

Application of Queuing Model in a Restaurant (A Case Study of Kalinga Restaurant)

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Abstract: In this work, to show that real life cases scenario when tested satisfied queuing theory, we obtain data from kalinga restaurant, Jodhpur. We then calculated the arrival rate, service rate, utilization rate, waiting time and the probability of potential customers to balk based on the collected data, Little's theorem and M/M/1 queuing model. The arrival rate during busiest period of the day is 1.42 customer per minute. The average number of customers in the restaurant is 70 and utilisation rate of 0.9859. We conclude the work by given the concluding remarks.

Keywords: Queue, Little's theorem, exponential distribution

1. Introduction

There are many factors for a restaurant to attract customers, the most important factors are; taste, cleanliness, [5] food quality, service quality restaurant environment and setting. [2] These factors, when managed carefully, will be able to attract plenty of customers. Furthermore, there is another important factor that needs to be considered, that is queuing time. Queuing theory is the study of queue or waiting lines. Some of the analysis that can be derived using queuing theory includes the expected queue length, the probability of the system to be in certain states, such as empty or full [1].

2. Queuing Theory

In 1903 FrJohannsen was appointed copenhengen telephone company director and he observed that the manual switch were not dimensioned in the right way. FrJohannsen published some work using the mathematical theory of probability from an economic point of view he stated that in one of his work "The overloading of the subscribers resulted in considerable extra expenses on account of the telephone operators having to make repeated attempt to establish a connection".

In order to develop more precise dimensioning FrJohannsen established scientific laboratory where he engaged the mathematician AgnerKrarupErlang. Erlang stated to work on the holding time in a telephone switch and then identified that the telephone conversations satisfied a Poisson distribution as well as the telephone holding time was exponentially distributed. Erlang work is the origin of the current network optimisation and queuing theory [6].

3. Little's Theorem

Little's theorem [3, 4] describes the relationship throughout arrival and service rate, cyclic time and work process. The theorem states that the expected numbers of customers L for a system in steady state can be derive from little's theorem;

$$L = \lambda T \quad \dots(1)$$

Where λ is the average customers arrival rate and T is the average service time for a customer.

Three fundamental relationships can be derive from little's theorem

- L Increase if λ or T increase
- λ Increase if L increase or T decrease
- T Increase if L increase or λ decrease

4. M/M/1 Queuing Model

M/M/1 queuing model means that arrival and service time are exponentially distributed or Poisson process.

The following variables will be investigated;

- λ : The mean customer's arrival rate
- μ : The mean service rate

$$\rho = \frac{\lambda}{\mu} : \text{Utilization factor} \quad \dots(2)$$

- P_n : The probability of having n customers

$$P_n = (1 - \rho) \rho^n \quad \dots(3)$$

- * Probability of zero customers

$$P_0 = 1 - \rho \quad \dots(4)$$

- L : The average number of customers

$$L = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda} \quad \dots(5)$$

- L_q : The average number of customers in the queue

$$L_q = L \times \rho = \frac{\rho^2}{1 - \rho} = \frac{\rho \lambda}{\mu - \lambda} \quad \dots(6)$$

- W_q : The average waiting time in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{1}{\mu - \lambda} \quad \dots(7)$$

• * W : The average time spent, including the waiting time

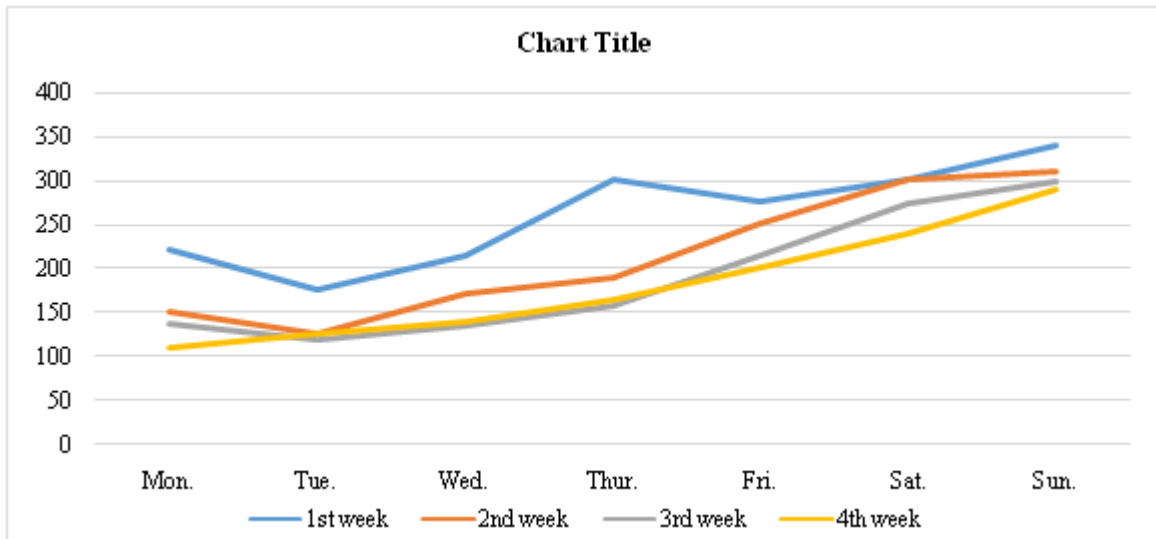
$$W = \frac{L}{\lambda} = \frac{1}{\mu - \lambda} \quad \dots(8)$$

5. Observation

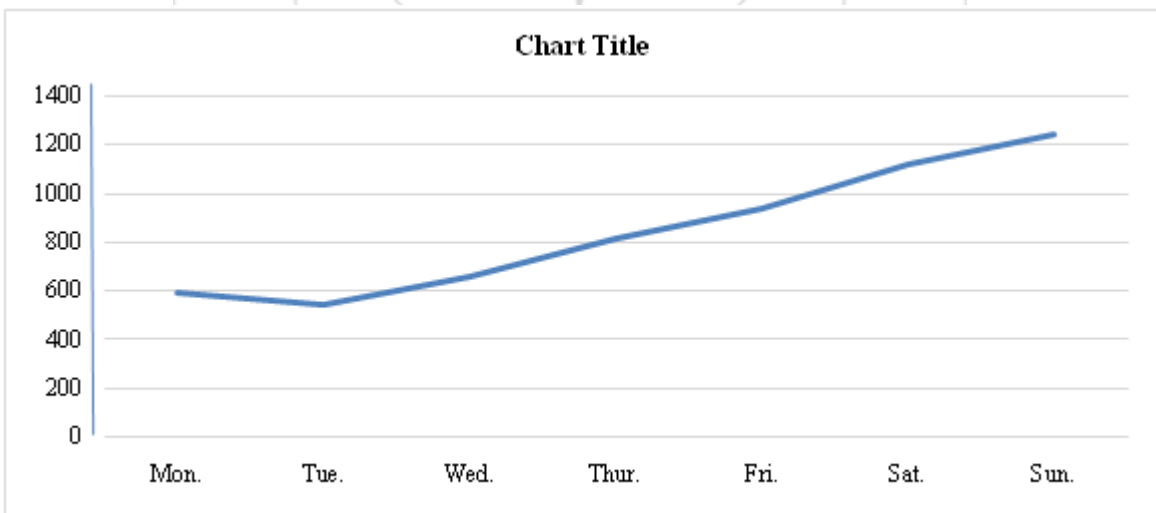
Below is one month daily customer data which was collected from the restaurant

Table One month customer count

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
1 st week	200	175	215	300	275	300	340
2 nd week	150	125	170	190	250	300	310
3 rd week	137	119	134	157	215	275	300
4 th week	110	125	140	165	200	240	290
Total	597	544	659	812	940	1115	1240



One month daily customers count



One month total Customers Count

6. Calculations

It was investigated that, after Friday, during the weekend there are on average 210 people coming to the restaurant in $2\frac{1}{2}$ hours' time of dinner. From this we obtain the arrival rate as;

$$\lambda = \frac{210}{150} = 1.4 \text{ Customers/minutes}$$

We also found out from observations and discussions with the manager that each customer spend 50 minutes on average in the restaurant (W) the queue length is around 25 people (L_q) on average and waiting time is around 15 minutes. It can be shown that by (7) that the observed actual waiting time does not differ much when compared with to the theoretical waiting time as shown below;

$$W_q = \frac{25 \text{ customers}}{1.4 \text{ cpm}} = 17.85714286$$

Next, we will calculate the average number of people in the restaurant using (6)

$$L = 1.4cpm \times 50 \text{ min}$$

$$= 70 \text{ customers}$$

Using (2) & (5) we can also calculate the service and utilization rate as shown below;

$$\mu = \frac{1.4(1 + 70)}{70}$$

$$= 1.42$$

Probability of customers going away

=P (more than 10 people in the queue)

=P (more than 10 people in the restaurant)

$$\therefore P_{111-130} = \sum_{n=111}^{130} P_n$$

$$= 0.01408 \sum_{n=111}^{130} (0.9859)^n$$

$$= 0.051157132$$

$$= 5.1157\%$$

7. Concluding Remarks

- The utilization rate at the restaurant is very large (0.9859). This is only utilization rate during lunch and dinner on Saturday and Sunday.
- In case the customer waiting time is less than 15mins, the number of customers that will be served per minute will increase. When the service rate is higher the utilization rate will be lower.
- The queue model formulation is much simpler than to provide simulation model.
- This research can help to increase the quality of service by anticipating if there are many customers in the queue.

References

- [1] Dharmawirya, M., & Adi, E., "Case study for restaurant queuing model" 2011 international conference on management and artificial intelligence. IPEDR vol.6 (2011), pp. 52-55.
- [2] Haghghi, M., Dorosti, A., Rahnama, A., & Hoseinpour, A. "Evaluation of factors affecting customer loyalty in restaurant industry". African Journal of Business Management, vol. 6(14), April, 2012, pp. 5041.
- [3] Little, J.D.C., "A proof for the queuing formula $L = \lambda W$ " Operations Research, vol. 9(3), 1961, pp. 383-387, doi: 10.2307/167570.
- [4] Little, J.D.C., "Little's law as viewed on its 50th anniversary" Operations Research vol. 59, No. 3 May-June 2011, pp. 536-549.
- [5] Mishra, G.D., Chauhan, V.S., & Chandra, N. "A study of service in restaurant by using queuing model". The Bulletin of society for mathematical service and standards vol. 5 (2013), pp. 14-18.
- [6] Stardahl, K. "The History behind the Probability Theory and Queuing Theory" elektronik 2.2007.