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Exact Solutions for the Mikhailov-Shabat Equation, and Classical Boussinesq Equation by Tan-Cot Method

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Abstract: ***In this paper, we established a travelling wave solution by using the proposed Tan-Cot function algorithm for non-linear partial differential equations. The method is used to obtain new solitary wave solutions for non-linear partial differential equations such as, for the Mikhailov-Shabat(MS) equation, and Classical Boussinesq (CB) equation, which are the important Soliton equations. Proposed method has been successfully implemented to establish new solitary wave solutions for the non-linear PDEs.***

Keywords: Non-linear PDEs, Tan-Cot function method, Mikhailov-Shabat (MS) equation, Classical Boussinesq (CB) equation

1. Introduction

Large varieties of physical, chemical, and biological phenomena are governed by non-linear partial differential equations. One of the most exciting advances of non-linear science and theoretical physics has been the development of methods to look for exact solutions of non-linear partial differential equations [1] .Exact solutions to non-linear partial differential equations play an important role in non-linear science, especially in non-linear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Non-linear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in non-linear wave equations.In recent years, quite a few methods for obtaining explicit travelling and solitary wave solutions of non-linear evolution equations have been proposed. A variety of powerful methods, such as, tanh-sech method [2,3, 4],extended tanh method [5,6,7],hyperbolic function method [8,9], Jacobi elliptic function expansion method [10], F-expansion method [11], and the First Integral method [12,13].The sine-cosine method [14,15,3] has been used to solve different types of non-linear systems of PDEs. In this paper,we applied the Tan-Cot method [6–8] to solve the Mikhailov-Shabat (MS) equation, and Classical Boussinesq(CB)equation given respectively by:

$$p_t = p_{xx} + (p+q)q_x - \frac{1}{6}(p+q)^3 ;$$
$$-q_t = q_{xx} - (p+q)p_x - \frac{1}{6}(p+q)^3 \qquad (1)$$
$$u_t + [(1+u)v]_x = -\frac{1}{4}v_{xxx} ;$$
$$v_t + vv_x + u_x = 0 \qquad (2)$$

2. The Tan-Cot Function Method

Consider the non-linear partial differential equation in the form

$$F(u, u_t, u_x, u_y, u_{xy}, u_{tt}, u_{xx}, u_{tx} \ldots\ldots\ldots)=0 \qquad (3)$$

where$u(x,y,t)$ is a travelling wave solution of non-linear partial differential equation Eq.(3).We use the transformations

$u(x,y,t) = f(\xi)$

where

$$\xi = x + y - \lambda t.$$

This enables us to use the following changes,

$$\frac{\partial}{\partial t}(.) = -\lambda\frac{d}{d\xi}(.), \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.), \frac{\partial}{\partial y}(.) = \frac{d}{d\xi}(.) \qquad (4)$$

Using Eq.(4) to transfer the non-linear partial differential equation Eq.(3) to non-linear ordinary differential equation

$$Q(f, f', f'', f''', \ldots\ldots) = 0 \qquad (5)$$

The ordinary differential equation (5) is then integrated as long as all terms contain derivatives, where we neglect the integration constants. The solutions of many non-linear equations can be expressed in the form:

$$f(\xi) = \alpha \tan^{\beta}(\mu\xi), |\xi| \le \frac{\pi}{2\mu}$$
$$f(\xi) = \alpha \cot^{\beta}(\mu\xi), |\xi| \le \frac{\pi}{2\mu} \qquad (6)$$

Where *α,μ,β parameters to be determined, μ and λ*are the wave number and the wave speed respectively

We use

$$f(\xi) = \alpha \tan^{\beta}(\mu\xi)$$
$$f' = \alpha\beta\mu\left[tan^{(\beta-1)}(\mu\xi) + tan^{(\beta+1)}(\mu\xi)\right]$$
$$f'' = \alpha\beta\mu^2\left[(\beta-1)tan^{(\beta-2)}(\mu\xi) + 2\beta\tan^{\beta}(\mu\xi) + (\beta+1)tan^{(\beta+2)}(\mu\xi)\right] \qquad (7)$$

And their derivatives or use,

$$f(\xi) = \alpha \cot^{\beta}(\mu\xi)$$
$$f' = -\alpha\beta\mu\left[cot^{(\beta-1)}(\mu\xi) + cot^{(\beta+1)}(\mu\xi)\right]$$
$$f'' = \alpha\beta\mu^2\left[(\beta-1)cot^{(\beta-2)}(\mu\xi) + 2\beta\cot^{\beta}(\mu\xi) + (\beta+1)cot^{(\beta+2)}(\mu\xi)\right] \qquad (8)$$

and so on. We substitute (7)or (8)into the reduced equation (5),balance the terms of the tan functions when (7) are used, or balance the terms of the cot functions when(8)are used ,and solve the resulting system of algebraic equations by using computerized symbolic packages. Next we collect all terms with the same power in $\tan^k(\mu\xi)$ or $\cot^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations with the unknowns α, β, μ and solve the subsequent system of equations.

3. Applications

3.1 The Mikhailov-Shabat (MS) Equation

In this section we deal with the Mikhailov-Shabat (MS)equations

$$p_t = p_{xx} + (p+q)q_x - \frac{1}{6}(p+q)^3$$

$$-q_t = q_{xx} - (p+q)p_x - \frac{1}{6}(p+q)^3 \qquad (9)$$

In order to solve *MS system* (9) we now introduce the transformation

$$u(x,t) = p(x,t) + q(x,t),$$

$$v(x,t) = q_x(x,t) - p_x(x,t) \qquad (10)$$

Then the *MS system* (9) becomes

$$u_t + v_x - u u_x = 0$$

$$v_t + (uv)_x - u^2 u_x + u_{xxx} = 0 \qquad (11)$$

Substituting

$$u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = x + \lambda t \qquad (12)$$

Where λ is a real constant.

Hence, substitute (12) in Eq.(11),we get the following ODEs

$$\lambda u' + v' - u u' = 0 \qquad (13)$$

$$\lambda v' + (uv)' - u^2 u' + u''' = 0 \qquad (14)$$

Integrating Eq.(13)and (14) once with zero constants to get:

$$\lambda u + v - \frac{u^2}{2} = 0 \qquad (15)$$

$$\lambda v + uv - \frac{u^3}{3} + u'' = 0 \qquad (16)$$

Assume the following solution in (7)

$$u(\xi) = \alpha_1 \tan^{\beta_1}(\mu\xi) \qquad (17)$$

$$v(\xi) = \alpha_2 \tan^{\beta_2}(\mu\xi) \qquad (18)$$

Substitute Eq.(17) and(18) and there derivatives in Eqs.(15) and (16)to get:

$$\lambda\alpha_1 \tan^{\beta_1}(\mu\xi) + \alpha_2 \tan^{\beta_2}(\mu\xi) - \frac{1}{2}\alpha_1{}^2 \tan^{2\beta_1}(\mu\xi) = 0 \qquad (19)$$

$$\lambda\alpha_2 \tan^{\beta_2}(\mu\xi) + \alpha_1\alpha_2 \tan^{(\beta_1+\beta_2)}(\mu\xi) - \frac{1}{3}\alpha_1{}^3 \tan^{3\beta_1}(\mu\xi) + \alpha_1\beta_1\mu^2\big[(\beta_1 - 1)tan^{(\beta_1-2)}(\mu\xi) + 2\beta_1 \tan^{\beta_1}(\mu\xi) + (\beta_1+1)tan^{(\beta_1+2)}(\mu\xi)\big] = 0 \qquad (20)$$

From Eqs.(19) and (20) we have

$2\beta_1 = \beta_2;$

$\beta_1 + \beta_2 = \beta_1 + 2$

Then, $\beta_2 = 2; \beta_1 = 1.$

From Equations (19) and (20) we get the following system

$$\alpha_2 - \frac{1}{2}\alpha_1^2 = 0 \qquad (21)$$

$$\alpha_1\alpha_2 - \frac{1}{3}\alpha_1^3 + 2\alpha_1\mu^2 = 0 \qquad (22)$$

Solving the system in Eq.(21) and (22),we get

$$\alpha_1 = 2\sqrt{3}i\mu; \ \alpha_2 = -6\mu^2 \qquad (23)$$

Then

$$u(x,t) = 2\sqrt{3}i\mu \ tan\{\mu(x+\lambda t)\} \qquad (24)$$

$$v(x,t) = -6\mu^2 \tan^2\{\mu(x+\lambda t)\} \qquad (25)$$

Figure (1) and (2)respectively represent $u(x,t)$ in (24) and$v(x,t)$ in (25)for$\lambda = 2; \mu = 1$ and $-10 \le x \le 10$; $-1 \le t \le 1$.

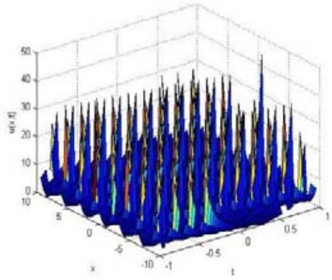

Figure 1: Presentation of $u(x,t)$ in (24) for $-10 \le x \le 10$ and $-1 \le t \le 1$.

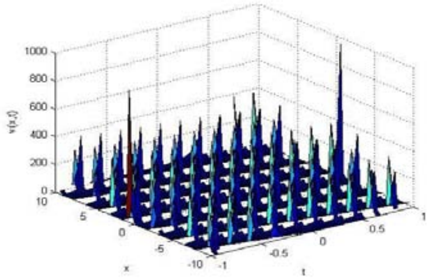

Figure 2: Presentation of $v(x,t)$ in (25) for $-10 \le x \le 10$ and $-1 \le t \le 1$.

3.2. The Classical Boussinesq (CB) equation

Now we deal with the Classical Boussinesq(CB) equations [18],

$$u_t + [(1+u)v]_x + \frac{1}{4}v_{xxx} = 0$$

$$v_t + v v_x + u_x = 0 \qquad (26)$$

In order to obtain travelling wave solutions of equation (26), we make the transformations

$$u(x,t) = u(\xi); \ v(x,t) = v(\xi); \ \xi = x + \lambda t \qquad (27)$$

Where λ is real constant

Hence, substitute Eq.(27) in Eq.(26),we get the following ODEs

$$\lambda u' + [(1+u)v]' + \frac{1}{4}v''' = 0 \qquad (28)$$

$$\lambda v' + v v' + u' = 0 \qquad (29)$$

Integrating Eq.(28)and (29)once with zero constants we have,

$$\lambda u + (1+u)v + \frac{1}{4}v'' = 0 \qquad (30)$$

$$\lambda v + \frac{v^2}{2} + u = 0 \qquad (31)$$

Assume the following solution in Eq.(7)

$$u(\xi) = \alpha_1 \tan^{\beta_1}(\mu\xi) \qquad (32)$$

$$v(\xi) = \alpha_2 \tan^{\beta_2}(\mu\xi) \qquad (33)$$

Substitute Eq.(32) and (33) and there derivativesinEqs.(30) and (31) we have,

$$\lambda\alpha_1 \tan^{\beta_1}(\mu\xi) + \big[\alpha_2 \tan^{\beta_2}(\mu\xi) + \alpha_1\alpha_2 tan^{\beta_1+\beta_2}(\mu\xi)\big] + \frac{1}{4}\alpha_2\beta_2\mu^2\big[(\beta_2 - 1)tan^{(\beta_2-2)}(\mu\xi) + 2\beta_2 \tan^{\beta_2}(\mu\xi) + (\beta_2+1)tan^{(\beta_2+2)}(\mu\xi)\big] = 0 \qquad (34)$$

$$\lambda\alpha_2 \tan^{\beta_2}(\mu\xi) + \frac{1}{2}\alpha_2{}^2 \tan^{2\beta_2}(\mu\xi) + \alpha_1 \tan^{\beta_1}(\mu\xi) = 0 \qquad (35)$$

From Eq.(34) and (35) we have,

$$\beta_1 + \beta_2 = \beta_2 + 2$$

$$\beta_1 = 2\beta_2$$

.Then, $\beta_1 = 2; \beta_2 = 1.$

From Equations (34) and (35) we get the following system

$$\alpha_1\alpha_2 + \frac{1}{2}\alpha_2\mu^2 = 0 \qquad (36)$$

$$\frac{1}{2}\alpha_2^2 + \alpha_1 = 0 \qquad (37)$$

Solving the system in Eq.(36) and (37),we get

$$\alpha_1 = -\frac{\mu^2}{2};\ \alpha_2 \qquad \mu \qquad (38)$$

Then

$$u(x,t) = -\frac{\mu^2}{2}\tan^2\{\mu(x+\lambda t)\} \qquad (39)$$

$$v(x,t) \qquad \mu\ \tan\ \{\mu(x+\lambda t)\} \qquad (40)$$

Figure (3) and (4) respectively represent $u(x,t)$ in (39) and$v(x,t)$ in (40)for$\lambda = 2$;$\mu = 1.5$ and $-1 \leq x \leq 1$;$0 \leq t \leq 1$.

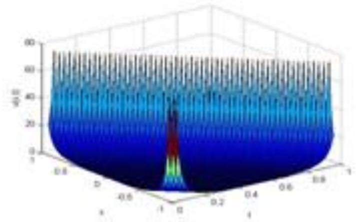

Figure 3: Presentation of $u(x,t)$ in (39) for $-1 \leq x \leq 1$ and $0 \leq t \leq 1$.

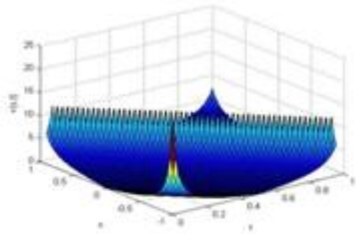

Figure 4: Presentation of $v(x,t)$ in (40) for $-1 \leq x \leq 1$ and $0 \leq t \leq 1$

4. Conclusions

In this paper, new method called the Tan-Cot function method has been successfully implemented to establishnew solitary wave solutions for the Mikhailov-Shabat (MS)equations and the Classical Boussinesq (CB) equations which are the non-linear PDEs.We can say that the new method can be extended to solve the problems of non-linear partial differential equations which arising in the theory ofsolitons and other areas; see [19-25].

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