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Non-Equilibrium Two Dimensional Electron-Lattice Interaction at Low Temperature

A. K. Ghorai

Physics Department, Kalimpong College, Darjeeling - 734301, West Bengal, India

Abstract: ***At low lattice temperature $(< 20\,\mathrm{K})$ a theory of the average rate of energy loss of non-equilibrium electrons in a two dimensional electron gas (2DEG) formed in semiconductor inversion layer has been developed here when the electrons interact only with the intravalley acoustic mode lattice vibrations. The results obtained from the theory both under low and high temperature conditions are compared and the utility of the theory in studying the high field electron transport in 2DEG is discussed.***

Keywords: Two dimensional electron gas, acoustic phonon, lattice temperature, electron temperature, electron energy loss

1. Introduction

With the advent of molecular beam epitaxy (MBE) and metal organic chemical vapour deposition (MOCVD), the fabrication of sophisticated semiconducting device structures whose dimensions are of the order of the de Broglie wavelength of the electrons has been realized. In such structures, electrons are confined in an active layer to form a two dimensional electron gas (2DEG) and electron motion normal to the layer is quantized. The study of electrical transport in 2DEG has assimilated much importance both from basic physics as well as device application of mesoscopic system. Such studies provide the way for practical realization of metal-oxide-semiconductor field effect transistor (MOSFET) having high mobility, better high frequency response and subsequently of high-speed logic circuits. Because of the advanced Si technology most of the studies made so far on 2DEG are devoted to the Si-SiO$_2$ system [1-12].

The transitions of a free carrier in surface layer are induced by different scattering sources and their relative importance is determined by the lattice temperature T_L and free carrier concentration N_i. At low temperatures the free carriers are dominantly scattered by the intravalley acoustic phonon and by impurity ions. The optical and intervalley phonon scattering can be important only at high temperatures when an appreciable number of corresponding phonons are excited or in the presence of a high electric field when the non-equilibrium electrons can emit high energy phonons. This apart, the scattering due to surface roughness may also arise because of non-planarity of the semiconductor interface. Of all these, the electron-phonon scattering is an intrinsic process and the scattering involving intravalley acoustic phonons is the most important mechanism in controlling the electrical transport at low lattice temperatures $(T_L < 20\,\mathrm{K})$ if the content of the impurity atoms in the system under study is relatively low [10]. It should be borne in mind that the possibility of obtaining the materials of higher and higher purity is not beyond the scope of the present day advanced semiconductor technology. Again at such low temperatures the electrons become hot in relatively weak fields of the order of only a few volts per centimeter. The electron transport under these conditions is limited by the acoustic scattering of the non-equilibrium carriers [13,14]. Thus the study of the problem of electrical transport in semiconductors particularly in 2DEG system at low lattice temperatures has become interesting.

Useful results on the study of the transport in 2DEG at low lattice temperatures have already been reported [7-12]. The 2DEG in GaAs has also been realized by Störmer et al. [15], who employed a GaAs-Ga$_x$Al$_{1-x}$As heterostructure and observed Shubnikov-de Haas oscillations around 4.2 K and reported the mobility values at the same temperatures. A theory of intravalley acoustic phonon scattering of the free carriers has been developed in 2DEG at low temperatures and the corresponding scattering rates are used to obtain the zero- field mobility characteristics in Si inversion layers with the help of Monte Carlo simulation of velocity autocorrelation function [10].

In this article the rate of increase of intravalley acoustic phonon due to scattering of the non-equilibrium electrons with acoustic mode lattice vibrations in a 2DEG system has been calculated. The rate of increase of phonon is then used to find out the average rate of energy loss of non-equilibrium electrons due to the acoustic phonon scattering of the free electrons. Once we know the average rate of energy loss, we may develop the carrier transport theory to find the high field electron mobility or field dependence of electron temperature in 2DEG formed in semiconducting materials. The same theory has already been developed in bulk semiconductor with the help of traditional practice [13] when the phonon energy can indeed be neglected as well as under the condition of low temperature when the phonon energy cannot be neglected in comparison to the carrier energy [16].

2. Theory

Under the application of electric field E the electron gains energy at the rate $e\mu E^2$, where e is the electronic charge and μ is the mobility of electrons. At low lattice temperature, a steady state may reach when the average rate of energy loss $\langle \frac{d\epsilon}{dt} \rangle_{ac}$ of the non-equilibrium electron due to acoustic phonon scattering is equal to the rate of energy gain from the field. Thus

$$e\mu E^2 = \langle \frac{d\epsilon}{dt} \rangle_{ac}. \tag{1}$$

Again if we know the rate of increase in acoustic phonon number $\left(\frac{\partial N_{\vec{q}}}{\partial t}\right)$ then we can calculate $\langle\frac{d\epsilon}{dt}\rangle_{ac}$ using the equation [13]

$$\langle\frac{d\epsilon}{dt}\rangle_{ac} = -\frac{1}{N_i s}\sum_{\vec{q}} \hbar u_l q \left(\frac{\partial N_{\vec{q}}}{\partial t}\right), \tag{2}$$

where s is the surface area, $\hbar$ is the Dirac constant, u_l is the acoustic phonon velocity and $\vec{q}$ is the phonon wave vector.

2.1 Rate of Increase in Acoustic Phonon

In 2DEG, we consider carrier transitions between two electronic states of wave vectors $\vec{k}$ and $\vec{k}+\vec{q}$ in the course of a collision accompanied with either emission or absorption of a phonon of wave vector $\vec{q}$, resulting an increase in the number of phonons $N_{\vec{q}}$. The rate of increase in the number of phonons can be written using the perturbation theory as [13]

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{2\pi}{\hbar}\sum_{\vec{k}}\Big[\left|\langle\vec{k}, N_{\vec{q}}+1\middle|H'_{ac}\middle|\vec{k}+\vec{q}, N_{\vec{q}}\rangle\right|^2 \\ \delta\left(\epsilon_{\vec{k}}, N_{\vec{q}}+1-\epsilon_{\vec{k}+\vec{q}}, N_{\vec{q}}\right) f_0\left(\vec{k}+\vec{q}\right) \\ -\left|\langle\vec{k}+\vec{q}, N_{\vec{q}}-1\middle|H'_{ac}\middle|\vec{k}, N_{\vec{q}}\rangle\right|^2 \\ \delta\left(\epsilon_{\vec{k}+\vec{q}}, N_{\vec{q}}-1-\epsilon_{\vec{k}}, N_{\vec{q}}\right) f_0\left(\vec{k}\right)\Big], \tag{3}$$

where the square of the matrix element of the electron-lattice scattering is given by [6]

$$\left|\langle\vec{k}+\vec{q}\middle|H'_{ac}\middle|\vec{k}\rangle\right|^2 = \left(\frac{\mathcal{E}_a^2\hbar q^2}{2sd\rho_v\omega_q}\right)\left(N_{\vec{q}}+\frac{1}{2}+\frac{1}{2}\delta N_{\vec{q}}\right),$$

where $\delta N_{\vec{q}} = +1$ for emission,
$\delta N_{\vec{q}} = -1$ for absorption.

Here $\mathcal{E}_a$ is the effective deformation potential constant which assumes a value larger than that for the bulk material for higher-order subbands [6]. Vass *et al* [17] developed a theory to determine the surface deformation potential constant $\mathcal{E}_a$ in terms of a bulk value of the deformation potential $\mathcal{E}_1$ and carrier concentration N_i in cm^{-2} as

$$\mathcal{E}_a = \mathcal{E}_1 + 2.5 \quad 10^{-8} \quad N_i^{2/3}\ \text{eV}. \tag{4}$$

The parameter d is the width of the layer of lattice atoms with which the electrons can interact, and ρ_v is the mass density. The frequency of the lattice vibration $\omega_q = u_l q$. The hot electron distribution function $f_0\left(\vec{k}\right)$ is given by the Maxwell-Boltzmann distribution at an effective electron temperature T_e as [13]

$$f_0\left(\vec{k}\right) = \frac{N_i}{N_C^{2D}}e^{-\epsilon_{\vec{k}}/k_B T_e}. \tag{5}$$

Here k_B is the Boltzmann constant and $\epsilon_{\vec{k}}$ is the energy of the electron which can be given for spherical constant energy surfaces as [6]

$$\epsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m_\parallel^*}, \tag{6}$$

where $m_\parallel^*$ is the effective mass of the electron parallel to the interface. The effective density of state N_C^{2D} in 2DEG can be given as[18]

$$N_C^{2D} = \frac{m_\parallel^*}{\pi\hbar^2}k_B T_e. \tag{7}$$

The summation over two-dimensional lattice wave vector $\vec{k}$ can be transformed into integral by the transformation [6]

$$\sum_{\vec{k}} \rightarrow \int d\theta \int \frac{s}{(2\pi)^2}kdk \tag{8}$$

Using the above transformation Eq.(3) may be written as

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{\mathcal{E}_a^2 m_\parallel^*}{2\pi d\rho_v u_l\hbar^2}\int_k\left\{\frac{\left(N_{\vec{q}}+1\right)f_0\left(\vec{k}+\vec{q}\right)dk}{\left[1-\left(\frac{q}{2k}\right)^2\left(1+\frac{2m_\parallel^* u_l}{\hbar q}\right)^2\right]^{1/2}} \\ -\frac{N_{\vec{q}}f_0\left(\vec{k}\right)dk}{\left[1-\left(\frac{q}{2k}\right)^2\left(1-\frac{2m_\parallel^* u_l}{\hbar q}\right)^2\right]^{1/2}}\right\}. \tag{9}$$

At low lattice temperature the electrons become hot and highly energetic under the application of even a relatively weak electric field and hence the phonon energy may be neglected in comparison to that of electron energy under such conditions. Thus the limits of integration over $\vec{k}$ as ascertained from the energy and momentum balance equations may be taken to be $q/2$ and ∞. If $f_0(\vec{k}+\vec{q})$ is expanded in a Taylor's series around $\vec{k}$ then one can obtain from Eq.(9), the rate of increase in the number of phonons as

$$\left(\frac{\partial N_{\vec{q}}}{\partial t}\right) = \frac{\mathcal{A}_{ac}N_i}{\sqrt{T_n}}\left[1+\left(N_{\vec{q}}+1\right)\frac{x}{T_n}\right]e^{-ax^2} \tag{10}$$

Here

$$\mathcal{A}_{ac} = \frac{\sqrt{\pi}\mathcal{E}_a^2}{2\hbar d\rho_v u_l^2}\left(\frac{\varepsilon_s}{k_B T_L}\right)^{1/2},$$

$$\varepsilon_s = \frac{1}{2}m_\parallel^* u_l^2,\ x = \hbar q u_l/k_B T_L,$$

$$T_n = T_e/T_L\ , a = \frac{k_B T_L}{16\varepsilon_s T_n}.$$

2.2 Average Rate of Energy Loss of an Electron

Once the rate of increase in the number of phonons is known then we may calculate the average rate of energy loss of a carrier to acoustic modes in 2DEG using the Eq.(2). Again the summation of Eq.(2) may be transformed into integral by the Eq.(8) and hence we the average rate of electron energy loss as

$$\langle\frac{d\epsilon}{dt}\rangle_{ac} = -\frac{\mathcal{A}_{ac}\hbar u_l}{2\pi\sqrt{T_n}}\int_q q^2 dq\left[1+\left(N_{\vec{q}}+1\right)\frac{x}{T_n}\right]e^{-ax^2} \tag{11}$$

In presence of electric field at low lattice temperature the limits of q can be taken as 0 to ∝, since $\left(\frac{\partial N_{\vec{q}}}{\partial t}\right)$ falls of rapidly for large q, the upper limit is taken to be ∝.

At low lattice temperatures the phonon distribution is given to a good approximation by the Laurent expansion of the form [18]

$$N_q(x) = \sum_{m=0}^{\infty}\frac{B_m}{m!}x^{m-1} \qquad ; \quad x \le \bar{x}, \\ \approx \ 0 \qquad ; \quad x > \bar{x}, \tag{12}$$

where B_m's are Bernoulli numbers and $\bar{x} < 2\pi$. For the practical purpose $\bar{x}$ may be taken to be 3.5. Now carrying out the integration in Eq.(11) one can obtain

$$\langle\frac{d\epsilon}{dt}\rangle_{ac} = -\frac{\mathcal{B}_{ac}}{\sqrt{T_n}}\left[\frac{\sqrt{\pi}}{4a^{3/2}}+\frac{1}{2a^2 T_n}+\sum_{m=0}^{\propto}\frac{B_m}{m!\,T_n}\left[\frac{\bar{x}^{(m+3)}}{2\left(a\bar{x}^2\right)^{\frac{m+3}{2}}} \\ \left\{\Gamma\left(\frac{m+3}{2}\right)-\Gamma\left(\frac{m+3}{2}, a\bar{x}^2\right)\right\}\right]\right], \tag{13}$$

where $\mathcal{B}_{ac} = \frac{\mathcal{A}_{ac}(k_B T_L)^3}{2\pi(\hbar u_l)^2}$.

When the equipartition law is assumed for N_q which is rather appropriate at higher lattice temperatures, the average rate of electron energy loss is obtained as

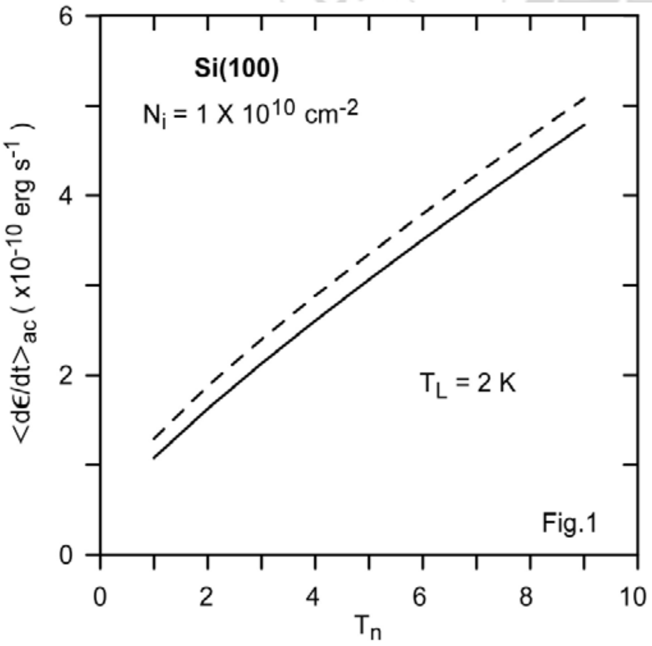
$$\langle\frac{d\epsilon}{dt}\rangle_{ac} = -\frac{\mathcal{B}_{ac}}{\sqrt{T_n}}\left[\frac{\sqrt{\pi}}{4a^{3/2}}\left(1+\frac{1}{T_n}\right)+\frac{1}{2a^2T_n}\right], \qquad (14)$$

3. Results and Discussion

At low lattice temperatures, Eq.(13) shows that the average rate of non-equilibrium electron energy loss due to interaction with deformation potential acoustic phonon in a 2DEG system depends upon the electron temperature in a very complex manner in comparison to the results obtained in Eq.(14) under the approximations of high lattice temperatures. For an application of the above theory, an n-channel (100) oriented Si inversion layer is considered with the material parameters [10]: $\mathcal{E}_1 = 9$ eV, $u_l = 9.037\ \ 10^3$ m s^{-1}, $\rho_v = 2.329\ \ 10^3$ kg m^{-3}, the permittivity $\epsilon_{sc} = 11.9$, longitudinal effective mass $m_l^* = 0.96m_0$, transverse effective mass $m_t^* = 0.19m_0$, m_0 being the free electron mass. At low lattice temperatures one may consider presumably the electrons occupy only the lowest subband when the layer thickness d is given by $(\hbar^2\epsilon_{sc}/2m_\perp^* e^2 N_i)^{1/3}\gamma_0$. Here γ_0 is the zeroth root of the Airy function $A_i(-\gamma_n)$.

For the (100) surface of Si the six valleys are not equivalent. The two equivalent valleys for which $m_\parallel^* = m_t^*$, $m_\perp^* = m_l^*$ occupy the lowest subband [3],[6].

In Fig.1 and Fig.2, we have plotted the electron temperature dependence of the average rate of energy loss of electron in 2DEG at different lattice temperatures.



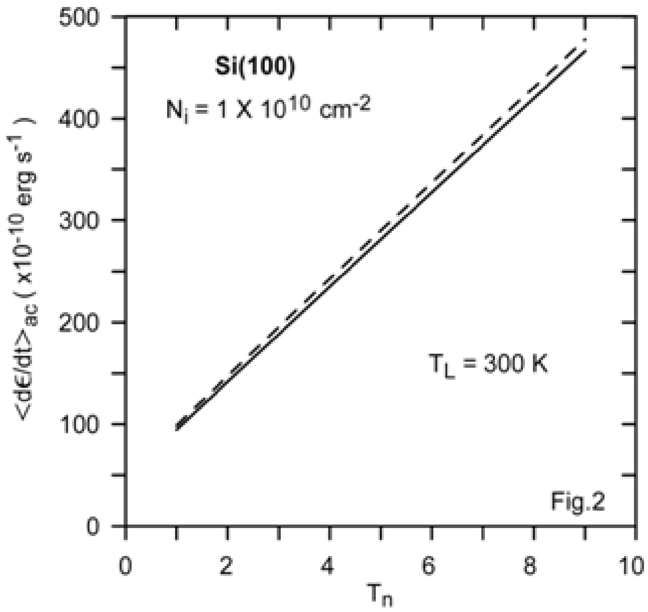


Fig.1 and Fig.2. Average rate of energy loss of non-equilibrium electron in 2DEG. Solid line and dashed line represent the results due to low temperature and high temperature approximations respectively.

The figures show that the average rate of loss of electron energy due to the interaction with the acoustic phonon depend upon electron temperature more or less in a same qualitative manner both for lower or higher lattice temperatures. But with higher lattice temperatures, however, the loss increases rapidly. This apart, as we see in Fig.1, at low lattice temperatures the discrepancy between the results shows that the true phonon distribution plays a role in developing the theory of electron transport. This sort of discrepancy is almost absent at higher temperatures what we see in Fig.2, and hence at high temperatures the phonon distribution may be approximated by equipartition law.

The theory developed here to obtain the average rate of energy loss of non-equilibrium electrons with the rise of electron temperature in 2DEG due to the applied electric field may be helpful to determine the field dependence of the electron temperature once we know the high field mobility data obtained experimentally. This can be done with the help of Eq.(1). This work would be reported elsewhere in future to justify the importance of the present theory.

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