International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Study of Identitie Involving and Special Case of Generalization of H-Functions

Payal Aloke Verma

Assistant Professor, Dept. of Mathematics, Kalinga University, New Raipur (C.G), Pin-492101

Abstract: ***The aim of this paper is to find the identities involving generalization of H-functions by suitable adjustment of functions.***

Keywords: Mellin transforms; Hypergeometric functions; Gamma function.

1. Introduction

The present research is devoted to certain identities and reduction formulae for generalized functions, which are of great interest and generalize many known and unknown results in literature especially the results given by Shweta and Srivastava [2006] and Cook [1981].

2. Notations and Results Used

${}_1(a_j\,;\,\alpha_j^{(k)})_p$ abbreviates the array of p parameters $(a_1\,;\,\alpha_1^{(k)})$, ..., $(a_p\,;\,\alpha_p^{(k)})$

${}_1(a_j\,;\,\alpha_j\,,A_j)_p$ stands for the array of p parameters $(a_1\,;\,\alpha_1\,,A_1)$,…., $(a_p\,;\,\alpha_p\,,A_p)$

${}_1(a_j\,;\,\alpha_j\,)_p$ abbreviates the array of p parameters $(a_1\,;\,\alpha_1\,)$,… , $(a_p\,;\,\alpha_p)$

$(a)_n$ stands for the product of n factors a(a+1)(a+2)…(a+n-1);$(a)_0 = 1$

Erdelyi [3,p.210(12)]

$$(a_p-a_1)\quad H^{m,n}_{p,q}\left[x\middle|\,\begin{matrix}{}_1(a_j\,)_p\\ {}_1(b_j)_q\end{matrix}\right] = H^{m,n}_{p,q}\left[x\middle|\,\begin{matrix}a_1-1,{}_2(a_j\,)_p\\ {}_1(b_j)_q\end{matrix}\right] + H^{m,n}_{p,q}\left[x\middle|\,\begin{matrix}{}_1(a_j\,)_{p-1}\,,a_p-1\\ {}_1(b_j)_q\end{matrix}\right]\ \ldots\ldots\ldots\ (1.1)$$

For $1\leq n\leq p-1$

Erdelyi [3, p.210(12)]

$$H^{1,n}_{p,q}\left[x\middle|\,\begin{matrix}{}_1(a_j\,)_p\\ {}_1(b_j)_q\end{matrix}\right] = \frac{\prod_{j=1}^{n}\Gamma(1+b_1-a_j)\,x^{b_1}}{\prod_{j=2}^{q}\Gamma(1+b_1-b_j)\prod_{j=n+1}^{p}\Gamma(a_j-b_1)}\ {}_pF_{q-1}\ {}^{1,n}_{p,q}\left[\begin{matrix}{}_1(1+b_1-a_j)_p;\\ {}_2(1+b_1-b_j)_q;\end{matrix}(-1)^{p-n-1}x\right]\ \ldots\ldots.(1.2)$$

For p≤q.

Taking m=1, b_1=0 and using (1.2); (1.1) reduces to the relation

$$(a_1\text{-}a_P)\,{}_pF_q\left[\begin{matrix}{}_1(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ x\right] = a_1\ {}_pF_q\left[\begin{matrix}a_1+1,{}_2(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ x\right] - a_P\ {}_pF_q\left[\begin{matrix}{}_1(a_j\,)_{p-1}\,,a_p+1\,;\\ {}_1(b_j)_q\,;\end{matrix}\ x\right]\ \ldots\ldots(1.3)$$

For $2\leq p\leq q+1$.

When p=2 and q=1; (1.3) agrees with a relation given by Erdelyi [3 , p.103 (32)].
Jeta Ram [5 , p.266 (3.11)]

$$\sum_{k=0}^{N}\binom{N}{k}\frac{\prod_{j=1}^{p}(a_j)_k}{\prod_{j=1}^{q}(b_j)_k}\ {}_{p+1}F_q\left[\begin{matrix}c+k\,,{}_1(a_j+k\,)_p;\\ {}_1(b_j+k)_q;\end{matrix}\ z\right]z^k = {}_{p+1}F_q\left[\begin{matrix}c+N\,,{}_1(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ z\right]\ \ldots\ldots.(1.4)$$

Taking N=1 and renaming c+1 as c (1.4) gives rise to the relation

$${}_{p+1}F_q\left[\begin{matrix}c\,,{}_1(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ z\right] = {}_{p+1}F_q\left[\begin{matrix}c-1\,,{}_1(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ z\right] + z\,\frac{\prod_{j=1}^{p}(a_j)}{\prod_{j=1}^{q}(b_j)}\ {}_{p+1}F_q\left[\begin{matrix}c\,,{}_1(a_j+1)_p;\\ {}_1(b_j+1)_q;\end{matrix}\ z\right]\ \ldots\ldots(1.5)$$

the relation (1.5) can be restated as

$${}_pF_q\left[\begin{matrix}{}_1(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ z\right] = {}_pF_q\left[\begin{matrix}a_1-1\,,{}_2(a_j\,)_p;\\ {}_1(b_j)_q;\end{matrix}\ z\right] + z\,\frac{\prod_{j=2}^{p}(a_j)}{\prod_{j=1}^{q}(b_j)}\ {}_pF_q\left[\begin{matrix}a_1\,,{}_2(a_j+1)_p;\\ {}_1(b_j+1)_q;\end{matrix}\ z\right]\ \ldots..(1.6)$$

3. Identities Involving Generalization of H-Functions Identity

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{M+m,N+m:\,M_1,N_1;\,M_2,N_2}_{P+m,Q+2m:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\middle|\ \begin{matrix}{}_1(\delta_i-r\,;\dot{\eta}_i,\dot{\eta}'_i)_m\,,{}_1(a_j;\alpha_j,A_j)_P\,,{}_1(c_j;C_j)_{P1}\,;{}_1(e_j\,,E_j)_{P2}\\ {}_1(v_i+r;\mu'_i,\mu'_i)_m\,,{}_1(b_j\,;\beta_j,B_j)_Q\,,{}_1(\varsigma_i-r;\lambda'_i\lambda'_i)_m:\ {}_1(d_j;D_j)_{Q1}\,;{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{M+m,N+m+1:\,M_1,N_1;\,M_2,N_2}_{P+m+1,Q+2m+1:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\middle|\ \begin{matrix}(\delta_1\,;\dot{\eta}_1,\dot{\eta}'_1),(\delta_1-r+1\,;\dot{\eta}_1,\dot{\eta}'_1),{}_2(\delta_i-r\,;\dot{\eta}_i,\dot{\eta}'_i)_m\,,{}_1(a_j;\alpha_j,A_j)._P:\ {}_1(c_j;C_j)_{P1}\,;{}_1(e_j\,,E_j)_{P2}\\ {}_1(v_i+r;\mu'_i,\mu'_i)_m\,,(b_j\,;\beta_j,B_j)_Q,(\delta_1+1\,;\dot{\eta}_1,\dot{\eta}'_1),{}_1(\varsigma_i+r;\lambda'_i\lambda'_i)_m:\ {}_1(d_j;D_j)_{Q1}\,;{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}H^{M+m,N+m:\,M_1,N_1;\,M_2,N_2}_{P+m,Q+2m:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\middle|\ \begin{matrix}(\delta_1-r\,;\dot{\eta}_1,\dot{\eta}'_1),{}_2(\delta_i-r-1\,;\dot{\eta}_i,\dot{\eta}'_i)_m\,,{}_1(a_j;\alpha_j,A_j)._P\,,:\ {}_1(c_j;C_j)_{P1}\,;{}_1(e_j\,,E_j)_{P2}\\ ,{}_1(v_i+r+1;\mu'_i,\mu'_i)_m\,,{}_1(b_j\,;\beta_j,B_j)_Q,{}_1(\varsigma_i+r;\lambda'_i\lambda'_i)_m:\ {}_1(d_j;D_j)_{Q1}\,;{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right];\ m>0$$

……… (2.1)

Proceeding similar lines to (2.1), for p=2m and q=m.

4. Special Case

Taking M=N=P=Q=0 ; M_2=1 ;N_2=P_2 ; E_j = F_j = 1 ; f_1=0 ; $\eta_i''=\mu_i''=\lambda_i''=0$with y→0, and renaming the parameters (3.2.7) gives rise to:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{m+k,n+k}_{P+k,q+2k}\left[x\,\middle|\,\begin{matrix}{}_1(\delta_i-r\,;\eta_i)_{\,k}\,,{}_1(a_j;\alpha_j,)_{\,P}\,,\\ {}_1(v_i+r;\mu_i)_{\,k}\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,{}_1(\varsigma_i-r,\lambda_i)_{\,k}\end{matrix}\right]$$

=

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\,{}^{m+k,n+k+1}_{p+k+1,q+2k+1}\left[x\,\middle|\,\begin{matrix}(\delta_1\,,\eta_1),(\delta_1-r+1\,;\eta_1),{}_2(\delta_i-r\,;\eta_i)_{\,k}\,,{}_1(a_j;\alpha_j,)_{\,P}\\ {}_1(v_i+r;\mu_i)_{\,k}\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,(\delta_1+1\,;\eta_1)\,,{}_1(\varsigma_i-r,\lambda_i)_{\,k}\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}$$

$$H^{m+k,n+k}_{p+k,q+2k}\left[x\middle|\,\begin{matrix}(\delta_1-r\,;\eta_1),{}_2(\delta_i-r-1\,;\eta_i)_{\,k}\,,{}_1(a_j;\alpha_j,)_{\,P}\\ {}_1(v_i+r+1;\mu_i)_{\,k}\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,{}_1(\varsigma_i-r-1\,,\lambda_i)_{\,k}\end{matrix}\right];\text{ where}$$

k>0 ……… (2.2)

On taking k=2, (2.10) reduces to the following identity:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}$$

$$H^{m+2,n+2}_{P+2,q+4}\left[x\,\middle|\,\begin{matrix}(\delta-r\,;\eta),(\sigma-r\,,\xi),{}_1(a_j;\alpha_j\,)_{\,P}\\ (v+r\,,\mu)\,,(\rho+r\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,(\varsigma-r,\lambda),(\tau-r\,,\zeta)\end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}$$

$$H^{m+2,n+3}_{p+3,q+3}\left[x\,\middle|\,\begin{matrix}(\delta\,,\eta),(\delta-r+1\,;\eta),(\sigma-r\,,\xi),{}_1(a_j;\alpha_j\,)_{\,P}\\ (v+r\,,\mu)\,,(\rho+r\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,(\delta+1\,,\eta)\,,(\varsigma-r,\lambda),(\tau-r\,,\zeta)\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}$$

H

$$^{m+2,n+2}_{p+2,q+4}\left[x\middle|\,\begin{matrix}(\delta-r;\eta),(\sigma-r-1\,,\xi),{}_1(a_j;\alpha_j\,)_{\,P}\,,\\ (v+r+1\,,\mu)\,,(\rho+r+1\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,,(\varsigma-r-1,\lambda),(\tau-r-1\,,\zeta)\end{matrix}\right]$$

……. (2.3)

When k=1, (2.11) gives rise to the identity:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{m+1,n+1}_{P+1,q+2}\left[x\,\middle|\,\begin{matrix}(\delta-r\,;\eta)\,,{}_1(a_j;\alpha_j\,)_{\,P}\\ (v+r\,,\mu)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,(\varsigma-r,\lambda)\end{matrix}\right]=$$

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\,{}^{m+1,n+2}_{p+2,q+3}\left[x\,\middle|\,\begin{matrix}(\delta\,,\eta),(\delta-r+1\,;\eta)\,,{}_1(a_j;\alpha_j\,)_{\,P}\\ (v+r\,,\mu)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,(\delta+1\,,\eta)\,,(\varsigma-r,\lambda),\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}\,H^{m+1,n+1}_{p+1,q+2}\left[x\middle|\,\begin{matrix}(\delta-r;\eta)\,,{}_1(a_j;\alpha_j\,)_{\,P}\,,\\ (v+r+1\,,\mu)\,,{}_1(b_j\,;\beta_j,)_{\,q}\,,,(\varsigma-r-1,\lambda)\end{matrix}\right];$$

……. (2.4)

References

[1] Erdélyi, A. **[1953]** : Higher Transcendental Functions, Vol. I , McGraw - Hill Book Company, New York.

[2] Erdélyi, A. **[1954]** : Tables of integral transforms, Vol. II , McGraw - Hill Book Company, New York.

[3] I.L.Hirchman and D.V.Widder.**[1955]** The convolution transforms. Princeton, New jersey: University Press,

[4] A.M. Erdelyi**,[1959]** Tables of integral transforms, vol.1,11, Mc-Graw-Hill,New York.

[5] Fox, C**.[1961]**. The G- and H-functions as symmetrical Fourier kernels.*Trans. Amer.Soc.98,p.395-429.*

[6] Gupta, K.C and Jain, U.C**[1966].** H-function,*ii.Proc. Nat.Acad. Sci. India Sect.A 36.p 594-609.*

[7] Gupta, K.C and Jain, U.C**[1968]**,On the derivative of the H-functions, *proc. Nat. Acad. Sci. India.a.38,189-192.*

[8] Sharma, B.L. **[1968]**. An integral involving product of G-function and generalized function of two variables.*Univ. Nac. Tucuman Rev. Ser.A 18, p.17-23.*

[9] Koul, C.L. **[1974]**. On certain integral relations and their applications. *Proc.Indian Aa.Sci.Sect A 79, p 56-66.*

[10] Cook, I.D. **[1981]** : The *H* - function and probability density functions of certain algebraic combinations of independent random variables with *H* – function probability, Ph.D. dissertation, Univ. of Texas, U.S.A.

[11] H. M. Srivastava and R.Panda,**[1989]** Some expansion theorems and generating relations for the H- Function of several complex variables.I, Comment.Math. Univ.St. Paul., 24, 119-137.

[12] Prasad, Y.N. and Gupta, R.K. **[1992]** : An expansion formula for *H* - function of two variables and its application, Vijnana Parishad Anusandhan Patrika, 19 , 39 - 45.

[13] Vasishta , S.K and Goyal, S.P **[1998]**On a generalized double L-H transform ,*Vijnana Parishad Anusandhan Patrika,20(1), 9-21.*

[14] Goyal, S.P.**[2006].**Multiple transformation of the H-function of two variables, *proc. Appl. Math. Sci.7,p.19-28.*