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Some Results on *I*-cordial Graph

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Abstract: ***An I-cordial labeling of a graph G (V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, respectively be an injective mapping such that f(u) + f(v) ≠ 0 and induces an edge labeling f^*: E → {0, 1} where, $f^*(uv)$ = 1 if f(u) + f(v) > 0 and $f^*(uv)$ = 0 otherwise, such that the number of edges labeled with1and the number of edges labeled with 0 differ atmost by 1. If a graph has I-cordial labeling, then it is called I-cordial graph. In this paper, we introduce the concept of I-cordial labeling and prove that some standard graphs that are I-cordial and some graph that are not I-cordial.***

Notation: Here [-x..x] = {t ⁄ t is an integer and |t| ≤ x} and $[-x..x]^*$ = [-x..x] - {0}.

Keywords: Cordial labeling; *I*-cordial labeling

AMS Mathematical Subject Classification (2010): 05C78

1. Introduction

By a graph we mean a finite undirected graph without loops and multiple edges. For terms not defined here we refer to Harary [9].

An *I*-cordial labeling of a graph G (V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, respectively, be an injective mapping such that f(u) + f(v) ≠ 0 and induces an edge labeling f^*: E → {0, 1} where, $f^*(uv)$ = 1 if f(u) + f(v) > 0 and $f^*(uv)$ = 0 otherwise such that the number of edges labeled with 1 and the number of edges labeled with 0 differ at most by 1. If a graph has *I*-cordial labeling, then it is called ***I*-cordial graph**. The concept of cordial graph originated from I.Cahit [1,2] in 1987 as a weaker version of graceful and harmonious graphs and was based on {0,1} binary labeling of vertices.

Let f: V → {0, 1} be a mapping that induces an edge labeling $\bar{f}$: E → {0, 1} defined by $\bar{f}(uv) = |f(u) - f(v)|$. Cahit called such a labeling cordial if the following condition is satisfied: $|v_f(0) - v_f(1)| \le 1$and $|e_f(0) - e_f(1)| \le 1$,where $v_f(i)$ and $e_f(i)$, i = 0, 1 are the number of vertices and edges of G respectively with label i (under f and $\bar{f}$ respectively). A graph G is called cordial if it admits cordial labeling.

In [1], Cahit showed that (i) every tree is cordial (ii) K_n is cordial if and only if n ≤ 3 (iii) $K_{r,s}$ is cordial for all r and s (iv) the wheel W_n is cordial if and only if n ≡ 3(mod4) (v) C_n is cordial if and only if n ≢2(mod4) (vi) an Eulerian graph is not cordial if its size is congruent to 2 modulo 4.

Du [4] investigated cordial complete k-partite graphs. Kuo et al. [13] determined all m and n for which mK_n is cordial. Lee et al. [14] exhibited some cordial graphs. Generalised Peterson graphs that are cordial are characterised in [7]. Ho et.al [6] investigated the construction of cordial graphs using Cartesian products and composition of graphs. Shee and Ho [7] determined the cordiality of $C_m^{(n)}$; the one–point union of n copies of C_m. Several constructions of cordial graphs were proposed in [10-12, 15-18]. Other results and open problems concerning cordial graph are seen in [2, 5]. Other types of cordial graphs were considered in [3, 4, 8, 20]. Vaidya et.al [21] has also discussed the cordiality of various graphs.

Definition 1.1 [23]
Let f be a binary edge labeling of graph G = {V, E}and the induced vertex labeling is given by$f(v) = \sum_{\forall u} f(u,v) (\mathrm{mod} 2)$ where v ϵ V and {u,v}ϵ E. f is called an **E-cordial labeling** of G if$|e_f(0) - e_f(1)| \le 1$ and $|v_f(0) - v_f(1)| \le 1$, where $e_f(0)$ and $e_f(1)$ denote the number of edges, and $v_f(0)$ and $v_f(1)$ denote the number of vertices with 0's and 1's respectively. The graph G is called **E-cordial** if it admits E-cordial labeling.

In 1997 Yilmaz and Cahit [23] have introduced E-cordial labeling as a weaker version of edge–graceful labeling. They proved that the trees with n vertices, K_n, C_n are E-cordial if and only if n≢2 (mod4) while $K_{m,n}$ admits E-cordial labeling if and only if m + n ≢ 2(mod4).

Definition 1.2 [20]
A **prime cordial labeling** of a graph G with vertex set V is a bijection f from V to {1, 2, 3, . . . , |V|} where each edge uv is assigned the label 1 if gcd (f(u), f(v)) = 1 and 0 if gcd (f(u), f(v)) > 1, such that the number of edges having label 0 and edges having label 1differ by at most 1.

Sundaram et.al. [19] introduced the notion of prime cordial labeling. They proved the following graphs are prime cordial: C_n if and only if n ≥ 6; P_n if and only if n ≠ 3 or 5; $K_{1,n}$(n, odd); the graph obtained by subdividing each edge of $K_{1,n}$ if and only if n ≥ 3; bi-stars; dragons; crowns; triangular snakes if and only if the snake has at least three triangles; ladders. J. Babujee and L.Shobana [22] proved the existence of prime cordial labeling for sun graph, kite graph and coconut tree and Y-tree, < $K_{1,n}$: 2 > (n ≥ 1); Hoffman tree, and $K_2 \Theta C_n$ (C_n)

In this paper we introduce the concept of integer *I*-cordial labeling and we prove that some standard graphs such as cycle C_n, Path P_n, Friendship F_n, Helm graph H_n, Closed graph CH_n, Double Fan DF_n, $n \geq 2$, are *I*-cordial; Wheel W_n and Fan graph f_n are *I*-cordial if and only if n is even and complete graph K_p is not *I*-cordial.

Notation.1.3 Here $[-x..x] = \{t / t$ is an integer and $|t| \leq x\}$ and $[-x..x]^* = [-x..x] - \{0\}$.

2. Main Results

Definition.2.1
Let G = (V,E) be a simple connected graph with p vertices. Let f: V $\rightarrow \left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd respectively be an injective mapping such that $f(u) + f(v) \neq 0$ and induces an edge labeling f^*: E $\rightarrow$ {0, 1} where f(uv) = 1, if f(u) + f(v) > 0 and f(uv) = 0 otherwise. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. f is said to be ***I*-cordial** if $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *I*-cordial if it admits a ***I*-cordial labeling**.

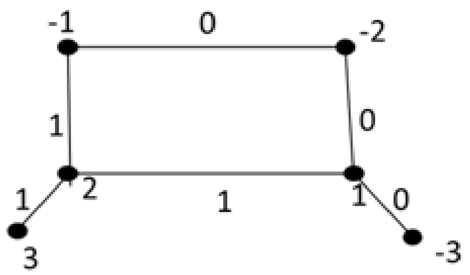

Figure 1: *I*-cordial Graph

Theorem 2.2 The cycle C_p is *I*-cordial.

***Proof*.** Let $v_1, v_2, \ldots v_p$ be the p vertices of the cycle C_p. Here q = p.

CASE 1.p is even.
Let p = 2n.We define f : V $\rightarrow[-n \ldots n]^*$as follows:$f(v_i) = -$ I ;$1 \leq i \leq n$; $f(v_{n+i}) = i$; $1 \leq i \leq n$

When $f(v_n) = -n$ and $f(v_{n+1}) = 1$, the edge labeling $f^*(v_n\, v_{n+1}) = -n + 1$.

This implies that $f^*(v_n\, v_{n+1}) < 0$.

Similarly, $f(v_1) = -1$ and $f(v_p) = n$ yield $f^*(v_1v_p) = n - 1$, which is positive when $n \geq 1$.

Therefore, we assign $f^*(v_1v_p) = 1$.Obviously, the sum of consecutive negative (positive) integers is negative (positive). As there are $\frac{q}{2}$ such negative labels and $\frac{q}{2}$ positive labels, $e_f(0) = e_f(1) = \frac{q}{2}$.

CASE 2. p is odd and p > 3.
Let p = 2n + 1, when n > 1.We define f : V $\rightarrow[-n \ldots n]$ as $f(v_i) = -i$, $1 \leq i \leq n - 1$; $f(v_{n-1+i}) = i + 1$,$1 \leq i \leq n - 1$and $f(v_n) = 0$

Let us consider the edge $v_n\, v_{n+1} \in E$ then $f(v_n) = -(n - 1)$ and $f(v_n) = 1$

That is, $f(v_n) + f(v_{n+1}) < 0$,which implies $f^*(v_n\, v_{n+1}) = 0$.
Similarly, for the edge, v_nv_1, $f^*(v_nv_1) = -1 + 0 = -1$, so that $f^*(v_nv_1)$ receives label 0. From the observation n + 1edges receive label 0 and n edges receive label 1.

Therefore, $e_f(0) = n + 1$ and $e_f(1) = n$. Thus $|e_f(0) - e_f(1)| = 1$.

The case when p = 3 does not yield any I-cordial labeling for C_3 by Theorem 2.3.

Thus C_p is *I*-cordial graph.

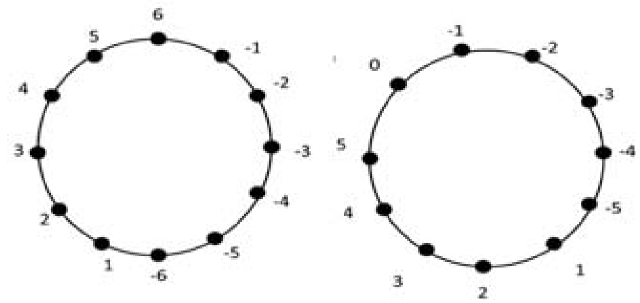

Figure 2: C_{12} and C_{11} are *I*-cordial graph.

Theorem 2.5 Path P_n n > 2 is *I*-cordial.

Proof. When n = 3, we label {-1, 0, 1} corresponding to the vertices $\{v_1, v_2, v_3\}$ which implies P_3 is I-cordial.

For the case n > 3, we labeling is similar to Theorem 2.4

Theorem2.6 Complete graph K_p is not *I*-cordial.
Proof holds from Theorem 2.3 and 2.4.

Theorem 2.7 The Wheel graph W_n ; n > 3 is *I*-cordial if and only if n is even.

Proof. Let u be the apex vertex and $v_1, v_2, \ldots, v_n$ be the rim vertices. Here |V| = n + 1 and |E| = 2n. Let us consider two cases.

Case 1. |V| is odd. (n is even)
Let n = 2m.We define f : V $\rightarrow$ [-m . . m] as f(u) = 0; $f(v_i) = -i$, $1 \leq i \leq m$ and
$f(v_{m+i}) = i$, $1 \leq i \leq m$.

Consider the vertex label $f(v_m) = -n$ and $f(v_{m+1}) = 1$ then, $f^*(v_mv_{m+1}) < 0$. Also $f^*(v_iv_{i+1}) < 0$ for all i = 1, 2, . . ., m and $f^*(v_i\, v_{i+1}) > 0$ for all i = m+1, . . ., 2m – 1. Also, $f(v_{2m}) = n$ and $f(v_1) = -1$.

Therefore, $f^*(v_{2m}v_1) > 0$.Now $f^*(uv_i) < 0$ for all i = 1, 2,. . .,m and $f^*(uv_i) > 0$ for all i = m + 1, m+2,. . .,2m. Hence, the q edges equally share label 0 and 1.That is, $e_f(0) = e_f(1) = \frac{q}{2}$ which imply $|e_f(0) - e_f(1)| = 1$. Thus from both the cases$|e_f(0) - e_f(1)| \leq 1$.

Hence W_n ; n > 3 is I-cordial.

Case 2.|V| is even
That is, when n is odd, by Theorem 2.4,W_n ; $n \geq 3$ is not *I*-cordial.

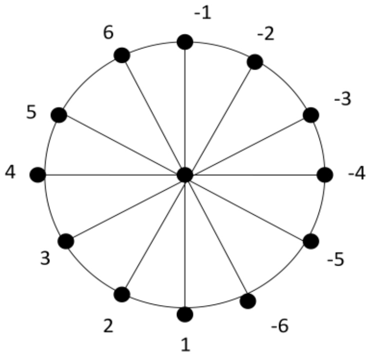

Figure 3: W_6 is *I*-cordial

Theorem 2.8 Helm graph H_n is *I*-cordial.

Proof. Let $H_n = G$, then $p = 2n + 1$ and $q = 3n$. Let v be the apex vertex, $v_1, v_2, \ldots, v_n$ be the rim vertices of the cycle and $u_1, u_2, \ldots, u_n$ be the pendant vertices corresponding to v_i' s.

CASE 1.n is odd.
Let $n = 2m + 1$.We define $f : V \rightarrow [-(2m + 1)..(2m + 1)]$ as$f(v) = 0$; $f(v_i)= -i$, $1 \leq i \leq m + 1$; $f(v_{(m+1+i)}) = i$, $1 \leq i \leq m$; $f(u_i) = -(m + 1 + i)$, $1 \leq i \leq m$ and $f(u_{m+i}) = (m + 1 - i)$, $1 \leq i \leq m$.

Let us consider the vertices $v_{m+1}$and v_{m+2}. We have $f(v_{m+1}) = -(m + 1)$ and $f(v_{m+1}) = 1$.Then $f^*(v_{m+1}v_{m+2}) < 0$. Also, $f^*(v_iv_{i+1}) < 0$, for all $i = 1, 2, 3, \ldots, m$. Similarly, consider the vertices v_1 and v_{2m+1}. Let $f(v_{2m+1}) = m$ and $f(v_1) = -1$, which implies, $f^*(v_{2m+1}v_1) > 0$. Also, $f^*(v_iv_{i+1}) > 0$, for all $i = m + 2, \ldots, 2m$. Thus $|e_f(0) - e_f(1)| = 1$.

Now let us consider the pendant vertices u_i's. Here $f^*(v_iu_i) < 0$ for all $i = 1, 2, \ldots m$ and $f^*(v_iu_i) > 0$ for all $i = m + 2, \ldots, 2m + 1$. Also, the vertex, $f(v_{m+1}) = -(m + 1)$ and $f(u_{2m+1}) = 2m + 1$. Thus, $f^*(v_{m+1}u_{2m+1}) > 0$. Here$|e_f(0) - e_f(1)| = 1$. Now, $f^*(vv_i) < 0$, for all $i = 1, 2, \ldots, m + 1$ and $f^*(vv_i) > 0$, for all $i = m + 2, \ldots, 2m + 1$ which implies, $|e_f(0) - e_f(1)| = 1$. From all the cases, $|e_f(0) - e_f(1)| \leq 1$.

CASE 2.n is even.
Let $n = 2m$.We define $f : V \rightarrow [-(2m + 1) \ldots (2m + 1)]$ as $f(v) = 0$; $f(v_i) = -i, 1 \leq i \leq m$; $f(v_{m+i}) = i, 1 \leq i \leq m$; $f(u_i) = -(m + i), 1 \leq i \leq m$ and $f(u_{m+i}) = (m + i), 1 \leq i \leq m$.

Since the apex vertex v, is labeled with 0, $f^*(vv_i) < 0$, for all $i = 1, 2, \ldots, m$ and $f^*(vv_i) > 0$, for all $i = m + 1, m + 2, \ldots, 2m$. Here $|e_f(0) - e_f(1)| = 0$.

Consider the vertex v_m and v_{m+1}. Now $f(v_m) = -m$ and $f(v_{m+1}) = 1$. Thus $f^*(v_mv_{m+1}) = -m + 1 < 0$. Also, $f^*(v_iv_{i+1}) < 0$ for all $i = 1, 2, \ldots, m$ and $f^*(v_iv_{i+1}) > 0$ for all $i = m + 1, \ldots, 2m - 1$. Also $f^*(v_{2m}v_1) = m - 1 > 0$. Here $|e_f(0) - e_f(1)| = 0$. Now, $f^*(v_iu_i) < 0$ for all $i = 1, 2, \ldots, m$ and $f^*(v_{i+1}u_{i+1}) > 0$ for all i $m + 1, m + 2, \ldots 2m$. Thus $|e_f(0) - e_f(1)| = 0$.
Hence, H_n is *I*-cordial.

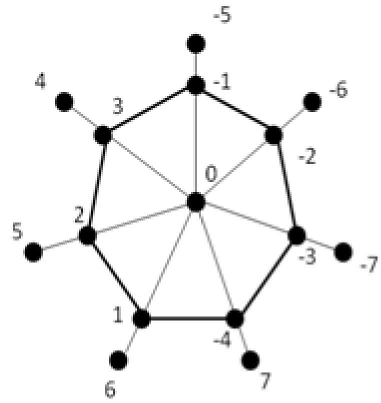

Figure 4: H_7 is *I*-cordial

Theorem 2.9 The closed helm graph CH_n is *I*-cordial.

Proof. Let $CH_n = G$. Then $p = 2n + 1$ and $q = 4n$. Let v be the apex vertex, $v_1, v_2, \ldots v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots u_n$ be the rim vertices of the outer cycle. The case when n is even follows from Theorem 2.7. Now suppose n is odd and $n = 2m+1$.

We define $f : V \rightarrow [-(2m + 1)..(2m + 1)]$ as $f(v) = -(m + 1)$; $f(v_i) = -i$, $1 \leq i \leq m$; $f(v_{m+1}) = 0$; $f(v_i) = 2m + 2 - i$, $1 \leq i \leq m$; $f(u_i) = -(m + 1 + i)$, $1 \leq i \leq m$; $f(u_{m+i}) = i$,$1 \leq i \leq m$.
Then
$f^*(vv_i) < 0$ for all $i = 1, 2, \ldots, m +1$ and $f^*(vv_i) > 0$ for all $i = m + 2, \ldots, 2m + 1$. Also $f^*(v_iu_i) < 0$ for all $i = 1, 2, \ldots, m$ and $f^*(v_iu_i) > 0$ for all $i = m + 1, \ldots, 2m + 1$.

Thus $|e_f(0) - e_f(1)| = 0$.

Now, $f(v_{m+1}) = 0$, $f(v_{m+2}) = n$ and $f(v_m) = -m$. Thus, $f^*(v_mv_{m+1}) < 0$ and $f^*(v_{m+1}v_{m+2}) > 0$. In the inner cycle, $f^*(v_iv_{i+1}) < 0$ for all $i = 1, 2, \ldots, m$, $f^*(v_iv_{i+1}) > 0$ for all $i = m + 1, \ldots, 2m$ and $f^*(v_{2m+1}v_1) > 0$. Here $|e_f(0) - e_f(1)| = 1$.

Considering the outer cycle $f^*(u_iu_{i+1}) < 0$ for all $i = 1, 2, \ldots, m$ and $f^*(u_iu_{i+1}) > 0$ for all $i = m+1, \ldots, 2m$. Also $f^*(u_{2m+1}u_1) < 0$ and $f^*(u_mu_{m+1}) < 0$. Here $|e_f(0) - e_f(1)| = 1$. From all the cases, $|e_f(0) - e_f(1)| \leq 1$.

Thus G is *I*-cordial.

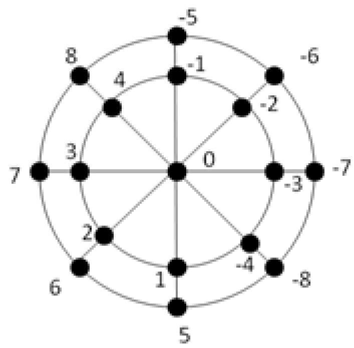

Figure 5: CH_8 is *I*-cordial.

Theorem 2.10 Friendship graph F_n , n > 1 is *I*-cordial.

Proof. Let v_0 be the central vertices of n triangles, consecutively of F_n. We note that p = 2n + 1 and q = 3n. We consider two cases.

CASE 1.n is even

Let n = 2m.Define f : V → [-2m . . 2m] as $f(v_i) = i$, $1 \leq i \leq 2m$, i is odd ; $f(v_{2m+i}) = -i$, $1 \leq i \leq 2m$, i is odd ; so that the edges of triangles C_i , i = 1, 2, . . . , m are all > 0 and the edges of triangles C_i, i = m + 1, . . . , 2m are all < 0. Hence, 3m edges shares positive and negative labels.

That is, $e_f(0) = e_f(1)$.

CASE 2.n is odd.

Let n = 2m + 1.We consider f : V → [-2m . . 2m] as$f(v_i) = i$, $1 \leq i \leq 2m + 1$, i is odd ; $f(v_{2m+1+i}) = - i$, $1 \leq i \leq 2m + 1$, i is odd ; so that the edges of triangles C_i , i = 1, 2, . . . , m are all > 0 and the edges of triangles C_i, i = m + 2, . . . , 2m + 1 are all < 0. Also in the triangle, C_m the edges $f^*(v_{2m+1}v_0) > 0$, $f^*(v_{2m+1}v_{2m+2}) > 0$ and $f^*(v_0v_{2m+2}) < 0$. Hence, $e_f(0) = 1$,$e_f(1) + 2$.

Thus, $|e_f(0) – e_f(1)| = 1$. Therefore, F_n is *I*-cordial.

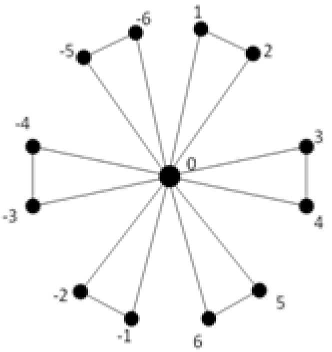

Figure 6: Fan graph F_6 is *I*-cordial

Theorem.2.11 The fan f_n ,n ≥ 3 is *I*-cordial if and only if n is even.

Proof. f_n has n + 1 vertices and 2n – 3 edges. Let u be the apex vertex with degree n. Let $v_1, v_2, v_3, . . ., v_n$ denote the path vertices adjacent to u in f_n.

Case 1.n is even

Let n = 2m. Define f : V → [-m . . m] as f(v) = 0 ; $f(v_i) = -i$; $1 \leq i \leq m$ and $f(v_{m+i}) = i$; $1 \leq i \leq m$ and $f(v_{m+i}) = i$; $1 \leq i \leq m$. Then $f^*(vv_i) < 0$ for all i = 1, 2, . . ., m and $f^*(vv_i) > 0$ for all i = m + 1, m + 2, . . ., 2m. Thus $|e_f(0) - e_f(1)| = 0$.Similarly, $f^*(v_iv_{i+1}) < 0$ for all i = 1, 2, . . ., m and $f^*(v_iv_{i+1}) > 0$ for all i = m + 1, m + 2, . . ., 2m – 1. Thus $|e_f(0) -$ ef1=1.Therefore from the above cases ef0−ef1≤1. Hence, F_n , n ≥ 3 is *I*-cordial.

When n is odd, by Theorem 2.2.3 f_n is not *I*-cordial.

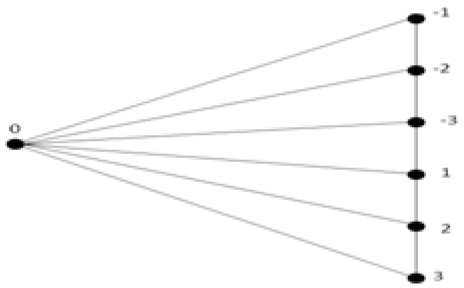

Figure 7: f_6 is *I*-cordial

Theorem 2.12 The double fan Df_n, n ≥ 2 is *I*-cordial.

Proof. Df_n has n + 2 vertices and 3n – 1 edges. Let a and b denote the apex vertices of degree n and $v_1, v_2, . . . , v_n$ be the path vertices adjacent to a and b in Df_n. Then $E(Df_n)$ =A U B U C where $A = \{av_i\}_{i=1}^{n}$; $B = \{bv_i\}_{i=1}^{n}$ and $C = \{v_iv_{i+1}\}_{i=1}^{n}$. We consider two cases:

CASE1. n is even

Let n = 2m.We define $f: V \rightarrow [-(m+1)..(m+1)]^*asf(a) = m + 1$; $f(b) = -(m+1)$; $f(v_i) = i$, $1 \leq i \leq m$ and $f(v_{m-1+i}) = - i$, $1 \leq i \leq m$. Consider $f^*(av_i) > 0$ for all i = 1, 2, . . ., 2m and $f^*(bv_i) < 0$ for all i = 1, 2, . . ., 2m. Thus $|e_f(0) - e_f(1)| = 0$, where e ϵ A U B.

Now, $f^*(v_iv_{i+1}) > 0$ for all i = 1, 2, . . ., m and $f^*(v_iv_{i+1}) < 0$ for all i = m + 1, m + 2,. . ., 2m – 1.

Thus $|e_f(0) - e_f(1)| = 1$.

CASE 2. n is odd

Let n = 2m + 1.We define f : V →[-(m+1)..(m+1)]. We label as follows: f(a) = - (m+1) ; f(b) = m + 1; $f(v_i) = -i$, $1 \leq i \leq m-1$; f(v) = 0 ; $f(v_{m+i}) = i$, , $1 \leq i \leq m$.Let us consider, $f^*(av_i) < 0$ for all i = 1, 2, . . ., 2m + 1.

Thus $|e_f(0) - e_f(1)| = 0$ for all e ∈ A U B.

Now, $f(v_iv_{i+1}) < 0$ for all i = 1,2,...m and $f^*(v_iv_{i+1}) > 0$ for all i = m + 1,. . ., 2m.

Thus $|e_f(0) - e_f(1)| = 0$ for all e ∈ C. Hence from all the cases $|e_f(0) - e_f(1)| \leq 1$.

Therefore double fan Df_n, n ≥ 2 is *I*-cordial.

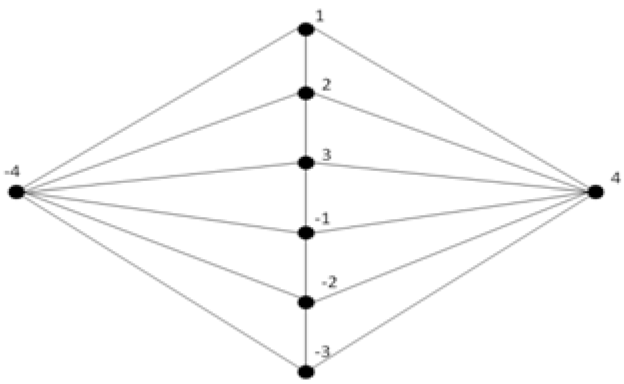

Figure 8: Df_6 is *I*-cordial.

Theorem 2.13 Double Wheel DW_n , n > 2 is *I*-cordial.

Proof. Let G = DW_n be the double wheel. Let $v_0, v_1, \ldots, v_n$ be the inner rim vertices and $v_1', v_2', \ldots, v_n'$ be the outer rim vertices of DW_n. Then p = 2n + 1 and q = 4n.

We define $f : V \rightarrow [-n \,.\,.\, n]$ as, $f(v_0) = 0$, $f(v_i) = i$, $1 \leq i \leq n$ and $f(v_i') = -i$, $1 \leq i \leq n$ so that $f^*(v_iv_{i+1}) > 0$ for all i = 1, 2, . . ., n – 1and $f^*(v_i'v_{i+1}') < 0$ for all i = 1, 2, . . ., n – 1. Here n edges equally shares negative and positive integers. Since, $f(v_0) = 0$ then $f^*(v_0v_i) > 0$ for all i = 1, 2, . . . , n and $f^*(v_0v_i') < 0$ for all i = 1, 2, . . ., n. Here also n edges shares negative and positive integers. That is, $e_f(0) = e_f(1)$. Hence, $|e_f(0) - e_f(1)| = 0$.

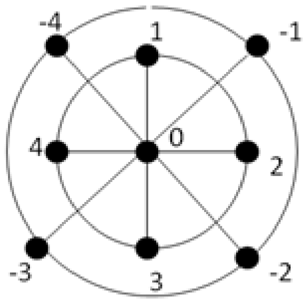

Figure 9: DW_4 is *I*-cordial.

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