An Approach of Internal and Fuzzy System of Linear Equations

Dr. S. Chandrasekaran[1], Christal Darly C.S.[2]

[1]Head & Associate Professor of Mathematics, Khadir Mohideen College, Adirampattinam

[2]Research Scholar, Khadir Mohideen College, Adirampattinam

Abstract: ***The system of linear equation has a great importance in many real life problem such as economics, optimization and in various engineering field. The system of linear equation, in general is solved for crisp unknowns. For the sake of simplicity or for fuzzy computation it is taken as crisp value. In actual case the parameters of the system of linear equations are modeled by taking the experimental or observation data. So the parameters of the system actually contain uncertainty rather than the crisp one. The uncertainties may be considered in term of interval or fuzzy number. Recently different authors have investigated these problems by various methods. These methods are described for system having various types of fuzzy and non fuzzy parameters. Although solution are obtained by these methods are good but sometimes the method requires lengthy Procedure and computationally not efficient.***

Keywords: System of linear equation, fuzzy number, fuzzy parameters.

1. Introduction

In 1965, Lotfi Jadeh, a professor of electrical engineering at the University of California (Berkley), published the first of his Papers on his new theory of fuzzy sets and systems. Since the 1980s, the mathematical theory of "unsharp amount" has been applied with great succession in many different fields.

The literature on fuzzy arithmetic and its application, often contains critical remarks as, "the standard fuzzy arithmetic does not take into account of all the information available, and the obtained results are more imprecise than necessary or in some cases, even incorrect"

The concept of a fuzzy number arises from phenomena which can be described quantitatively. These phenomena do not lend themselves to being characterized.

System of linear equations has various applications. Equations of this type are necessary to solve for getting the involved parameters. It is simple and straight forward when the variables involving the system of equations are crisp number. But in actual case the system variables cannot be obtained as crisp.

Fuzzy solution of system of linear equation by known method :

The n X n linear systems of equations may be written as

$a_{11} x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = Y_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}X_n = Y_2$

$\cdot\ \cdot$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = Y_n$

$$\rightarrow (1)$$

Now if the system (1) is fuzzy, then i[th] equation of the system may be written as

$$a_i 1\, x_1 + \ldots + a_{ii}\, x_i + \ldots + a_{in}\, x_n = y_i$$

We have

$$a_{i1} x_1 + \ldots + a_{ii} x_i + \ldots + a_{in} x_n = Y_i\, \alpha$$

$$a_i 1x_1 + \ldots + a_{ii} x_i + \ldots + a_{in} x_n = Y_i\ \alpha \quad i \le i \le \alpha \rightarrow (2)$$

Where notation α is considered for α -cut of fuzzy number. From (2) we have two crisp n X n linear systems for all i that can be extended to a 2n x 2n crisp linear system as follows.

SX = Y

$$\begin{bmatrix} S \ge 0 & S_2 \le 0 \\ S_2 \le 0 & S_1 \ge 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} \rightarrow (3)$$

We have

$a_{i1}x_1 + \ldots + a_{ii}x_i + \ldots + a_{in}x_n = Y_i\ (\alpha)$

$a_{i1}x_1 + \ldots + a_{ii}x_i + \ldots + a_{in}x_n = Y_1\ (\alpha)$

$1 \le i \le \alpha$ - (2)

Where notation α is considered for α-cut of fuzzy number. From (2) we have two crisp n x n linear systems for all i that can be extended to a 2n x 2n crisp linear system as follows.

SX = Y

$$\Rightarrow \begin{bmatrix} S_1 \ge 0 & S_2 \le 0 \\ S_2 \le 0 & S_1 \ge 0 \end{bmatrix} \begin{bmatrix} X \\ X \end{bmatrix} = \begin{bmatrix} Y \\ Y \end{bmatrix} \text{--> (3)}$$

$$\left.\begin{aligned} & S_1 X + S_2 X \\ & S_2 X + S_1 X \\ & S_1 X - S_2 X = Y \\ & -S_2 X - S_1 X = Y \end{aligned}\right\} \text{--> 4}$$

Now let X = $\left(x_i(\alpha), x_i(\alpha) \right),\ 1 \le i \le n$

Denote the unique solution of SX = Y

The fuzzy number vector

U = $\left(u_i(\alpha), u_i(\alpha) \right),\ 1 \le i \le n$

defined by

$$u_i(\alpha) = \min\left(x_i(\alpha), x_i(\alpha), x_i(1) \right)$$

$\underline{u}_i(\alpha) = \min\left(\underline{x}_i(\alpha), \overline{x}_i(\alpha), \underline{x}_i(1)\right)$ Is the fuzzy solution of $\tilde{S}\tilde{X} = \tilde{Y}$. If $(\underline{x}_i(\alpha), \overline{x}_i(\alpha)), 1 \le i \le n$ are all triangular fuzzy number then

$$\underline{u}_i(\alpha) = \underline{x}_i(\alpha)$$
$$\overline{u}_i(\alpha) = \overline{x}_i(\alpha) \qquad 1 \le i \le n$$

And U is called a strong fuzzy solution. Otherwise, U is a weak fuzzy solution.

First Method

Let the system of equation be :

$$(\underline{a}_{11}, a_{11}, \overline{a}_{11})(\underline{x}_1, x_1, \overline{x}_1) + (\underline{a}_{12}, a_{12}, \overline{a}_{12})(\underline{x}_2, x_2, \overline{x}_2) + \ldots + (\underline{a}_{1n}, a_{1n}, \overline{a}_{1n})(\underline{x}_n, x_n, \overline{x}_n) = (\underline{r}_1, r_1, \overline{r}_1)$$
$$(\underline{a}_{21}, a_{21}, \overline{a}_{21})(\underline{x}_1, x_1, \overline{x}_1) + (\underline{a}_{22}, a_{22}, \overline{a}_{22})(\underline{x}_2, x_2, \overline{x}_2) + \ldots + (\underline{a}_{2n}, a_{2n}, \overline{a}_{2n})(\underline{x}_n, x_n, \overline{x}_n) = (\underline{r}_2, r_2, \overline{r}_2)$$
$$\vdots$$
$$(\underline{a}_{n1}, a_{n1}, \overline{a}_{n1})(\underline{x}_1, x_1, \overline{x}_1) + (\underline{a}_{n2}, a_{n2}, \overline{a}_{n2})(\underline{x}_2, x_2, \overline{x}_2) + \ldots + (\underline{a}_{nn}, a_{nn}, \overline{a}_{nn})(\underline{x}_n, x_n, \overline{x}_n) = (\underline{r}_n, r_n, \overline{r}_n)$$

where all a_{ij} are in R^+, the above equation may be written equivalently as

$$\underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 + \ldots + \underline{a}_{1n}\underline{x}_n = \underline{r}_1$$
$$\overline{a}_{11}\overline{x}_1 + \overline{a}_{12}\overline{x}_2 + \ldots + \overline{a}_{1n}\overline{x}_n = \overline{r}_1$$

$$\underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 + \ldots + \underline{a}_{2n}\underline{x}_n = \underline{r}_2$$
$$\overline{a}_{21}\overline{x}_1 + \overline{a}_{22}\overline{x}_2 + \ldots + \overline{a}_{2n}\overline{x}_n = \overline{r}_2 \quad \vdots$$

$$\underline{a}_{n1}\underline{x}_1 + \underline{a}_{n2}\underline{x}_2 + \ldots + \underline{a}_{nn}\underline{x}_n = \underline{r}_n$$
$$\overline{a}_{n1}\overline{x}_1 + \overline{a}_{n2}\overline{x}_2 + \ldots + \overline{a}_{nn}\overline{x}_n = \overline{r}_n$$

--> (6)

equation (6) can now be written in matrix form as:

$$\begin{bmatrix} \underline{a}_{11} & \overline{a}_{11} & \cdots & \underline{a}_{1n} & \overline{a}_{1n} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & a_{11} & a_{12} & & & a_{1n} \\ \underline{a}_{21} & \overline{a}_{21} & \cdots & \underline{a}_{2n} & \overline{a}_{2n} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & a_{21} & a_{22} & & & a_{2n} \\ \vdots & & & & & & & & \vdots \\ \underline{a}_{n1} & \overline{a}_{n1} & \cdots & \underline{a}_{nn} & \overline{a}_{nn} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & a_{n1} & a_{n2} & & & a_{nn} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \overline{x}_1 \\ x_1 \\ \underline{x}_2 \\ \overline{x}_2 \\ x_2 \\ \vdots \\ \underline{x}_n \\ \overline{x}_n \\ x_n \end{bmatrix} = \begin{bmatrix} \underline{r}_1 \\ \overline{r}_1 \\ r_1 \\ \underline{r}_2 \\ \overline{r}_2 \\ r_2 \\ \vdots \\ \underline{r}_n \\ \overline{r}_n \\ r_n \end{bmatrix}$$

---> (7)

For clear understanding we now give the procedure with 2 equation and 2 unknowns. So for 2 equations and 2 unknowns we have

$$(\underline{a}_{11}, a_{11}, \overline{a}_{11})(\underline{x}_1, x_1, \overline{x}_1) + (\underline{a}_{12}, a_{12}, \overline{a}_{12})(\underline{x}_2, x_2, \overline{x}_2) = (\underline{r}_1, r_1, \overline{r}_1) \quad \text{-> (8)}$$
$$(\underline{a}_{21}, a_{21}, \overline{a}_{21})(\underline{x}_1, x_1, \overline{x}_1) + (\underline{a}_{22}, a_{22}, \overline{a}_{22})(\underline{x}_2, x_2, \overline{x}_2) = (\underline{r}_2, r_2, \overline{r}_2) \quad \text{-> (9)}$$

Similar to equation (7) one may write (8) and (9) as

$$\begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} & \overline{a}_{11} & \overline{a}_{12} \\ 0 & 0 & a_{11} & a_{12} \\ \underline{a}_{21} & \underline{a}_{22} & \overline{a}_{21} & \overline{a}_{22} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \overline{x}_1 \\ x_1 \\ \underline{x}_2 \\ \overline{x}_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} \underline{r}_1 \\ \overline{r}_1 \\ r_1 \\ \underline{r}_2 \\ \overline{r}_2 \\ r_2 \end{bmatrix}$$

This is a crisp system of equation and the above matrix equation may easily be solved now as below.

From equation (8) we have

$$\underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 = \underline{r}_1 \text{ -> (10)}$$
$$a_{11}x_1 + a_{12}x_2 = r_1 \text{ -> (11)}$$

from equation (0) we have

$$\underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 = \underline{r}_2 \text{ -> (12)}$$
$$a_{21}x_1 + a_{22}x_2 = r_2 \text{ -> (13)}$$

Solving (10) and (12) we have

$$\underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 = \underline{r}_1$$
$$\underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 = \underline{r}_2$$

We have

$$\Delta = \begin{vmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{vmatrix}$$
$$\Delta = \underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}$$
$$\Delta_{\underline{x}_1} = \begin{vmatrix} \underline{r}_1 & \underline{a}_{12} \\ \underline{r}_2 & \underline{a}_{22} \end{vmatrix}$$
$$\Delta_{\underline{x}_1} = \underline{a}_{22}\underline{r}_1 - \underline{a}_{12}\underline{r}_2$$
$$\Delta_{\underline{x}_2} = \begin{vmatrix} \underline{a}_{11} & \underline{r}_1 \\ \underline{a}_{21} & \underline{r}_2 \end{vmatrix}$$
$$\Delta_{\underline{x}_2} = \underline{a}_{11}\underline{r}_2 - \underline{a}_{21}\underline{r}_1$$

since $\Delta \neq 0$, it has unique solution.

$$\Delta = \underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}$$
$$\Delta_{\underline{x}_1} = \underline{a}_{22}\underline{r}_1 - \underline{a}_{12}\underline{r}_2$$
$$\Delta_{\underline{x}_2} = \underline{a}_{11}\underline{r}_2 - \underline{a}_{21}\underline{r}_1$$

By Cramer's rule

$$\underline{x}_1 = \frac{\Delta_{\underline{x}_1}}{\Delta} = \frac{\underline{a}_{22}\underline{r}_1 - \underline{a}_{12}\underline{r}_2}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$
$$\underline{x}_2 = \frac{\Delta_{\underline{x}_2}}{\Delta} = \frac{\underline{a}_{11}\underline{r}_2 - \underline{a}_{21}\underline{r}_1}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$

solving (11) and (13) we get

$$a_{11}x_1 + a_{12}x_2 = r_1 \text{ -> (11)}$$
$$a_{21}x_1 + a_{22}x_2 = r_2 \text{ -> (13)}$$

we have

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$
$$\Delta = a_{11}a_{22} - a_{21}a_{12}$$
$$\Delta_{x_1} = \begin{vmatrix} r_1 & a_{12} \\ r_2 & a_{22} \end{vmatrix}$$
$$\Delta_{x_1} = a_{22}r_1 - a_{12}r_1$$
$$\Delta_{x_2} = \begin{vmatrix} a_{11} & r_1 \\ a_{21} & r_2 \end{vmatrix}$$
$$\Delta_{x_2} = a_{11}r_2 - a_{21}r_1$$

since $\Delta \neq 0$, it has unique solution.

By Cramer's rule

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{\ldots}{a_{11}a_{22} - a_{21}a_{12}}$$

$$x_2 = \frac{\Delta_2}{\Delta} = \frac{\ldots}{a_{11}a_{22} - a_{21}a_{12}}$$

Therefore,

$$\underline{x}_1 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\underline{x}_2 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\overline{x}_1 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\overline{x}_2 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

Second method

In this method equation (8) and (9) are first written taking.

Left

$$a_{11}\underline{x}_1 + a_{12}\underline{x}_2 = \underline{r}_1$$
$$a_{21}\underline{x}_1 + a_{22}\underline{x}_2 = \underline{r}_2$$

Right :

$$a_{11}x_1 + a_{12}x_2 = r_1$$

$$a_{21}x_1 + a_{22}x_2 = r_2$$

By using Cramer's rule we have

$$\underline{x}_1 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\underline{x}_2 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\overline{x}_1 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\overline{x}_2 = \frac{\ldots}{a_{11}a_{22} - a_{12}a_{21}}$$

Although methods 1 and 2 are same but this is shown in the above methods that we may solve for the left and right individually too.

Third Method

Here we first consider the equations (10) and (12).

$$a_{11}\underline{x}_1 + a_{12}\underline{x}_2 = \underline{r}_1$$
$$a_{21}\underline{x}_1 + a_{22}\underline{x}_2 = \underline{r}_2$$

write the above as

AX = b

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \begin{pmatrix} \underline{r}_1 \\ \underline{r}_2 \end{pmatrix} \quad \text{--> (14)}$$

from (14) we may have (if the inverse of the coefficient matrix exists)

Here

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$|A| = a_{11}a_{22} - a_{12}a_{21} \neq 0$$

Since A is non-singular, A^{-1} exists.

We know that $A^{-1} = \frac{1}{|A|}\, adj\, A$

$$\left(A_{ij}\right) = \begin{pmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{pmatrix}$$

$$\text{adj A} = \left(A_{ij}\right)' = \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

The solution is $X = A^{-1}B$

$$\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}\begin{pmatrix} \underline{r}_1 \\ \underline{r}_2 \end{pmatrix}$$

$$= \frac{1}{a_{11}a_{22} - a_{12}a_{21}}\begin{pmatrix} a_{22}\underline{r}_1 - a_{12}\underline{r}_2 \\ -a_{21}\underline{r}_1 + a_{11}\underline{r}_2 \end{pmatrix}$$

$$\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \begin{bmatrix} \dfrac{\ldots}{a_{11}a_{22} - a_{12}a_{21}} \\ \dfrac{\ldots}{a_{11}a_{22} - a_{12}a_{21}} \end{bmatrix} \quad \text{--> (15)}$$

following the same way equation (11) and (13) may be used to get the solution.

$$\begin{pmatrix} \overline{x}_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} \dfrac{\ldots}{a_{11}a_{22} - a_{12}a_{21}} \\ \dfrac{\ldots}{a_{11}a_{22} - a_{12}a_{21}} \end{bmatrix} \quad \text{--> (16)}$$

2. Numerical Example

In this problem taking three crisp equations with three unknowns

$$0.4\,\tilde{x}_1 + 1.4\,\tilde{x}_2 + 0.3\,\tilde{x}_3 = 0.1$$
$$0.15\,\tilde{x}_1 + 0.14\,\tilde{x}_2 + 6.1\,\tilde{x}_3 = 0.14$$
$$5.1\,\tilde{x}_1 + 0.3\,\tilde{x}_2 + 0.2\,\tilde{x}_3 = 0.14$$

These equations we can write as

$$[0.4, 0.4]\,\tilde{x}_1 + [1.4, 1.4]\,\tilde{x}_2 + [0.3, 0.3]\,\tilde{x}_3 = [0.1, 0.1]$$

$$0.15, 0.15\ \underline{x}_1, x_1 + 0.14, 0.14\ \underline{x}_2, x_2 + 6.1, 6.1\ \underline{x}_3, x_3 = 0.14, 0.14$$
$$5.1, 5.1\ \underline{x}_1, x_1 + 0.3, 0.3\ \underline{x}_2, x_2 + 0.2, 0.2\ \underline{x}_3, x_3 = 0.14, 0.14$$

1. First Method

$$\begin{bmatrix} 0.4 & 1.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.14 & 6.1 \\ 5.1 & 0.3 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.1 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.4 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 \\ 5.1 & 0.3 & 0.2 \end{vmatrix}$$

= 0.4 (0.028 – 1.83) – 1.4(0.03 – 31.11)
+ 0.3(0.0450-0.7140)
= -0.72 + 43.512 – 0.2007
= 42.5913

$$\Delta_{\underline{x}_1} = \begin{vmatrix} 0.1 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 \\ 0.14 & 0.3 & 0.2 \end{vmatrix}$$

= 0.1 (0.0280-1.83) – 1.4 (0.0280-0.854)
+ 0.3(0.0420-0.0196)
= -0.1802 + 1.1564 + 0.0067
= 0.9829

$$\Delta_{\underline{x}_2} = \begin{vmatrix} 0.4 & 0.1 & 0.3 \\ 0.15 & 0.14 & 6.1 \\ 5.1 & 0.14 & 0.2 \end{vmatrix}$$

= 0.4(0.0280 – 0.8540) – 0.1(0.03-3.11)
+ 0.3 (0.0210-0.7140)
= -0.3304 + 3.1 – 0.2079
= 2.5617

$$\Delta_{\underline{x}_2} = \begin{vmatrix} 0.4 & 0.1 & 0.3 \\ 0.15 & 0.14 & 6.1 \\ 5.1 & 0.14 & 0.2 \end{vmatrix}$$

= 0.4(0.0280 – 0.8540) – 0.1 (0.03-3.11)
+ 0.3(0.0210-0.7140)
= -0.3304 + 3.1 – 0.2079
= 2.5617

$$\Delta_{\underline{x}_3} = \begin{vmatrix} 0.4 & 1.4 & 0.1 \\ 0.15 & 0.14 & 0.14 \\ 5.1 & 0.3 & 0.14 \end{vmatrix}$$

= 0.4 (0.0196 – 0.0420) – 1.4 (0.0210 – 0.7140)
+ 0.1 (0.0450-0.7140)
= 0.0196 + 0.9702 – 0.0669
= 0.9229

$$\underline{x}_1 = \frac{\Delta_{\underline{x}_1}}{\Delta} = \frac{0.9829}{42.5913} = 0.0231$$

$$\underline{x}_2 = \frac{\Delta_{\underline{x}_2}}{\Delta} = \frac{2.5617}{42.5913} = 0.0603$$

$$\underline{x}_3 = \frac{\Delta_{\underline{x}_3}}{\Delta} = \frac{0.9229}{42.5913} = 0.0210$$

$$\underline{x}_1 = 0.0231$$
$$\underline{x}_2 = 0.0603$$
$$\underline{x}_3 = 0.0210$$

Similarly,

$$x_1 = 0.0231$$
$$x_2 = 0.0603$$
$$x_3 = 0.0210$$

2. Second Method

$$0.4\ \underline{x}_1 + 1.4\ \underline{x}_2 + 0.3\ \underline{x}_3 = 0.1$$
$$0.4\ x_1 + 1.4\ x_2 + 0.3\ x_3 = 0.1$$
$$0.15\ \underline{x}_1 + 0.14\ \underline{x}_2 + 6.1\ \underline{x}_3 = 0.14$$
$$0.15\ x_1 + 0.14\ x_2 + 6.1\ x_3 = 0.14$$
$$5.1\ \underline{x}_1 + 0.3\ \underline{x}_2 + 0.2\ \underline{x}_2 = 0.14$$
$$5.1\ x_1 + 0.3\ x_2 + 0.2\ x_2 = 0.14$$
$$\underline{x}_1 = 0.0231$$
$$\underline{x}_2 = 0.0603$$
$$\underline{x}_3 = 0.0210$$
$$x_1 = 0.0231$$
$$x_2 = 0.0603$$
$$x_3 = 0.0210$$

3. Third Method

AX = b

$$A = \begin{bmatrix} 0.4 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 \\ 5.1 & 0.3 & 0.2 \end{bmatrix} \quad X = \begin{bmatrix} 0.1 \\ 0.14 \\ 0.14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\ A$$

|A| = 42.5913

$$adjA = \begin{bmatrix} -1.8020 & 31.08 & -0.6690 \\ -0.19 & -1.45 & 7.02 \\ 8.498 & -2.395 & -0.1540 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1.8020 & -0.19 & 8.498 \\ 31.08 & -1.45 & -2.395 \\ -0.6690 & 7.02 & -0.1540 \end{bmatrix}$$

$$A^{-1} = \frac{1}{42.5913} \begin{bmatrix} -1.8020 & -0.19 & 8.498 \\ 31.08 & -1.45 & -2.395 \\ -0.6690 & 7.02 & -0.1540 \end{bmatrix}$$

$$= \frac{1}{42.5913} \begin{bmatrix} -1.8020 & -0.19 & 8.498 \\ 31.08 & -1.45 & -2.395 \\ -0.6690 & 7.02 & -0.1540 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.14 \\ 0.14 \end{bmatrix}$$

$$= \frac{1}{42.5913}\begin{bmatrix} -0.1802 & -0.0266 & +1.1897 \\ 3.1080 & -0.2030 & -0.3353 \\ -0.0669 & +0.9828 & -0.0216 \end{bmatrix}$$

$$= \frac{1}{42.5913}\begin{bmatrix} 0.9829 \\ 2.5697 \\ 0.8943 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.0231 \\ 0.0603 \\ 0.0210 \end{bmatrix}$$

Similarly,

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.0231 \\ 0.0603 \\ 0.0210 \end{bmatrix}$$

3. Conclusion

Present work demonstrates a new method for interval and fuzzy solution of fuzzy system of linear equations. This is applied first in a known problem of circuit analysis. As discussed earlier, the concepts of fuzzy number (that is, triangular fuzzy number, trapezoidal fuzzy number), α-cut, have been used here to solve the numerical problems of system of linear equations. Few other example problems are also solved to have the efficiency of the proposed method.

Three different cases are considered in the above circuit analysis problem taking source and resistance as:
Case I : Crisp
Case II : Interval
Case III : Fuzzy.
Where, current is taken as fuzzy in case I and case III and crisp in case II

As mentioned above the other example problems are solved with interval, triangular fuzzy number and trapezoidal fuzzy number to have the reliability and powerfulness of the proposed methods.

The investigation gives a new idea of solving the interval / fuzzy system of linear equations with simple computations.

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