Study of Identitie Involving and Special Case of Generalization of H-Functions

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Abstract: ***The aim of this paper is to find the identities involving generalization of H-functions by suitable adjustment of functions.***

Keywords: Mellin transforms; Hypergeometric functions; Gamma function.

1. Introduction

The present research is devoted to certain identities and reduction formulae for generalized functions, which are of great interest and generalize many known and unknown results in literature especially the results given by Shweta and Srivastava [2006] and Cook [1981].

2. Notations and Results Used

${}_1(a_j\,;\,\alpha_j^{(k)})_p$ abbreviates the array of p parameters $(a_1\,;\,\alpha_1^{(k)})$, ..., $(a_p\,;\,\alpha_p^{(k)})$

${}_1(a_j\,;\,\alpha_j\,,A_j)_p$ stands for the array of p parameters $(a_1\,;\,\alpha_1\,,A_1)$,…., $(a_p\,;\,\alpha_p\,,A_p)$

${}_1(a_j\,;\,\alpha_j)_p$ abbreviates the array of p parameters $(a_1\,;\,\alpha_1)$,… , $(a_p\,;\,\alpha_p)$

$(a)_n$ stands for the product of n factors a(a+1)(a+2)…(a+n-1);$(a)_0 = 1$

Erdelyi [3,p.210(12)]

$$(a_p-a_1)\quad H^{m,n}_{p,q}\left[x\Big|\begin{matrix}{}_1(a_j)_p\\ {}_1(b_j)_q\end{matrix}\right] = H^{m,n}_{p,q}\left[x\Big|\begin{matrix}a_1-1,\,{}_2(a_j)_p\\ {}_1(b_j)_q\end{matrix}\right] + H^{m,n}_{p,q}\left[x\Big|\begin{matrix}{}_1(a_j)_{p-1},\,a_p-1\\ {}_1(b_j)_q\end{matrix}\right] \;\ldots\ldots\ldots\;(1.1)$$

For $1\le n\le p-1$

Erdelyi [3, p.210(12)]

$$H^{1,n}_{p,q}\left[x\Big|\begin{matrix}{}_1(a_j)_p\\ {}_1(b_j)_q\end{matrix}\right] = \frac{\prod_{j=1}^{n}\Gamma(1+b_1-a_j)\,x^{b_1}}{\prod_{j=2}^{q}\Gamma(1+b_1-b_j)\prod_{j=n+1}^{p}\Gamma(a_j-b_1)}\;{}_pF_{q-1}\;{}^{1,n}_{p,q}\left[\begin{matrix}{}_1(1+b_1-a_j)_p;\\ {}_2(1+b_1-b_j)_q;\end{matrix}(-1)^{p-n-1}x\right]\;\ldots\ldots.(1.2)$$

For p≤q.

Taking m=1, b_1=0 and using (1.2); (1.1) reduces to the relation

$$(a_1-a_P)_pF_q\left[\begin{matrix}{}_1(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,x\right] = a_1\;{}_pF_q\left[\begin{matrix}a_1+1,\,{}_2(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,x\right] - a_P\;{}_pF_q\left[\begin{matrix}{}_1(a_j)_{p-1},a_p+1;\\ {}_1(b_j)_q;\end{matrix}\,x\right]\;\ldots\ldots(1.3)$$

For $2\le p\le q+1$.

When p=2 and q=1; (1.3) agrees with a relation given by Erdelyi [3 , p.103 (32)].
Jeta Ram [5 , p.266 (3.11)]

$$\sum_{k=0}^{N}\binom{N}{k}\frac{\prod_{j=1}^{p}(a_j)_k}{\prod_{j=1}^{q}(b_j)_k}\;{}_{p+1}F_q\left[\begin{matrix}c+k,\,{}_1(a_j+k)_p;\\ {}_1(b_j+k)_q;\end{matrix}\,z\right]z^k = {}_{p+1}F_q\left[\begin{matrix}c+N,\,{}_1(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,z\right]\;\ldots\ldots.(1.4)$$

Taking N=1 and renaming c+1 as c (1.4) gives rise to the relation

$${}_{p+1}F_q\left[\begin{matrix}c,\,{}_1(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,z\right] = {}_{p+1}F_q\left[\begin{matrix}c-1,\,{}_1(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,z\right] + z\,\frac{\prod_{j=1}^{p}(a_j)}{\prod_{j=1}^{q}(b_j)}\;{}_{p+1}F_q\left[\begin{matrix}c,\,{}_1(a_j+1)_p;\\ {}_1(b_j+1)_q;\end{matrix}\,z\right]\ldots\ldots(1.5)$$

the relation (1.5) can be restated as

$${}_pF_q\left[\begin{matrix}{}_1(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,z\right] = {}_pF_q\left[\begin{matrix}a_1-1,\,{}_2(a_j)_p;\\ {}_1(b_j)_q;\end{matrix}\,z\right] + z\,\frac{\prod_{j=2}^{p}(a_j)}{\prod_{j=1}^{q}(b_j)}\;{}_pF_q\left[\begin{matrix}a_1,\,{}_2(a_j+1)_p;\\ {}_1(b_j+1)_q;\end{matrix}\,z\right]\;\ldots..(1.6)$$

3. Identities Involving Generalization of H-Functions Identity

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{M+m,N+m:\,M_1,N_1;\,M_2,N_2}_{P+m,Q+2m:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\Big|\begin{matrix}{}_1(\delta_i-r\,;\dot{\eta}_i,\dot{\eta}'_i)_m,\,{}_1(a_j;\alpha_j,A_j)_P\,,\,{}_1(c_j;C_j)_{P1}\,;\,{}_1(e_j\,,E_j)_{P2}\\ {}_1(v_i+r;\,\mu'_i,\mu'_i)_m\,,\,{}_1(b_j\,;\beta_j,B_j)_Q\,,\,{}_1(\varsigma_i-r;\lambda'_i\lambda'_i)_m:\,{}_1(d_j;D_j)_{Q1}\,;\,{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{M+m,N+m+1:\,M_1,N_1;\,M_2,N_2}_{P+m+1,Q+2m+1:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\Big|\begin{matrix}(\delta_1\,;\dot{\eta}_1,\dot{\eta}'_1),(\delta_1-r+1\,;\dot{\eta}_1,\dot{\eta}'_1),\,{}_2(\delta_i-r\,;\dot{\eta}_i,\dot{\eta}'_i)_m,\,{}_1(a_j;\alpha_j,A_j).{}_P:\,{}_1(c_j;C_j)_{P1}\,;\,{}_1(e_j\,,E_j)_{P2}\\ {}_1(v_i+r;\,\mu'_i,\mu'_i)_m\,,(b_j\,;\beta_j,B_j)_Q,(\delta_1+1\,;\dot{\eta}_1,\dot{\eta}'_1),\,{}_1(\varsigma_i+r;\lambda'_i\lambda'_i)_m:\,{}_1(d_j;D_j)_{Q1}\,;\,{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}H^{M+m,N+m:\,M_1,N_1;\,M_2,N_2}_{P+m,Q+2m:\,P_1,Q_1;\,P_2,Q_2}\left[\begin{matrix}x\\ y\end{matrix}\Big|\begin{matrix}(\delta_1-r\,;\dot{\eta}_1,\dot{\eta}'_1),\,{}_2(\delta_i-r-1\,;\dot{\eta}_i,\dot{\eta}'_i)_m,\,{}_1(a_j;\alpha_j,A_j).{}_P\,,:\,{}_1(c_j;C_j)_{P1}\,;\,{}_1(e_j\,,E_j)_{P2}\\ ,{}_1(v_i+r+1;\,\mu'_i,\mu'_i)_m\,,\,{}_1(b_j\,;\beta_j,B_j)_Q,\,{}_1(\varsigma_i+r;\lambda'_i\lambda'_i)_m:\,{}_1(d_j;D_j)_{Q1}\,;\,{}_1(f_j\,,F_j)_{Q2}\end{matrix}\right];\ m>0$$

……… (2.1)

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Proceeding similar lines to (2.1), for p=2m and q=m.

4. Special Case

Taking $M=N=P=Q=0$; $M_2=1$;$N_2=P_2$; $E_j = F_j = 1$; $f_1=0$; $\eta_i''=\mu_i''=\lambda_i''=0$ with $y\to 0$, and renaming the parameters (3.2.7) gives rise to:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{m+k,n+k}_{P+k,q+2k}\left[x \,\middle|\, \begin{matrix} {}_1(\delta_i-r\,;\eta_i)_k,{}_1(a_j;\alpha_j,)_P\,, \\ {}_1(v_i+r;\mu_i)_k\,,{}_1(b_j\,;\beta_j,)_q\,,{}_1(\varsigma_i-r,\lambda_i)_k \end{matrix}\right]$$

$$=$$

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\,{}^{m+k,n+k+1}_{p+k+1,q+2k+1}\left[x \,\middle|\, \begin{matrix} (\delta_1,\eta_1),(\delta_1-r+1\,;\eta_1),{}_2(\delta_i-r\,;\eta_i)_k,{}_1(a_j;\alpha_j,)_P \\ {}_1(v_i+r;\mu_i)_k\,,{}_1(b_j\,;\beta_j,)_q\,,(\delta_1+1\,;\eta_1)\,,{}_1(\varsigma_i-r,\lambda_i)_k \end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}$$

$$H^{m+k,n+k}_{p+k,q+2k}\left[x \,\middle|\, \begin{matrix} (\delta_1-r\,;\eta_1),{}_2(\delta_i-r-1\,;\eta_i)_k,{}_1(a_j;\alpha_j,)_P \\ {}_1(v_i+r+1;\mu_i)_k\,,{}_1(b_j\,;\beta_j,)_q\,,{}_1(\varsigma_i-r-1\,,\lambda_i)_k \end{matrix}\right];\ \text{where}$$

k>0 ……… (2.2)

On taking k=2, (2.10) reduces to the following identity:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}$$

$$H^{m+2,n+2}_{P+2,q+4}\left[x \,\middle|\, \begin{matrix} (\delta-r\,;\eta),(\sigma-r\,,\xi),{}_1(a_j;\alpha_j)_P \\ (v+r\,,\mu)\,,(\rho+r\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_q\,,(\varsigma_.-r,\lambda_.),(\tau-r\,,\zeta) \end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}$$

$$H^{m+2,n+3}_{p+3,q+3}\left[x \,\middle|\, \begin{matrix} (\delta\,,\eta),(\delta-r+1\,;\eta),(\sigma-r\,,\xi),{}_1(a_j;\alpha_j)_P \\ (v+r\,,\mu)\,,(\rho+r\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_q\,,(\delta+1\,,\eta)\,,(\varsigma_.-r,\lambda_.),(\tau-r\,,\zeta) \end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}$$

H

$$^{m+2,n+2}_{p+2,q+4}\left[x \,\middle|\, \begin{matrix} (\delta-r;\eta),(\sigma-r-1\,,\xi),{}_1(a_j;\alpha_j)_P\,, \\ (v+r+1\,,\mu)\,,(\rho+r+1\,,\gamma)\,,{}_1(b_j\,;\beta_j,)_q\,,,(\varsigma_.-r-1,\lambda_.),(\tau-r-1\,,\zeta) \end{matrix}\right]$$

……. (2.3)

When k=1, (2.11) gives rise to the identity:

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}H^{m+1,n+1}_{P+1,q+2}\left[x \,\middle|\, \begin{matrix} (\delta-r\,;\eta)\,,{}_1(a_j;\alpha_j)_P \\ (v+r\,,\mu)\,,{}_1(b_j\,;\beta_j,)_q\,,(\varsigma_.-r,\lambda_.) \end{matrix}\right]=$$

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\,{}^{m+1,n+2}_{p+2,q+3}\left[x \,\middle|\, \begin{matrix} (\delta\,,\eta),(\delta-r+1\,;\eta)\,,{}_1(a_j;\alpha_j)_P \\ (v+r\,,\mu)\,,{}_1(b_j\,;\beta_j,)_q\,,(\delta+1\,,\eta)\,,(\varsigma_.-r,\lambda_.), \end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}H^{m+1,n+1}_{p+1,q+2}\left[x \,\middle|\, \begin{matrix} (\delta-r;\eta)\,,{}_1(a_j;\alpha_j)_P\,, \\ (v+r+1\,,\mu)\,,{}_1(b_j\,;\beta_j,)_q\,,,(\varsigma_.-r-1,\lambda_.) \end{matrix}\right];$$

……. (2.4)

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