On $f\omega$ -Continuity in Fine-Topological Space

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Abstract: ***In this paper, the authors have introduced a new class of functions called total fine-continuity, strongly fine—continuity, contra fine –continuity and investigated some of their properties. Also, we define $f\omega$ –homeomorphism in fine-topological space and investigated some properties.***

Keywords: $f\omega$ –open sets, $f\omega$ –open sets, fine-open sets

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1. Introduction

Continuous functions are the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years. Some of them are totally continuous functions ([3]), strontly continuous functions ([5]), contra continuous functions ([2]). Its importance is significant in various areas of mathematics and related sciences. As generalization of closed sets, the concept of ω-closed sets were introduced and studied by Sundaram and Sheik John [11]. Rajesh N. introduced some new types of continuity called totally $\omega-$ continuity, strongly $\omega-$continuity and contra $\omega-$continuity etc. (cf. [8]). Also, the author has defined contra pre$-\omega-$open maps, contra pre $\omega-$ closed maps, contra ω^*- homeomorphism and ω^*c-homeomorphism. Powar P. L. and Rajak K. [7] have introduced fine-topological space which is a special case of generalized topological space. This new class of fine-open sets contains all α –open sets, β –open sets, semi-open sets, pre-open sets, regular open sets etc. and fine-irresolute mapping includes pre-continuous function, semi-continuous functions, α –continuous function, β –continuous function, α –irresolute and β –irresolute functions.

The aim of this paper is to give some new types of continuity called totally $f\omega$ -continuity, strongly $f\omega$ -continuity and contra $f\omega$-continuity. In this connection, four classes of maps were formed namely contra pre $f\omega$-open maps, contra pre $f\omega$ -closed maps, contra $f\omega$ -homeomorphisms and $f\omega$-homeomorphisms.

2. Preliminaries

Throughout this paper (X,τ) and (Y,τ') represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $Cl(A)$, $Int(A)$ and A^c denote the closure, the interior and the complement of A in (X,τ), respectively. We recall the following definitions, which are useful in the sequel.

Definition 2.1 A subset A of a space (X,τ) is called a semi-open ([4]) (resp. pre-open ([6])) if $A \subseteq Cl(Int(A))$ (resp.$A \subseteq Int(Cl(A))$). The complement of semi-open set is called semi-closed. The intersection of all semi-closed sets of (X,τ) containing A is called the semi-closure ([1]) of A and is denoted by $sclX(A)$.

Definition 2.2 A subset A of a space (X,τ) is called an ω-closed ([11]) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ). The complement of ω-closed set is called ω-open.

Definition 2.3 A function $f:(X,\tau) \to (Y,\tau')$ is called:
(i) totally continuous ([3]) if the inverse image of every open subset of (Y,τ) is a clopen subset of (X,τ').
(ii) strongly continuous ([5]) if the inverse image of every subset (Y,τ') is a clopen subset of (X,τ).
(iii) contra-continuous ([2]) if the inverse image of every open subset of (Y,τ') is a closed subset of (X,τ).
(iv) ω-continuous ([11]) if the inverse image of every open subset of (Y,τ') is ω-open in (X,τ).

Definition 2.4 A function $f:(X,\tau) \to (Y,\tau')$ is said to be totally ω-continuous if the inverse image of every open subset of (Y,τ') is a ω-clopen (i.e., ω-open and ω-closed) subset of (X,τ) (cf. [8]).
It is evident that every totally continuous function is totally ω-continuous.

Definition 2.5 A function $f:(X,\tau) \to (Y,\tau')$ is said to be strongly ω-continuous if the inverse image of every open subset of (Y, τ') is a ω-clopen subset of (X,τ) (cf. [8]).

Definition 2.6 A function $f:(X,\tau) \to (Y,\tau')$ is called contra-ω-continuous if $f^{-1}(V)$ is ω-open in (X,τ) for every closed set V in (Y,τ') (cf. [8]).

Definition 2.7 A function $f:(X,\tau) \to (Y,\tau')$ is said to be:
(i) contra pre ω-open ([8]) if $f(U)$ is ω-closed in (Y,τ') for every ω-open set U of (X,τ);
(ii) contra pre ω–closed ([8]) if f(F) is ω-open in (Y, τ') for every ω-closed set F of (X, τ);
(iii) ω-irresolute ([9]) if $f^{-1}(F)$ is ω-closed in (X,τ) for every ω-closed set F of (Y,τ');
(iv) ω-closed (resp. ω-open) ([9]) if $f(F)$ is ω-closed (resp. ω-open) in (Y,τ') for every ω-closed (resp. ω-open) set $F$of (X,τ').

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Definition 2.8 (i) A function $f : (X,\tau) \to (Y,\tau')$ is said to be an ω-homeomorphism ([9]) if f and f^{-1} both are bijective and ω-irresolute (cf. [8]).
(ii) A function $f : (X,\tau) \to (Y,\tau')$ is said to be a contra ω-homeomorphism ([8]) if f is bijective and contra pre ω-open and f^{-1} is contra pre ω-open (cf. [8]).

Definition 2.9 A topological space (X,τ) is said to be ω-normal ([9]) if each pair of non-empty disjoint closed sets can be separated by disjoint ω-open sets.

Definition 2.10 A topological space (X,τ) is said to be ultra normal ([10]) if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Definition 2.11 Let (X,τ) be a topological space we define
$\tau(A_\alpha) = \tau_\alpha(say) = \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where J is the index set.$\}$
Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha \in J\}} \{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [7]).

Definition 2.12 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [7]).

Definition 2.13 Let A be a subset of a fine space X, we say that a point $x \in X$ is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [7]).

Definition 2.14 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{int}. (cf. [7]).

Definition 2.15 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [7]).

Definition 2.16 A function $f: (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [7]).

Definition 2.17 A function $f: (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is said to be αf −irresolute if $f^{-1}(V)$ is αf −open in X for every αf −open set V of Y (cf. [7]).

Definition 2.18 A function $f: (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is said to be βf − irresolute if $f^{-1}(V)$ is βf −open in X for every βf − open set V of Y (cf. [7]).

Definition 2.19 A function $f: (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is said to be pf −irresolute if $f^{-1}(V)$ is pf −open in X for every pf −open set V of Y (cf. [7]).

Definition 2.20 A function $f: (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is said to be sf −irresolute if $f^{-1}(V)$ is sf −open in X for every sf −open set V of Y (cf. [7]).

3. Totally and Strongly fω -Continuous Functions

In this section, we define totally and strongly $f\omega-$ continuous functions by using the concept of $f\omega$ −open sets, $f\omega$ −closed sets, fine semi-open sets.

Definition 3.1 A subset A of a fine-topological space (X, τ,τ_f) is called an $f\omega$-closed if $f_{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is fine-semi-open in X. The complement of $f\omega$-closed set is called $f\omega$-open.

Example 3.1 Let $X = \{a,b,c\}$ and $\tau = \{X,\phi,\{a\}\}$, $\tau_f = \{X,\phi,\{a\},\{a,b\},\{a,c\}\}$. It may be easily checked that, the only $f\omega$ −closed sets are $\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}$.

Remark 3.1 Every ω −closed sets are $f\omega$ −closed sets.

Definition 3.2 (i) Let A be a subset of a fine topological space (X,τ,τ_f). The intersection of all $f\omega$-closed sets containing A is called the $f\omega$-closure of A and is denoted by $f\omega Cl(A)$.

Definition 3.3 A function $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is called totally fine-continuous if the inverse image of every fine-open subset of Y is a f-clopen subset of X.

Example 3.2 Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{a,b\},\{a,c\}\}$, $\tau_f = \{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y = \{1,2,3\}$, $\tau' = \{X,\phi,\{1\}\}$, $\tau_f' = \{X,\phi,\{1\},\{1,2\},\{1,3\}\}$. We define a map $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ by $f(a) = 1, f(b) = 2, f(c) = 3$. It may be easily checked that, the only fine-open sets of Y are $\phi, X, \{1\}, \{1,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{a,b\}, \{a,c\}$ which are f-clopen in X. Hence, f is totally fine-continuous.

Definition 3.4 A function $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is called strongly fine-continuous if the inverse image of every fine-closed subset Y is a f −clopen subset of X.

Example 3.3 Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{b\},\{a,b\},\{b,c\}\}$, $\tau_f = \{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y = \{1,2,3\}$, $\tau' = \{X,\phi,\{2\}\}$ $\tau_f' = \{X,\phi,\{2\},\{1,2\},\{2,3\}\}$. We define a map $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ by $f(a) = 1, f(b) = 2, f(c) = 3$. It may be easily checked that, the only fine-closed sets of Y are $\phi, Y, \{1,3\}, \{3\}, \{1\}$ and their pre-images are $X, \phi, \{1,c\}, \{c\}, \{1\}$ which are f-clopen in X. Hence, f is strongly fine-continuous.

Definition 3.5 A function $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is called contra-fine-continuous if the inverse image of every fine-open subset of Y is a fine-closed subset of X.

Example 3.4 Let $X = \{a,b,c\}$, $\tau = \{X,\phi, \{c,a\}\}$, $\tau_f = \{X,\phi,\{c\},\{a\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y = \{1,2,3\}, \tau' = \{X,\phi,\{2\}\}, \tau_f' = \{X,\phi,\{2\},\{2,3\},\{1,2\}\}$. We define a map

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$f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only fine-open sets of Y are $\phi, Y, \{2,3\}, \{2\}, \{1,2\}$ and their pre-images are $X, \phi, \{b\}, \{b,c\}, \{a,b\}$ which are f-closed in X. Hence, f is contra-fine-continuous.

Definition 3.6 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is called $f\omega$-continuous if the inverse image of every fine-open subset of (Y, τ', τ_f') is $f\omega$-open in (X,τ).

Example 3.5 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a,b\}\}$, $\tau_f=\{X,\phi,\{a\}, \{b\},\{a,b\},\{a,c\},\{b,c\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{1\}\}$ $\tau_f'=\{X,\phi,\{1\},\{1,2\},\{1,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only fine-open sets of Y are $\phi, X, \{1\}, \{1,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{a,b\}, \{a,c\}$ which are $f\omega$-open in X. Hence, f is $f\omega$-continuous.

Definition 3.7 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is said to be totally $f\omega$- continuous if the inverse image of every fine-open subset of Y is a $f\omega$-clopen (i.e., $f\omega$ -open and $f\omega$-closed) subset of X.

Remark 3.2 It is evident that every totally fine-continuous function is totally $f\omega$-continuous.

Example 3.6 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$, $\tau_f=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{3\}\}\tau_f'=\{X,\phi,\{3\},\{3,2\},\{1,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only fine-open sets of Y are $\phi, X, \{3\}, \{3,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{c\}, \{c,b\}, \{a,c\}$ which are $f\omega$-clopen in X. Thus, f is totally $f\omega$ −continuous.

Definition 3.8 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is said to be strongly $f\omega$- continuous if the inverse image of every fine-closed subset of Y is a $f\omega$-clopen subset of X.

Example 3.7 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{b\},\{a,b\},\{b,c\}\}$, $\tau_f=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{1\}\}$, $\tau_f'=\{X,\phi,\{2\},\{1,2\},\{2,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only fine-closed sets of Y are $\phi, X, \{1\}, \{3\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{c\}, \{a,b\}$ which are $f\omega$-clopen in X. Hence, f is strongly $f\omega$ −continuous.

Definition 3.9 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is called contra-$f\omega$-continuous if $f^{-1}(V)$ is $f\omega$-open in (X, τ) for every fine-closed set V in Y.

Example 3.8 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$, $\tau_f=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{1\}\}\tau_f'=\{X,\phi,\{1\},\{1,2\},\{1,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only fine-closed sets of Y are $\phi, X, \{1\}, \{1,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{a,b\}, \{a,c\}$ which are $f\omega$-open in X.

Definition 3.10 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is said to be $f\omega$-irresolute if $f^{-1}(F)$ is $f\omega$-closed in X for every $f\omega$-closed set F of Y.

Example 3.9 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\}\}$, $\tau_f=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{1\}\}$, $\tau_f'=\{X,\phi,\{1\},\{1,2\},\{1,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only $f\omega$ −closed sets of Y are $\phi, X, \{1\}, \{1,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{a,b\}, \{a,c\}$ which are $f\omega$ −closed in X. Hence, f is $f\omega$ −irresolute.

Remark 3.3 From definition 2.16 and above definitions it can be observe that

⇒ Totally continuous
⇒ Strongly continuous
⇒ ω −continuous

Fine-irresolute mapping

⇒ Contra continuous
⇒ Totally ω −continuous
⇒ Strongly ω −continuous
⇒ Contra ω −continuous
⇒ ω −irresolute

4. fω-Homeomorphism

In this section we define $f\omega$-homeomorphism in fine-topological space.

Definition 4.1 A function $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ is said to be an $f\omega$-homeomorphism if

- f is bijective
- f and f^{-1} is $f\omega$-irresolute.

Example 4.1 Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\}\}$, $\tau_f=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$ and $Y=\{1,2,3\}$, $\tau'=\{X,\phi,\{1\}\}$, $\tau_f'=\{X,\phi,\{1\},\{1,2\},\{1,3\}\}$. We define a map $f:(X,\tau,\tau_f)\rightarrow(Y,\tau',\tau_f')$ by $f(a)=1, f(b)=2, f(c)=3$. It may be easily checked that, the only $f\omega$ − closed sets of Y are $\phi, X, \{1\}, \{1,2\}, \{1,3\}$ and their pre-images are $X, \phi, \{a\}, \{a,b\}, \{a,c\}$ which are $f\omega$ −closed in X. Thus, f is $f\omega$ −irresolute. It may also checked that, f^{-1} is $f\omega$ − irresolute. Hence, f is $f\omega$ −homeomorphism.

Definition 4.2 A fine topological space X is said to be $f\omega - T_0$ space if for each pair of distinct points x and y of X, there exists a $f\omega$−open set containing one point but not the other.

Definition 4.3 A fine-topological space X is said to be $f\omega$−T_1 space if for any pair of distinct point x and y, there exists $f\omega$−open sets G and H such that $x\in G, y\notin G$ and $x\notin H, y\in H$.

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Definition 4.4 A fine-topological space X is said to be $f\omega - T_2$ space if for any pair of distinct points x and y, there exists disjoint $f\omega-$open sets G and H such that $x \in G$ and $y \in H$.

Definition 4.5 A fine-topological space X is said to be $f\omega-$regular if for each closed set F and each point $x \notin F$, there exist disjoint f$\omega-$open sets U and V such that $x \in U$ and $F \subset V$.

Definition 4.6 A fine-topological space X is said to be $f\omega$ $-$normal if for each pair of non-empty disjoint $f\omega$ $-$closed sets can be separated by disjoint $f\omega$ $-$open sets.

Definition 4.7 A fine-topological space X is said to be fine-ultra normal if for each pair of non-empty disjoint fine$-$closed sets can be separated by disjoint f $-$clopen sets.

Definition 4.8 A pair (A, B) of non-empty subsets of a topological space (X,τ) is said to be an $f\omega-$seperation relative to X if $f\omega cl(A) \cap B = \phi$ and $A \cap f\omega cl(B) = \phi$ holds. For a point $x \in X$, the set of all ponts y in X such that x and y cannot be separated by an $f\omega$ $-$seperation of X is said to be the quasi $f\omega-$ component containing x in (X,τ,τ_f).

Theorem 4.1 Let $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ be a totally $f\omega$ $-$continuous function and Y be a T_1 $-$space. Then, f is constant on each quasi $f\omega$ $-$component in X.

Proof. Let x and y be points in (X,τ,τ_f) such that x and y lie in the same quasi $f\omega$-component of X with $x \neq y$. Assume that $f(x) \neq f(y)$, say $a = f(x)$ and $b = f(y)$. It follows from assumptions that $f^{-1}(\{a\})$ and $f^{-1}(Y \setminus \{a\})$ are disjoint $f\omega$ -clopen subsets of (X,τ,τ_f) and $x \in f^{-1}(\{a\}), y \in f^{-1}(Y \setminus \{a\})$. There exists an $f\omega$-separation $(f^{-1}(\{a\}))$, $f^{-1}(Y \setminus \{a\})$ relative to X such that x and y are separated by $(f^{-1}(\{a\}), f^{-1}(Y \setminus \{a\}))$. Thus, x and y don't lie the same quasi $f\omega$ -component in (X,τ,τ_f). This contradicts to the assumption. Therefore, we have that $f(x) = f(y)$ for any points x and y which belong to the same quasi-$f\omega$-component.

Theorem 4.2. Let $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ be a totally $f\omega$-continuous function and $(Y,\tau',\tau_f'$ is a T_1-space. Let A be an open and semi-closed subset of (X,τ,τ_f). If A is an $f\omega$-connected subset of (X,τ,τ_f), then $f(A)$ is a single point.

Proof. Since $A \neq \emptyset$, there exists a point $b \in f(A)$. Since singleton $\{b\}$ is closed in (Y,σ), $f^{-1}(\{b\})$ is a subset of A and it is $f\omega$-clopen in (X,τ,τ_f). In general, we can prove that:
(1) If $B \subset H \subset X$ and B is $f\omega$-open in (X,τ,τ_f) and H is open and semi-closed in (X,τ,τ_f), then B is $f\omega$-open in a subspace $(H,\tau|H)$. (2) If $B \subset H \subset X$ and B is $f\omega$-closed in (X,τ,τ_f), then B is $f\omega$-closed in a subspace $(H,\tau|H)$. By (1) (resp. (2)), $f^{-1}(\{b\})$ is an $f\omega$-open set (resp. $f\omega$-closed set) in $(A,\tau|A)$. Namely, there exists a non-empty $f\omega$-clopen set $f^{-1}(\{b\})$ of a subspace $(A,\tau|A)$. Since A is $f\omega$-connected set (i.e., a subspace $(A,\tau|A)$ is ω-connected as a topological space), we conclude that $f^{-1}(\{b\}) = A$. Therefore, we have that $f(A) = \{b\}$.

Theorem 4.3 If $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is a contra-$f\omega$-continuous, fine-closed injection and Y is ultra- fine normal, then X is $f\omega$-normal.

Proof. Let F_1 and F_2 be disjoint $f\omega$-closed subsets of X. Since f is fine-closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint fine-closed subsets of (Y,τ'). Since $(Y,\tau')$is ultra fine-normal, $f(F_1)$ and $f(F_2)$ are seperated by disjoint fine-clopen sets V_1 and V_2 respectively. Hence, $F_i \subset f^{-1}(V_i), f^{-1}(V_i) \in \omega(X,\tau,\tau_f)$ for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus, (X,τ,τ_f) is $f\omega$-normal.

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