\bar{\emptyset}_{2i+2}(y)dy}{(y-\xi)(y-a_{2i+1})^{\gamma-\lambda+\rho}}\right\}\right]-\sum_{i=k/2}^{\frac{n-2}{2}}\frac{\sin(1-\gamma+\lambda-\rho)\pi}{\pi}.$$

$$\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{m(\xi)d\xi}{(a_{2i+1}-\xi)^{-\gamma+\lambda-\rho}}.\int_{a_{2i+1}}^{a_{2i+2}}\frac{\bar{\emptyset}_{2i+2}(y)dy}{(y-\xi)(y-a_{2i+1})^{\gamma-\lambda+\rho}}$$

$$+\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(\gamma-\lambda+\rho)^{-1}}\sum_{i=0}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}}dt$$

$$.\int_t^{a_{2i+2}}\frac{\bar{\emptyset}_{2i+2}(y)dy}{(y-t)^{1+\gamma-\lambda+\rho}}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{s(\sigma,t)d\xi}{(x-\sigma)^{\rho}}.$$

$$m(x)\bar{\emptyset}_k(x)=A_k(x)-\int_{a_{k-1}}^{a_k}\bar{\emptyset}_k(y)L_k(x,y)dy$$

$$-\sum_{i=0}^{(k-4)/2}\int_{a_{2i+1}}^{a_{2i+2}}\bar{\emptyset}_{2i+2}(y)M_{2i+2}(x,y)dy$$

$$-\sum_{i=k/2}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}}\bar{\emptyset}_{2i+2}(y)N_{2i+2}(x,y)dy$$

$$-\sum_{i=0}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}}\bar{\emptyset}_{2i+2}(y)O_{2i+2}(x,y)dy \quad (37)$$

For all k= 2,4,6,……n.

where,

$$L_k(x,y)=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}.\frac{1}{(y-a_{2i+1})^{\gamma-\lambda+\rho}}$$

$$.\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{2\rho+\gamma-\lambda}m(\xi)d\xi}{(x-\xi)(y-\xi)} \quad (38)$$

$$M_{2i+2}(x,y)=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}\left[\frac{1}{(y-a_{2i+1})^{\gamma-\lambda+\rho}}.\right.$$

$$.\int_0^{a_{2i+1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)(a_{2i+1}-\xi)^{\gamma-\lambda+\rho}d\xi}{(x-\xi)(y-\xi)}-(\gamma-\lambda+\rho).$$

$$\left..\int_{\xi}^{a_{2i+2}}\frac{dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}.\int_t^{a_{2i+2}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-\xi)(y-t)^{1+\gamma-\lambda+\rho}}d\xi\right] \quad (39)$$

$$N_{2i+2}(x,y)=\frac{\sin(1-\gamma+\lambda-\rho)\pi}{\pi}\left[\frac{\sin(1-\rho)\pi}{\pi(x-a_{k-1})^{\rho}}.\frac{1}{(y-a_{2i+1})^{\gamma-\lambda+\rho}}\right.$$

$$.\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)(a_{2i+1}-\xi)^{\gamma-\lambda+\rho}d\xi}{(x-\xi)(y-\xi)}+\frac{1}{(y-a_{2i+1})^{\gamma-\lambda+\rho}}.$$

$$\left.\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{m(\xi)d\xi}{(y-\xi)(a_{2i+1}-\xi)^{-\gamma+\lambda-\rho}}.\right] \quad (40)$$

$$O_{2i+2}(x,y)=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(\gamma-\lambda+\rho)^{-1}}.$$

$$.\int_{a_{2i+2}}^{y}\frac{dt}{(y-t)^{1+\gamma-\lambda+\rho}}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{s(\sigma,t)d\sigma}{(x-\sigma)^{\rho}}. \quad (41)$$

Substituting k = 2,4,6,…., n. in equation (37) we will get n/2simultaneous Fredholm Integralequations of the second kind. With the help of these n/2 simultaneous equations we can calculate $\bar{\emptyset}_2(x),\bar{\emptyset}_4(x),\ldots\ldots\bar{\emptyset}_n(x)$ and Then the values of $\emptyset`_2(t),\emptyset`_4(t),\ldots.\emptyset`_n(t)$ can be determined .After all these calculations we cancompute the coefficient A_m with the help of equation (22).

(ii) Equations of the second kind:

Let us assume

$$\sum_{m=0}^{\infty}D_m\,j_m(\alpha,\gamma;\sigma)=\Psi_i(\sigma),a_{i-1}<x<a_i \quad (42)$$

where, i = 1,3,5,…., n − 1. and $\Psi_i(x)$ are unknown functions in the given interval.

Using orthogonality relation it follows from equations (6) and (42).

$$D_m=\Delta_m^2\sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}}\Psi_{2i+1}(\sigma)+\int_{a_{2i+1}}^{a_{2i+2}}g_{2i+2}(\sigma)\right\}$$

$$.\sigma^{\gamma-1}(1-\sigma)^{\alpha-\gamma}j_m(\alpha,\gamma;\sigma).\,d\sigma \quad (43)$$

$$g_{2i+1}'(\sigma)=(1-\sigma)^{\alpha-\gamma}g_{2i+1}(\sigma) \quad (44)$$

i = 0,1,2,………(n-2)/2

$$\Psi_{2i+1}'(\sigma)=(1-\sigma)^{\alpha-\gamma}\Psi_{2i+1}(\sigma) \quad (45)$$

i = 0,1,2,………(n-2)/2

we get,

$$D_m=\Delta_m^2\sum_{i=0}^{\frac{n-2}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}}\Psi_{2i+1}'(\sigma)+\int_{a_{2i+1}}^{a_{2i+2}}\Psi_{2i+1}'(\sigma)\right\}$$

$$.\sigma^{\gamma-1} j_m(\alpha,\gamma;\sigma).\, d\sigma \qquad \textbf{(46)}$$

Substituting this expression for D_m in equation (5) we get on interchanging the order of integration and summation,

$$\sum_{i=0}^{\frac{n-2}{2}}\left[\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)\{K(\sigma,t)+S(\sigma,t)\}dt + \int_{a_{2i+1}}^{a_{2i+2}} g'_{2i+1}(t)\{K(\sigma,t)+S(\sigma,t)\}dt\right]$$

$$=\frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\quad \sigma^{1-\lambda}} g_j(\sigma);\, a_{j-1}<\sigma<a_j \qquad \textbf{(47)}$$

$$j = 1,3,5,\ldots,n-1.$$

Let $j=k$ where k is an odd number $\&1\le k\le n-1$.

$$\sum_{i=0}^{\frac{k-3}{2}}\left[\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)\, K_t(\sigma,t)dt + \int_{a_{k-1}}^{\sigma} \Psi'_k(t) K_t(\sigma,t)dt + \int_{\sigma}^{a_k} \Psi'_k(t) K_\sigma(\sigma,t)dt\right] + \sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)\, S(\sigma,t)dt + \sum_{i=(k+1)/2}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)\, K_\sigma(\sigma,t)dt = M_k(\sigma)$$

$$a_{k-1}<\sigma<a_k \qquad \textbf{(48)}$$

where,

$$N_k(\sigma)=\frac{\Gamma\gamma\Gamma\rho\Gamma(\gamma-\lambda+\rho)}{\Gamma\lambda\quad\sigma^{1-\lambda}} g_k(\sigma) - \sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}} g'_{2i+1}(t)\,\{K(\sigma,t)+S(\sigma,t)\}dt \textbf{(49)}$$

Using equation (13) in equation (48) , we get

$$\int_{a_{k-1}}^{\sigma}\Psi'_k(t)\int_0^t m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\,dx\,dt + \int_{\sigma}^{a_k}\Psi'_k(t)\int_0^{\sigma} m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\,dx\,dt$$

$$= N_K(\sigma) - \sum_{i=0}^{\frac{k-3}{2}}\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t) .\int_0^t m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\,dx\,dt - \sum_{i=(k+1)/2}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t) .\int_0^{\sigma} m(x)(\sigma-x)^{\rho-1}(t-x)^{\gamma-\lambda+\rho-1}\,dxdt$$

$$-\sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t)S(\sigma,t)dt$$

Inverting the order of integration, we obtain

$$\int_{a_{k-1}}^{\sigma}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_x^{a_k}\frac{\Psi'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \int_0^{a_{k-1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_{a_{k-1}}^{a_k}\frac{\Psi'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} = N_K(\sigma)$$

$$-\sum_{i=0}^{\frac{k-3}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_x^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \int_0^{a_{2i+1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}}\right\} - \sum_{i=k+1/2}^{(n-2)/2}\left\{\int_0^{\sigma}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}}\right\} + \sum_{i=0}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t)S(\sigma,t)dt \textbf{(50)}$$

Assuming,

$$\int_x^{a_k}\frac{\Psi'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} = \overline{\Psi}_k(x) \qquad \textbf{(51)}$$

for allk $= 1,3,5,\ldots,\mathrm{n}-1$.

Now equation (50) can be written as,

$$\int_{a_{k-1}}^{\sigma}\frac{m(x)\overline{\Psi}_k(x)dx}{(\sigma-x)^{1-\rho}} = N_K(\sigma) - \int_0^{a_{k-1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}.\int_{a_{k-1}}^{a_k}\frac{\Psi'_k(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} - \sum_{i=0}^{\frac{k-3}{2}}\left\{\int_{a_{2i}}^{a_{2i+1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}.\int_x^{a_{2i+1}}\frac{\emptyset'_{2i+1}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} + \int_0^{a_{2i+1}}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}.\int_{a_{2i}}^{a_{2i+1}}\frac{\emptyset'_{2i+2}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}}\right\}$$

$$-\ \sum_{i=k+1/2}^{(n-2)/2}\int_0^{\sigma}\frac{m(x)dx}{(\sigma-x)^{1-\rho}}\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-x)^{1-\gamma+\lambda-\rho}} - \sum_{i=0}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t)S(\sigma,t)dt \textbf{(52)}$$

This is an Abel type integral equation and when $0<\rho<1$.then we can solve it as,

$$m(x)\overline{\Psi}_k(x) = C_k(x) - \frac{\sin(1-\rho)\pi}{\pi}\left[\int_0^{a_{k-1}}\frac{m(\xi)}{(\sigma-\xi)^{1-\rho}}d\xi .\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}\int_{a_{k-1}}^{a_k}\frac{\Psi'_k(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \sum_{i=0}^{(k-3)/2}\left\{\int_0^{a_{2i+1}} m(\xi)d\xi .\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \int_{a_{2i}}^{a_{2i+1}} m(\xi)d\xi .\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}\int_{\xi}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\right\} + \sum_{i=(k+1)/2}^{(n-2)/2}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}}\int_0^{\sigma}\frac{m(\xi)}{(\sigma-\xi)^{1-\rho}}d\xi\int_{a_{2i}}^{a_{2i+1}}\frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} - \sum_{i=0}^{(n-2)/2}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}}.\int_{a_{2i}}^{a_{2i+1}}\Psi'_{2i+1}(t)s(\sigma,t)dt\right] \textbf{(53)}$$

where,

$$C_k(x) = \frac{\sin(1-\rho)\pi}{\pi}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{N_K(\sigma)d\sigma}{(x-\sigma)^{\rho}} \qquad \textbf{(54)}$$

For all k = 1,3,5,……n-1.

Changing the order of integration in equation (53)

$$m(x)\overline{\Psi}_k(x) = C_k(x) - \frac{\sin(1-\rho)\pi}{\pi}\left[\int_0^{a_{k-1}} m(\xi)d\xi .\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}\int_{a_{k-1}}^{a_k}\frac{\Psi'_k(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \sum_{i=0}^{(k-3)/2}\left\{\int_0^{a_{2i+1}} m(\xi)d\xi .\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}}\right.\right.$$

$$\int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} + \int_{a_{2i}}^{a_{2i+1}} m(\xi)d\xi.$$

$$\frac{d}{dx}\int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{\xi}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\Bigg\} +$$

$$\sum_{i=(k+1)/2}^{(n-2)/2}\Bigg\{\int_0^{a_{k-1}} m(\xi)d\xi \frac{d}{dx}\int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}$$

$$+\frac{d}{dx}\int_{a_{k-1}}^{x} m(\xi)d\xi.\int_{\xi}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}(\sigma-\xi)^{1-\rho}} \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\Bigg\}$$

$$-\sum_{i=0}^{(n-2)/2} \frac{d}{dx}\int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}}.\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)s(\sigma,t)dt]$$

Using the equations (32) and (33) we get ,

$$m(x)\overline{\Psi}_k(x) = C_k(x) - \frac{\sin(1-\rho)\pi}{\pi(x-a_{k-1})^{\rho}}.$$

$$\Bigg[\int_0^{a_{k-1}} \frac{m(\xi)(a_{k-1}-\xi)^{\rho}}{(x-\xi)}d\xi\,.\int_{a_{k-1}}^{a_k} \frac{\Psi'_k(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} +$$

$$\sum_{i=0}^{(k-3)/2}\Big\{\int_0^{a_{2i+1}} \frac{m(\xi)(a_{k-1}-\xi)^{\rho}}{(x-\xi)}d\xi.\int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}$$

$$+\int_{a_{2i}}^{a_{2i+1}} \frac{m(\xi)(a_{k-1}-\xi)^{\rho}}{(x-\xi)}d\xi \int_{\xi}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}$$

$$+\sum_{i=(k+1)/2}^{(n-2)/2}\Bigg\{\int_0^{a_{k-1}} \frac{m(\xi)(a_{k-1}-\xi)^{\rho}}{(x-\xi)}d\xi \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}\Bigg\}\Bigg]$$

$$-\sum_{i=(k+1)/2}^{(n-2)/2}\Big\{\frac{d}{dx}\int_{a_{k-1}}^{x} m(\xi)d\xi \int_{a_{2i}}^{a_{2i+1}} \frac{\Psi'_{2i+1}(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}$$

$$-\frac{\sin(1-\rho)\pi}{\pi}\sum_{i=0}^{(n-2)/2} \frac{d}{dx}\int_{a_{k-1}}^{x} \frac{d\sigma}{(x-\sigma)^{\rho}}.\int_{a_{2i}}^{a_{2i+1}} \Psi'_{2i+1}(t)s(\sigma,t)dt]$$

(55)

Equation (51) is also an Abel type integral equation and its solution is given by

$$\Psi'_k(t) = \frac{-\sin(1-\gamma+\lambda-\rho)\pi}{\pi}\frac{d}{dt}\int_t^{a_k} \frac{\overline{\Psi}_k(x)dx}{(x-t)^{\gamma-\lambda+\rho}} \quad \textbf{(56)}$$

For all k= 1,3,5,……n-1.

Therefore,

$$\int_{a_{k-1}}^{a_k} \frac{\Psi'_k(t)dt}{(t-\xi)^{1-\gamma+\lambda-\rho}} = \frac{-\sin(1-\gamma+\lambda-\rho)\pi}{\pi(a_{k-1}-\xi)^{-\gamma+\lambda-\rho}} \int_{a_{k-1}}^{a_k} \frac{\overline{\Psi}_k(x)dx}{(x-\xi)(x-a_{k-1})^{\gamma-\lambda+\rho}}$$

For all k= 1,3,5,……n-1.**(57)**

Applying the equation (57) in equation (55) and also applying the Leibnitz theorem, we get

$$m(x)\overline{\Psi}_k(x) = C_k(x) - \frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}.$$

$$\Big[\int_0^{a_{k-1}} \frac{(a_{k-1}-\xi)^{2\rho+\gamma-\lambda}m(\xi)d\xi}{(x-\xi)}.\int_{a_{k-1}}^{a_k} \frac{\overline{\Psi}_k(y)dy}{(y-\xi)(y-a_{k-1})^{\gamma-\lambda+\rho}} +$$

$$\sum_{i=0}^{(k-3)/2}\Big\{-\frac{1}{(\gamma-\lambda+\rho)^{-1}}\int_{a_{2i}}^{a_{2i+1}} \frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-\xi)}d\xi\;.$$

$$\int_{\xi}^{a_{2i+1}} \frac{dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}.\int_{t}^{a_{2i+1}} \frac{\overline{\Psi}_{2i+1}(y)dy}{(y-t)^{1+\gamma-\lambda+\rho}} +$$

$$\int_0^{a_{2i+1}} \frac{(a_{k-1}-\xi)^{\rho}m(\xi)d\xi}{(x-\xi)(a_{2i}-\xi)^{-\gamma+\lambda-\rho}}.\int_{a_{2i}}^{a_{2i+1}} \frac{\overline{\Psi}_{2i+1}(y)dy}{(y-\xi)(y-a_{2i})^{\gamma-\lambda+\rho}}\Bigg\} +$$

$$\sum_{i=(k+1)/2}^{(n-2)/2}\left\{\int_0^{a_{k-1}} \frac{(a_{k-1}-\xi)^{\rho} m(\xi)d\xi}{(a_{2i}-\xi)^{-\gamma+\lambda-\rho}(x-\xi)}.\right.$$

$\int_{a_{2i}}^{a_{2i+1}} \frac{\Psi_{2i+1}(y)dy}{(y-\xi)(y-a_{2i})^{\gamma-\lambda+\rho}}\}]$ $-\sum_{i=(k+1)/2}^{\frac{n-2}{2}} \frac{\sin(1-\gamma+\lambda-\rho)\pi}{\pi}$.

$$\frac{d}{dx}\int_{a_{k-1}}^{x} \frac{m(\xi)d\xi}{(a_{2i}-\xi)^{-\gamma+\lambda-\rho}}.\int_{a_{2i}}^{a_{2i+1}}\frac{\overline{\Psi}_{2i+2}(y)dy}{(y-\xi)(y-a_{2i})^{\gamma-\lambda+\rho}}$$

$+\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(\gamma-\lambda+\rho)^{-1}}\sum_{i=0}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}}dt$

$$.\int_{t}^{a_{2i+1}}\frac{\overline{\Psi}_{2i+1}(y)dy}{(y-t)^{1+\gamma-\lambda+\rho}}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{s(\sigma,t)dt}{(x-\sigma)^{\rho}}.$$

or,

$$m(x)\overline{\Psi}_k(x)=C_k(x)-\int_{a_{k-1}}^{a_k}\overline{\Psi}_k(y)P_k(x,y)dy$$

$-\sum_{i=0}^{(k-3)/2}\int_{a_{2i}}^{a_{2i+1}}\overline{\Psi}_{2i+1}(y)Q_{2i+1}(x,y)dy$
$-\sum_{i=(k+1)/2}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}}\overline{\Psi}_{2i+1}(y)R_{2i+1}(x,y)dy$
$-\sum_{i=0}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}}\overline{\Psi}_{2i+1}(y)S_{2i+1}(x,y)dy$ **(58)**

For all k= 1,3,5,……n-1.
where,

$P_k(x,y)=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}.\frac{1}{(y-a_{k-1})^{\gamma-\lambda+\rho}}$
$.\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{2\rho+\gamma-\lambda}m(\xi)d\xi}{(x-\xi)(y-\xi)}$ **(59)**

$Q_{2i+1}(x,y)$
$=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(x-a_{k-1})^{\rho}}\Big[\frac{1}{(y-a_{2i})^{\gamma-\lambda+\rho}}.$
$.\int_0^{a_{2i+1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)(a_{2i}-\xi)^{\gamma-\lambda+\rho}d\xi}{(x-\xi)(y-\xi)}-(\gamma-\lambda+\rho).$
$.\int_{\xi}^{a_{2i+1}}\frac{dt}{(t-\xi)^{1-\gamma+\lambda-\rho}}.\int_t^{a_{2i+1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)}{(x-\xi)(y-t)^{1+\gamma-\lambda+\rho}}d\xi\Big]$ **(60)**

$R_{2i+1}(x,y)$
$=\frac{\sin(1-\gamma+\lambda-\rho)\pi}{\pi}\Big[\frac{\sin(1-\rho)\pi}{\pi(x-a_{k-1})^{\rho}}.\frac{1}{(y-a_{2i})^{\gamma-\lambda+\rho}}$
$.\int_0^{a_{k-1}}\frac{(a_{k-1}-\xi)^{\rho}m(\xi)(a_{2i}-\xi)^{\gamma-\lambda+\rho}d\xi}{(x-\xi)(y-\xi)}+\frac{1}{(y-a_{2i})^{\gamma-\lambda+\rho}}.$
$\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{m(\xi)d\xi}{(y-\xi)(a_{2i}-\xi)^{-\gamma+\lambda-\rho}}.\Big]$ **(61)**

$$S_{2i+1}(x,y)=\frac{\sin(1-\rho)\pi\sin(1-\gamma+\lambda-\rho)\pi}{\pi^2(\gamma-\lambda+\rho)^{-1}}.$$

$$.\int_{a_{2i}}^{y}\frac{dt}{(y-t)^{1+\gamma-\lambda+\rho}}\frac{d}{dx}\int_{a_{k-1}}^{x}\frac{s(\sigma,t)d\sigma}{(x-\sigma)^{\rho}}.\quad\textbf{(62)}$$

Substituting $k=1,3,5,\dots,n-1$. in equation (58) we will get n/2simultaneous Fredholm Integralequations of the second kind. With the help of these n/2 simultaneous equations we can calculate $\overline{\Psi}_2(x),\overline{\Psi}_4(x),\dots\dots.\overline{\Psi}_n(x)$ and Then the values of $\Psi`_2(t),\Psi`_4(t),\dots.\Psi`_n(t)$ can be determined. After all these calculations we cancompute the coefficient D_m with the help of equation (46).

5. Particular Cases

With the help of the result of these N-series equation it is easy to find the solution of corresponding dual ,triple, quadruple etc. series equations.

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