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Some New Characterizations of Spacelike Curves According to Type-2 Bishop Frame in Eculidean 4-Space E_1^4

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Abstract: ***In this study, some characterizations of the Space Curves in Euclidean 4- Space E_1^4 with constant curvatures spanned by subspaces of E_1^4 according to type-2 Bishop Frame were given after we investigated the position vectors of space curve with constant curvatures***

Keywords: Euclidean SpaceE_1^4; Second type Bishop Frame; *Space Curves*

1. Introduction

The characterizations of space curves with respect to Frenet and Bishop Frames have been studied by many authors. In [7] M. A. Akgun, A. I. Sivridag were give some characterizations for spacelike curves by studied the positionvectors of a spacelike curve on some subspaces in Minkowski 4-space R_1^4

In [5] The author gave us the sufficient conditions of null curves to be osculating curves in terms of their curvature functions by obtaining some relations between null rectifying curves in E_1^4 and null osculating curves with some example of null osculating after maked relations between null normal curves and null osculating curves. In [2] Fathi studied the position vectors of space curves in Euclidean 3-Space depending on the Type-1 Bishop Frame with constant Curvatures. In [8] According to type-2 Bishop frame the authors gave us some characterizations of spacelike curve with principal normal vector, in addition to that position vector of spacelike curves on Lorentzian sphere with respect to the type-2 Bishop curvatures was obtained and also they established some relations among Frenet apparatus. In [6] the authors were discussome parametrizations of rectifying curves after they gave thecharacterize of non–null and null rectifying curves which is lying fully in the Minkowski3–space E_1^3. The general helix position vector with respect to Frenet frame in Euclidean 3-Space studied by A. T. Ali in [1] and in addition to that the natural representation of a general helix in terms of the curvature and torsion was deduced. In [4] the authors were study and characterize the rectifying curves in E^4 after they defined it in the Euclidean 4-space as position vector always lies in orthogonal complement. In [3] the author used Laplacian operator and Levi-Civita connection to obtained some characterizations of timelike curves in mMinkowski 3-space E_1^3 according to Bishop frame and also some characterization of timelike curves was studied according to the Bishop Darboux vector and the normal Bishop Darboux vector by gave it is general differential equations . In [9] some characterizations of closed dual curves of constant breadth were presented in dual Euclidean space and discussed by the author according to Bishop Frame in addition to that third order vectorial differential equation has been obtained in dual Euclidean 3-space.

2. Preliminaries

In this paper E_1^4denote to the Minkowski 4-space together with a metric $\langle , \rangle$ of sign nature $(-,+,+,+)$.

For each
$X = (a_1, a_2, a_3, a_4) \in E_1^4 \quad , Y = (b_1, b_2, b_3, b_4) \in E_1^4$
The standard scalar product in the Euclidean 4- space E_1^4 given by:

$$\langle X, Y \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 - a_4 b_4 \qquad (2.1)$$

Since a vector X is said to be Timelike if$\langle X, X \rangle < 0$, Spacelike if $\langle X, X \rangle > 0$ and null(Lightlike) if $\langle X, X \rangle = 0$ and $X \neq 0$. Where $\|X\| = \sqrt{|\langle X, X \rangle|}$ denoted to the norm of a vector X

If we take an arbitrary curve$\alpha : I \subset R \to E_1^4$ in E_1^4 . Recall that the curve α is said to be of unit speed (or parameterized by arc length function s) if$\langle \alpha'(s), \alpha'(s) \rangle = 1$
Let $\{T, N_1, N_2, B\}$ Denoted the type-2 Bishop moving frame along the unit Speed curveα. Where the vectors T,N_1, $N_2 and B$ are mutually orthogonal vectors satisfying $\langle T, T \rangle = \langle N_1, N_1 \rangle = \langle N_2, N_2 \rangle = 1, \langle B, B \rangle = -1.$

Then the type-2 Bishop formula [3] . And satisfying α are:

$$\begin{pmatrix} T \\ N_1 \\ N_2 \\ B \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -k_1 \\ 0 & 0 & 0 & -k_2 \\ 0 & 0 & 0 & -k_3 \\ k_1 & k_2 & k_3 & 0 \end{pmatrix} \begin{pmatrix} T \\ N_1 \\ N_2 \\ B \end{pmatrix} \quad (2.2)$$

Where k_1 , k_2 and k_3 are the first, second and third Bishop curvature.

3. Some NEW Characterizations of a Spacelike Curves with constant Curvatures According to Type-2 Bishop Frame in Euclidean 4-Space E_1^4

In this section we will give some investigation to characterize the Spacelike curves that is lie on some subspaces of E_1^4 . Let $\propto$ be a Spacelike curve in E_1^4 with the type-2 Bishop frame$\{T, N_1, N_2, B\}$.

Case 1: Firstly we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{T, N_1\}$. So we can write the position vector as:

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) \qquad (3.1)$$

Where $\lambda = \lambda(s) and \mu = \mu(s)$The equation(3.1) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_1 + \mu N_1' \qquad (3.2)$$

From theequations(2.2), we get the following:

$$\left.\begin{array}{c} \lambda' = 1 \\ \mu' = 0 \\ \lambda k_1 + \mu k_2 = 0 \end{array}\right\} \qquad (3.3)$$

So by solving these equations if $k_1 = k_2 = 0$ we find:

$$\mu(s) = c and \lambda = s + c_1,$$

$$\propto(s) = [s + c_1]T(s) + cN_1(s) \qquad (3.4)$$

Theorem 3.1: The Spacelike curve $\propto \subset E_1^4$ lies on the subspace spanned by $\{T, N_1\}$ if and only if it is in the form

$$\propto(s) = [s + c_1]T(s) + cN_1(s) \qquad (3.4)'$$

Case 2: Secondly suppose that the Spacelike curve $\propto$ lies on the subspace spanned by$\{T, N_2\}$. So the position vector can be written as:

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_2(s) \qquad (3.5)$$

Where$\lambda = \lambda(s) and \mu = \mu(s)$. The equation(3.5)after differentiation with respect to $s$became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_2 + \mu N_2' \qquad (3.6)$$

From the equations (2.2), we get the following:

$$\left.\begin{array}{c} \lambda' = 1 \\ \mu' = 0 \\ \lambda k_1 + \mu k_3 = 0 \end{array}\right\} \qquad (3.7)$$

The solution of these equations lead to:

$\mu(s) = c and \lambda = s + c_1$,if $k_1 = k_3 = 0$

So we find

$$\propto(s) = (s + c_1)T(s) + cN_1(s) \qquad (3.8)$$

Theorem 3.2: The Spacelike curve $\propto \subset E_1^4$ lies on the subspace spanned by $\{T, N_2\}$ if and only if it is in the form

$$\propto(s) = (s + c_1)T(s) + cN_1(s) \qquad (3.8)'$$

Case 3: Thirdly suppose that the Spacelike curve $\propto$ lies on the subspace spanned by$\{T, B\}$. So we can defined the position vector as:

$$\propto(s) = \lambda(s)T(s) + \mu(s)B(s) \qquad (3.9)$$

Where $\lambda = \lambda(s) and \mu = \mu(s)$. The equation(3.9) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' B(s) + \mu B(s)' \qquad (3.10)$$

From the equations (2.2), we get the following:

$$\left.\begin{array}{c} \lambda' + k_1\mu = 1 \\ k_2\mu = 0 \\ k_3\mu = 0 \\ \acute{\mu} - \lambda k_1 = 0 \end{array}\right\} \qquad (3.11)$$

If:$k_2 = k_3 = 0$, these equations lead to

$$\lambda = c_1\cos[k_1 s] - c_2\sin[k_1 s] \qquad (3.12)$$

$$\mu = c_2\cos[k_1 s] + \frac{\cos[k_1 s]^2}{k_1} + c_1\sin[k_1 s] + \frac{\sin[k_1 s]^2}{k_1} \qquad (3.13)$$

So

$$\propto(s) = [c_1\cos[k_1 s] - c_2\sin[k_1 s]]T(s) + \left[c_2\cos[k_1 s] + \frac{\cos[k_1 s]^2}{k_1} + c_1\sin[k_1 s] + \frac{\sin[k_1 s]^2}{k_1}\right]B(s) \qquad (3.14)$$

Theorem 3.3: Suppose that$\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{T, B\}$ if and only if it is in the form

$$\propto(s) = [c_1\cos[k_1 s] - c_2\sin[k_1 s]]T(s) + \left[c_2\cos[k_1 s] + \frac{\cos[k_1 s]^2}{k_1} + c_1\sin[k_1 s] + \frac{\sin[k_1 s]^2}{k_1}\right]B(s) \qquad (3.14)'$$

Case 4: In this case suppose that the Spacelike curve $\propto$ lies on the subspace spanned by$\{N_1, N_2\}$. So we can defined the position vector as:

$$\propto(s) = \lambda(s)N_1(s) + \mu(s)N_2(s) \qquad (3.15)$$

Where$\lambda = \lambda(s) and \mu = \mu(s)$. The equation(3.15) after differentiation with respect to s became:

$$\propto' = \lambda' N_1 + \lambda N_1' + \mu' N_2 + \mu N_2' \qquad (3.16)$$

From the equations (2.2) we find that:

$$\left.\begin{array}{c} \lambda' = 1 \\ \mu' = 0 \\ \lambda k_2 + \mu k_3 = 0 \end{array}\right\} \qquad (3.17)$$

If:$k_2 = k_3 = 0$ these equations lead to

$$\mu(s) = s + c and \lambda = s + c_1$$

Then:

$$\propto (s) = [s + c_1]N_1(s) + [s + c]N_2(s) \qquad (3.18)$$

Theorem 3.4: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{N_1, N_2\}$ if and only if it is in the form

$$\propto (s) = [s + c_1]N_1(s) + [s + c]N_2(s) (3.18)'$$

Case 5: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{N_1, B\}$. So we can defined the position vector as:

$$\propto (s) = \lambda(s)N_1(s) + \mu(s)B \qquad (3.19)$$

Where $\lambda = \lambda(s) and \mu = \mu(s)$. The equation (3.19) after differentiation with respect to s became:

$$\propto' = \lambda' N_1 + \lambda N_1' + \mu' B + \mu B' \qquad (3.20)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' + k_2\mu = 0 \\ k_1\mu = 1 \\ k_3\mu = 0 \\ \mu' - \lambda k_2 = 0 \end{aligned}\right\} \qquad (3.21)$$

So by solving these equations if $k_2 = k_3 = 0$ we find that:

$$\mu(s) = \frac{1}{k_1}, \quad \lambda(s) = \frac{k_2}{k_1}s + c$$

Then

$$\propto (s) = \left[\frac{k_2}{k_1}s + c\right] N_1(s) + \left[\frac{1}{k_1}\right] B \qquad (3.22)$$

Theorem 3.5: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{N_1, B\}$ if and only if it is in the form

$$\propto (s) = \left[\frac{k_2}{k_1}s + c\right] N_1(s) + \left[\frac{1}{k_1}\right] B (3.22)'$$

Case 6: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{N_2, B\}$. So we can defined the position vector as:

$$\propto (s) = \lambda(s)N_2(s) + \mu(s)B \qquad (3.23)$$

Where $\lambda = \lambda(s) and \mu = \mu(s)$. The equation (3.23) after differentiation with respect to s became:

$$\propto' = \lambda' N_2 + \lambda N_2' + \mu' B + \mu B' \qquad (3.24)$$

From the equations (2.2) we find that:

$$\left.\begin{aligned} \lambda' + k_3\mu = 0 \\ k_1\mu = 1 \\ k_2\mu = 0 \\ \mu' - \lambda k_3 = 0 \end{aligned}\right\} \qquad (3.25)$$

So by solving these equations if $k_3 = k_2 = 0$, we find:

$\mu = \frac{1}{k_1}$ and $\lambda(s) = \frac{-k_3}{k_1}s + c$

Then:

$$\propto (s) = \left[\frac{-k_3}{k_1}s + c\right] N_2(s) + \left[\frac{1}{k_1}\right] B \qquad (3.26)$$

Theorem 3.6: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{N_2, B\}$ if and only if it is in the form

$$\propto (s) = \left[\frac{-k_3}{k_1}s + c\right] N_2(s) + \left[\frac{1}{k_1}\right] B (3.26)'$$

Case 7: In this case suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{T, N_1, N_2\}$. So we can defined the position vector as:

$$\propto (s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) \qquad (3.27)$$

Where $\lambda = \lambda(s)$, $\mu = \mu(s) and \nu = \nu(s)$. The equation (3.27) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_1 + \mu N_1' + \nu' N_2 + \nu N_2' \qquad (3.28)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' = 1 \\ \mu' = 0 \\ \nu' = 0 \\ \mu k_2 + \lambda k_1 + \nu k_3 = 0 \end{aligned}\right\} \qquad (3.29)$$

If $k_1 = 0$, these equations lead to

$$\lambda = s + c \quad \mu = c_1 \quad , \nu = c_2$$

$$\propto (s) = [s + c]T(s) + c_1N_1(s) + c_2N_2(s) \qquad (3.30)$$

Theorem 3.7: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{T, N_1, N_2\}$ if and only if it is in the form

$$\propto (s) = [s + c]T(s) + c_1N_1(s) + c_2N_2(s) (3.30)'$$

Case 8: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{T, N_1, B\}$. So we can defined the position vector as:

$$\propto (s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)B(s) (3.31)$$

Where $\lambda = \lambda(s)$, $\mu = \mu(s) and \nu = \nu(s)$. The equation (3.31) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_1 + \mu N_1' + \nu' B + \nu B' \qquad (3.32)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' + \nu k_1 = 1 \\ \mu' + \nu k_2 = 0 \\ \nu k_3 = 0 \\ \lambda k_1 + \mu k_2 + \nu' = 0 \end{aligned}\right\} \qquad (3.33)$$

If $k_1 = 0$ these equations lead to

$$\lambda(s) = -\frac{k_1^{\,2}\ e^{-2ms}(-1+e^{2ms})(1+e^{2ms})}{4m^3} + \frac{(k_1^{\,2}k_2^{\,2})e^{-ms}(-1+e^{ms})^2(e^{ms}-e^{-ms}-2ms)}{4m^5} + \frac{e^{-ms}(k_1^{\,2}+k_1^{\,2}e^{2ms}+2k_2^{\,2}e^{ms})(k_1^{\,2}e^{ms}-k_1^{\,2}e^{-ms}+2mk_2^{\,2}s)}{4m^5} + \frac{c_1\ e^{-ms}(k_1^{\,2}+e^{2ms}k_1^{\,2}+2e^{ms}k_2^{\,2})}{2m^2} + \frac{k_1k_2c_2\ e^{-ms}(-1+e^{ms})^2}{2m^2} - \frac{k_1c_3\ e^{-ms}(-1+e^{2ms})}{2m} \quad (3.34)$$

$$\mu(s) = -\frac{k_1k_2e^{-2ms}(-1+e^{2ms})(1+e^{2ms})}{4m^3} + \frac{e^{-ms}k_1k_2(2e^{ms}k_1^{\,2}+k_2^{\,2}+e^{2ms}k_2^{\,2})(e^{ms}-e^{-ms}-2ms)}{4m^5} + \frac{k_1k_2\ e^{-ms}(-1+e^{ms})^2(me^{ms}-me^{-ms}+2k_2^{\,2}s)}{4m^4} + \frac{k_1k_2c_1\ e^{-ms}(-1+e^{ms})^2}{2m^2} + \frac{c_2\ e^{-ms}(2e^{ms}k_1^{\,2}+n^2+e^{2ms}n^2)}{2m^2} - \frac{k_2c_3e^{-ms}(-1+e^{2ms})}{2m} \quad (3.35)$$

$$\nu(s) = \frac{k_1\ e^{-2ms}(1+e^{2ms})^2}{4m^2} - \frac{k_1k_2^{\,2}e^{-ms}(-1+e^{2ms})(e^{ms}-e^{-ms}-2ms)}{4m^4} - \frac{k_1\ e^{-ms}(-1+e^{2ms})(k_1^{\,2}e^{ms}-k_1^{\,2}e^{-ms}+2mk_2^{\,2}s)}{4m^4} - \frac{k_1c_1e^{-ms}(-1+e^{2ms})}{2m} - \frac{k_2c_2\ e^{-ms}(-1+e^{2ms})}{2m} + \frac{c_3}{2}e^{-ms}(1+e^{2ms}) \quad (3.36)$$

Where $m = \sqrt{k_1^{\,2}+k_2^{\,2}}$

Theorem 3.8: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{T, N_1, B\}$ if and only if it is in the form

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)B(s) \quad (3.37)$$

Where $\lambda(s), \mu(s)\ and\ \nu(s)$ as an equations $(3.34), (3.35)\ and\ (3.36)$ respectively

Case 9: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{T, N_2, B\}$. So we can defined the position vector as:

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_2(s) + \nu(s)B(s) \quad (3.38)$$

Where $\lambda = \lambda(s)$, $\mu = \mu(s)\ and\ \nu = \nu(s)$. The equation (3.38) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_2 + \mu N_2' + \nu' B + \nu B' \quad (3.39)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' + \nu k_1 &= 1 \\ \mu' - \nu k_3 &= 0 \\ \nu k_2 &= 0 \\ \lambda k_1 + \mu k_3 + \nu' &= 0 \end{aligned}\right\} \quad (3.40)$$

If $k_2 = 0$ these equations lead to

$$\lambda(s) = \frac{k_1^{\,2}e^{-2ns}(-1+e^{2ns})(1+e^{2ns})}{4n(-n)^2} - \frac{k_3^{\,2}k_1^{\,2}\ e^{-ns}(-1+e^{ns})^2(e^{ns}-e^{-ns}-2ns)}{4n(-n)^4} - \frac{1}{4(-n)^4}e^{-ns}(-2e^{ns}k_3^{\,2}+k_1^{\,2}+e^{2ns}k_1^{\,2})\left(\frac{e^{-ns}k_1^{\,2}}{n} - \frac{e^{ns}k_1^{\,2}}{n} + 2k_3^{\,2}s\right) - \frac{c_1e^{-ns}(-2e^{ns}k_3^{\,2}+k_1^{\,2}+e^{2ns}k_1^{\,2})}{2(-n)^2} - \frac{k_1k_3c_2\ e^{-k_2s}(-1+e^{k_2ns})^2}{2(-n)^2} + \frac{k_1nc_3e^{-ns}(-1+e^{2ns})}{2(-n)^2} \quad (3.41)$$

$$\mu(s) = -\frac{k_1k_3e^{-2ns}(-1+e^{2ns})(1+e^{2ns})}{4(-n)^2n} + \frac{k_1k_3n\ e^{-ns}}{4(-n)^4}(k_3^{\,2}+e^{2ns}k_3^{\,2}-2e^{ns}k_1^{\,2})(e^{ns}-e^{-ns}-2ns) + \frac{k_1k_3\ e^{-ns}(-1+e^{ns})^2(e^{-ns}k_1^{\,2}-e^{-ns}k_1^{\,2}+2nk_3^{\,2}s)}{4n(-n)^4} + \frac{k_1k_3c_1e^{-ns}(-1+e^{ns})^2}{2(-n)^2} + \frac{c_2\ e^{-ns}(k_3^{\,2}+k_3^{\,2}e^{2ns}-2k_1^{\,2}e^{ns})}{2(-n)^2} - \frac{k_3nc_3\ e^{-ns}(-1+e^{2ns})}{2(-n)^2} \quad (3.42)$$

$$\nu(s) = \frac{k_1\ e^{-2ns}(1+e^{2ns})^2}{4n^2} + \frac{k_3^{\,2}k_1e^{-ns}}{4(-n)^4}(-1+e^{2ns})(e^{ns}-e^{-ns}-2ns) + \frac{k_1\ e^{-ns}}{4(-n)^4}(-1+e^{2ns})(k_1^{\,2}e^{-ns}-k_1^{\,2}e^{ns}+2k_3^{\,2}ns) + \frac{k_1nc_1\ e^{-ns}(-1+e^{2ns})}{2(-n)^2} + \frac{k_3nc_2e^{-ns}(-1+e^{2ns})}{2(-n)^2} + \frac{c_3}{2}e^{-ns}(1+e^{2ns}) \quad (3.43)$$

Then

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_2(s) + \nu(s)B(s) \quad (3.44)$$

Where $\lambda(s), \mu(s)\ and\ \nu(s)$ as in equations $(3.41), (3.42)\ and\ (3.43)$ respectively

Where $n = \sqrt{k_1^{\,2} - k_3^{\,2}}$

Theorem 3.9: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{T, N_2, B\}$ if and only if it is in the form (3.44)

Case 10: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{N_1, N_2, B\}$. So we can defined the position vector as:

$$\propto(s) = \lambda(s)N_1(s) + \mu(s)N_2(s) + \nu(s)B(s) \qquad (3.45)$$

Where $\lambda = \lambda(s)$, $\mu = \mu(s)$ and $\nu = \nu(s)$. The equation (3.45) after differentiation with respect to s became:

$$\propto' = \lambda' N_1 + \lambda N_1' + \mu' N_2 + \mu N_2' + \nu' B + \nu B' \qquad (3.46)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' + \nu k_2 &= 0 \\ \acute{\mu} + \nu k_3 &= 0 \\ \nu k_1 &= 1 \\ \lambda k_2 + \mu k_3 + \nu' &= 0 \end{aligned}\right\} \qquad (3.47)$$

These equations lead to

$$\nu(s) = \frac{1}{k_1} \qquad (3.48)$$

$$\lambda(s) = \frac{k_2}{k_1}s + c \qquad (3.49)$$

$$\mu = \frac{-k_2}{k_1}s + c_1 \qquad (3.50)$$

So

$$\propto(s) = \left[\frac{k_2}{k_1}s + c\right]N_1(s) + \left[\frac{-k_2}{k_1}s + c_1\right]N_2(s) + \left[\frac{1}{k_1}\right]B(s) \qquad (3.51)$$

Theorem 3.10: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{N_1, N_2, B\}$ if and only if it is in the form

$$\propto(s) = \left[\frac{k_2}{k_1}s + c\right]N_1(s) + \left[\frac{-k_2}{k_1}s + c_1\right]N_2(s) + \left[\frac{1}{k_1}\right]B(s) \qquad (3.51)'$$

Case 11: In this case we suppose that the Spacelike curve $\propto$ lies on the subspace spanned by $\{T, N_1, N_2, B\}$. So we can defined the position vector as:

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) + \nu(s)B(s) \qquad (3.52)$$

Where $\lambda = \lambda(s)$, $\mu = \mu(s)$ and $\nu = \nu(s)$. The equation (3.52) after differentiation with respect to s became:

$$\propto' = \lambda' T + \lambda T' + \mu' N_1 + \mu N_1' + \nu' N_2 + \nu N_2' + \delta' B + \delta B' \qquad (3.53)$$

From the equations (2.2) we get the following:

$$\left.\begin{aligned} \lambda' + \delta k_1 &= 1 \\ \acute{\mu} + \delta k_2 &= 0 \\ \nu' + \delta k_3 &= 0 \\ \delta' - \lambda k_1 - \mu k_2 - \nu k_3 &= 0 \end{aligned}\right\} \qquad (3.54)$$

These equations lead to

$$\lambda = \frac{k_1^{\,2} e^{-2rs}(-1+e^{2rs})(1+e^{2rs})}{4r^3} + \frac{1}{4r(-r)^4}(-1+e^{rs})^2(e^{rs} - e^{-rs} - 2rs)\left(k_3^{\,2}k_1^{\,2}e^{-rs} + k_1^{\,2}k_2^{\,2}e^{-rs}\right) + \frac{e^{-rs}}{4r(-r)^4}(2k_3^{\,2}e^{rs} + k_1^{\,2} + e^{2rs}k_1^{\,2} + 2k_2^{\,2}e^{rs})(k_1^{\,2}e^{rs} - k_1^{\,2}e^{-rs} + 2rk_3^{\,2}s + 2rk_2^{\,2}s) + \frac{c_1 e^{-rs}}{2(-r)^2}(2e^{rs}k_3^{\,2} + k_1^{\,2} + e^{2rs}k_1^{\,2} + 2e^{rs}k_2^{\,2}) + \frac{(-1+e^{rs})^2(k_3k_1c_2e^{-rs} + k_1k_2c_3e^{-rs})}{2(-r)^2} - \frac{k_1c_4e^{-rs}(-1+e^{2rs})}{2r} \qquad (3.55)$$

$$\mu = \frac{k_1k_2e^{-2rs}(-1+e^{2rs})(1+e^{2rs})}{4r^3} + \frac{k_3^{\,2}k_1k_2e^{-rs}}{4r(-r)^4}(-1+e^{rs})^2(e^{rs} - e^{-rs} - 2rs) + \frac{k_1k_2e^{-rs}}{4(-r)^4}(2k_3^{\,2}e^{rs} + 2k_1^{\,2}e^{rs} + k_2^{\,2} + e^{2rs}k_2^{\,2})\left(-\frac{e^{-rs}}{r} + \frac{e^{rs}}{r} - 2s\right) + \frac{k_1k_2e^{-rs}}{4(-r)^4}(-1+e^{rs})^2\left(-\frac{e^{-rs}k_1^{\,2}}{r} + \frac{e^{rs}k_1^{\,2}}{r} + 2k_3^{\,2}s + 2k_2^{\,2}s\right) + \frac{k_1k_2c_1e^{-rs}(-1+e^{rs})^2}{2(-r)^2} + \frac{k_3k_2c_2e^{-rs}(-1+e^{rs})^2}{2(-r)^2} + \frac{c_3e^{-rs}}{2(-r)^2}(2e^{rs}k_3^{\,2} + 2e^{rs}k_1^{\,2} + k_2^{\,2} + e^{2rs}k_2^{\,2}) - \frac{k_2c_4e^{-rs}(-1+e^{2rs})}{2r} \qquad (3.56)$$

$$\nu = \frac{k_3k_1e^{-2rs}(-1+e^{2rs})(1+e^{2rs})}{4r^3} + \frac{k_3k_1k_2{}^2e^{-rs}}{4r(-r)^4}(-1+e^{rs})^2(e^{rs}-e^{-rs}-2rs) + \frac{k_3k_1e^{-rs}}{4r(-r)^4}(k_3{}^2+k_3{}^2e^{2rs}+2k_1{}^2e^{rs} + 2k_2{}^2e^{rs})(e^{rs}-e^{-rs}-2rs) + \frac{k_3k_1e^{-rs}}{4r(-r)^4}(-1+e^{rs})^2(k_1{}^2e^{rs}-k_1{}^2e^{-rs}+2rk_3{}^2s+2rk_2{}^2s) + \frac{k_3k_1c_1e^{-rs}(-1+e^{rs})^2}{2(-r)^2} + \frac{c_2e^{-rs}}{2(-r)^2}(k_3{}^2+e^{2rs}k_3{}^2+2e^{rs}k_1{}^2+2e^{rs}k_2{}^2) + \frac{k_3k_2c_3e^{-rs}(-1+e^{rs})^2}{2(-r)^2} - \frac{k_3c_4e^{-rs}(-1+e^{2rs})}{2r} \quad (3.57)$$

$$\delta(s) = -\frac{k_1e^{-2rs}(1+e^{2rs})^2}{4(r)^2} + \frac{(-1+e^{2rs})(e^{rs}-e^{-rs}-2rs)(k_3{}^2k_1e^{-rs}+k_1k_2{}^2e^{-rs})}{4r^2(-r)^2} + \frac{k_1e^{-rs}(-1+e^{2rs})(e^{rs}k_1{}^2-e^{-rs}k_1{}^2+2rk_3{}^2s+2rk_2{}^2s)}{4r^2(-r)^2} + \frac{(-1+e^{2rs})(k_1c_1e^{-rs}+k_3c_2e^{-rs}+k_2c_3e^{-rs})}{2r} + \frac{c_4e^{-rs}}{2}(1+e^{2rs}) \quad (3.58)$$

So

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) + \nu(s)B(s) \quad (3.59)$$

Where $\lambda(s), \mu(s) and \nu(s)$ as in equations $(3.56), (3.57) and\ (3.58)$ respectively

And $r = \sqrt{-(k_1{}^2+k_2{}^2+k_3{}^2)}$

Theorem 3.4: Suppose that $\propto \subset E_1^4$ be Spacelike curve lies on the subspace spanned by $\{T, N_1, N_2, B\}$ if and only if it is in the form

$$\propto(s) = \lambda(s)T(s) + \mu(s)N_1(s) + \nu(s)N_2(s) + \nu(s)B(s) \quad (3.59)'$$

Where $\lambda(s), \mu(s) and \nu(s)$ as in equations $(3.55), (3.56), (3.57) and (3.58)$ respectively

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