Coupled Heat and Mass Transfer by Free Convection from an Inclined Surface in a Fluid-Saturated Porous Medium using Variable Prandtl Number

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Abstract: ***The heat and mass transfer characteristics of natural convection over an inclined surface immersed in a porous medium subject to variable Prandtl number is considered. The Darcy effects including buoyancy ratio and inclination of porous plate are also included in this analysis. The governing partial differential equations are converted into ordinary differential equations using an appropriate similarity transformation and then the resulting equations are solved numerically. Nachtsheim-Swigert shooting method is applied to perform this computation. A representative set of numerical results of the present investigation for the velocity, temperature and concentration profiles are illustrated graphically to show interesting features of Prandtl number, Darcy number, buoyancy parameter and angle of inclination of stretching sheet. In addition, the Nusselt number and Sherwood number are also derived and discussed numerically in tabular form.***

Keywords: Heat and mass transfer, Free convection, Inclined surface, Porous medium, Prandtl number

1. Introduction

In recent years, considerable attention has been devoted to the work of combined heat and mass transfer in a porous media due to its extensively use in thermal insulation engineering, geophysics, geothermal reservoirs, exploration of petroleum and gas fields, prevention of subsoil water pollution, water movements in geothermal reservoirs, etc. A number of studies have been reported in the literature focusing on the problem of coupled heat and mass transfer (or double-diffusion) driven by buoyancy that arises due to temperature and concentration variations in a fluid-saturated porous medium. Natural convection in heat and mass transfer processes in porous media are often encountered in the study of the chemical industry, grain storage, evaporation cooling and solidification.

Ahammad and Mollah (2011) discussed the MHD free convection flow and mass transfer problem over a stretching sheet taking into account Dufour & Soret effects with magnetic field. Heat and mass transfer flow through a porous medium with variable permeability and periodic suction has been investigated by Hamza et al. (2011). Alam et al. (2011) reported the effects of variable chemical reaction and variable electric conductivity on free convective flow with heat and mass transfer over a stretching sheet considering Dufour and Soret effects. Considering non-darcy porous medium El-Hakiem et al. (2014) performed the effect of magnetic field and double dispersion on mixed convection heat and mass transfer. The flow and heat transfer of a fluid through a porous medium over a stretching surface was investigated by Cortell (2005) with the internal heat generation/absorption and suction/blowing effects. Moorthy and Senthilvadivu (2012) have presented Soret and Dufour effects on free convection flow past a vertical surface in a porous medium with variable viscosity. RamReddy et al. (2013) observed the outcome of viscous dissipation on free convection in a non-darcy porous medium saturated with nanofluid in the presence of magnetic field. Esmaeil Khaje et al. (2013) analyzed the effect of heat generation on free convection boundary-layer flow over an arbitrarily impermeable inclined surface in a saturated porous medium. Local similarity solution for mixed convective and mass transfer flow past a semi-infinite vertical plate has been computed by Kafoussias (1990). Lavanya and Leela Ratnam (2014) analyzed the Dufour and soret effects on steady MHD free convective flow past a vertical porous plate embedded in a porous medium with chemical reaction, radiation heat generation and viscous dissipation. Murali et al. (2013) investigated unsteady Magnetohydrodynamic Free Convective Flow Past a Vertical Porous Plate. Ravikumar et al. (2013) presented a steady free convective and mass transfer flow of an electrically conducting viscous fluid through a porous medium bounded by two vertical plates. The influence of forced convective flow and heat transfer over a porous plate in a Darcy-Forchheimer porous medium in presence of radiation has been focused in the work of Mukhopadhyay et al. (2012). Samad et al. (2010) reported natural convection flow through a porous medium considering the effect of magnetic field with thermal radiation, viscous dissipation and variable suction. EL-Kabeir et al. (2007) performed natural convection from a permeable sphere embedded in a variable porosity porous medium due to thermal dispersion.

This paper is devoted to study the problem of steady natural convection with heat and mass transfer from an inclined plate embedded in fluid saturated porous medium.

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2. Formulation of the Problem

Consider the free convection flow of an electrically conducting fluid from inclined surface embedded in a fluid saturated Darcy porous medium with heat and mass transfer. Also, the fluid flow is assumed to be steady, laminar, incompressible and two dimensional. The coordinate system is chosen such that x-axis is along the plate in the upward direction and y-axis is normal to the plate. A uniform magnetic field of strength B_0 is applied normal to the plate that produces magnetic force $F = \frac{\sigma B_0^2 u}{\rho}$ in x-direction, where σ is the electrical conductivity. It is considered that the induced magnetic field is negligible. The porous medium is supposed to be uniform with a constant porosity and permeability. The fluid has constant properties with the exception of the density in the buoyancy term of the balance of momentum equation. The plate and the fluid are taken into account in the beginning at the same temperature T while C is the concentration all over the place in the fluid. The surface of the plate is kept at a uniform constant temperature T_w (> T_∞) and concentration C_w (> C_∞), where T_∞ and C_∞ are the corresponding values respectively sufficiently far away from the flat surface.

Taking the Darcy-Forchhemier flow model together with the Boussnesq's and the usual boundary-layer approximations the present problem is governed by the continuity, momentum, energy and concentration equations respectively are expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon}{k}u - \frac{b}{k}u^2 \tag{2}$$

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \lambda \frac{\partial^2 T}{\partial y^2} + Q_0 \left(T - T_\infty \right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u, v are the fluid velocity components that acts along the x and y directions respectively, υ is the kinematic viscosity, g is the gravitational acceleration, ρ is the density, k is the Darcy permeability constant, β is the volumetric coefficient of thermal expansion, σ is the electrical conductivity, B_0 is the uniform magnetic field strength, b is the stretching rate, λ is the thermal conductivity of fluid, c_p is the specific heat at constant pressure, T is the fluid temperature inside the boundary layer, T_∞ is the fluid temperature in the free-stream, C is the concentration of the fluid within the boundary layer, D_m is the molecular diffusivity of the species concentration and Q_0 is the heat generative volumetric rate.

The boundary conditions for this problem are defined as:

$$\left. \begin{aligned} &u = bx,\ v = v_w(x),\ T = T_w,\ C = C_w \ \text{ at } \ y = 0, \\ &u = 0, \qquad\qquad T = T_\infty,\ C = C_\infty \ \text{ as } \ y \to \infty. \end{aligned} \right\} \tag{5}$$

where b is a constant called stretching rate and $v_w(x)$ represents the permeability of the porous surface where its negative sign indicates suction and positive indicates injection.

Now, the following appropriate similarity variables reported earlier by Acharya et al. (1999) are introduced:

$$\left. \begin{aligned} &u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \\ &\psi = (\upsilon b)^{1/2} x f(\eta),\ \ \eta = (b/\upsilon)^{1/2} y \\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \ \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \tag{6}$$

Then using the non-dimensional transformations (6) in Eqs. (2)-(4), we get the following system of non-dimensional equations:

$$f''' + ff'' + \gamma\theta - (f')^2 - Mf' - \frac{1}{Da\,\mathrm{Re}} f' - \frac{Fs}{Da}(f')^2 = 0 \tag{7}$$

$$\theta'' + Prf\theta' + PrQ\theta = 0 \tag{8}$$

$$\varphi'' + Scf\varphi' = 0 \tag{9}$$

where the dimensionless parameters can be expressed as follows:

$\gamma = \frac{Gr}{\mathrm{Re}^2}$ is the temperature buoyancy parameter, $M = \frac{\sigma B_0^2}{\rho b}$ is the magnetic field parameter, $Da = \frac{k}{x^2}$ is the Darcy number, $\mathrm{Re} = \frac{u_w(x)x}{\upsilon}$ is the Reynolds number, $Fs = \frac{b}{x}$ is the Forchhemier number, $\mathrm{Pr} = \frac{\upsilon\rho c_p}{\lambda}$ is the Prandtl number, $Q = \frac{Q_0}{b\rho c_p}$ is the heat generating parameter and $Sc = \frac{\upsilon}{D_m}$ is the Schmidt number.

Then the associated boundary conditions (5) becomes

$$\left. \begin{aligned} &f = f_w,\ f' = 1,\ \theta = 1,\ \varphi = 1 \ \text{ at } \ \eta = 0 \\ &f' = 0,\ \theta = 0,\ \varphi = 0 \ \text{ as } \ \eta \to \infty \end{aligned} \right\} \tag{10}$$

where $f_w = - v_w/(b\,\nu)^{1/2}$ stands for the non-dimensional wall mass transfer coefficient such that $f_w > 0$ indicates wall suction and $f_w < 0$ indicates wall injection.

The physical quantities of interests are the local skin-friction coefficient, the local Nusselt number and the local Sherwood number which are given respectively as below:

$$\frac{1}{2} Cf_x (\mathrm{Re}_x)^{\frac{1}{2}} = f''(0)$$

$$Nu_x (\mathrm{Re}_x)^{-\frac{1}{2}} = \frac{1}{\theta(0)}$$

$$Sh_x (\mathrm{Re}_x)^{-\frac{1}{2}} = \frac{1}{\varphi(0)}$$

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3. Method of Solution

The reduced system of equations (7-9) are nonlinear, coupled, ordinary differential equations, which possess no closed-form solution. Therefore, they must be solved numerically subject to the boundary conditions (10). Nachtsheim-Swigert (1965) shooting iteration technique with sixth-order Runge-Kutta integration scheme was employed in the present work with step size $\Delta\eta = 0.01$. The whole procedure is repeated until we get the converged results within a tolerance limit of 10^{-6}.

In order to verify the accuracy of our code , the numerical results are compared and found to be in good agreement with previously published results Kafoussias (1990), for a special case ($M = Df = Sr = 0$) of the present investigation.

Table 1: Comparison of local skin-friction coefficient (Cf_x) and local Nusselt number (Nu_x) between Kafoussias (1990) and present work for $M = Df = Sr = 0$

γ	ζ	Kafoussias[1990] (Cf_x)	Present (Cf_x)	Kafoussias[1990] (Nu_x)	Present (Nu_x)
10.0	0.05	6.8389	6.8378	0.6449	0.6441
10.0	0.10	6.8715	6.8709	0.6461	0.6455
10.0	0.20	6.9366	6.9351	0.6487	0.6479

4. Results and Discussion

The problem of heat and mass transfer past an inclined porous plate through a fluid-saturated porous medium with different types of fluid is considered. As a result of the numerical computations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behavior have been discussed with the help of figures (1)-(4) for variations in the pertinent parameters such as Prandtl number Pr, Darcy number Da, buoyancy parameter γ and inclination of porous plate α. Moreover, the wall share stress, the rate of heat transfer and the rate of mass transfer have been reported in terms of local skin friction coefficient, local Nusselt number and local Sherwood number respectively for variable Prandtl number in table-2. Note that there are three parts in each of Figure (1)-(4), where part (a), (b), (c) indicates respectively velocity, temperature and concentration profiles.

The default parameter values throughout the computations of current study are as Pr =0 .71, Da = 0 . 5, f_w = 0 . 5, M = 0 .5, Re = 100, F_s = 0.1, γ = 10, Sc = 0.22, Q =0.75 and α = 30^0. Figure 1 depicts the typical velocity, temperature and concentration fields for various values of Prandtl number Pr (1.0, 1.5, 2.0), while all other parameters are kept at above mentioned fixed values. The effect of enhancing value of Pr

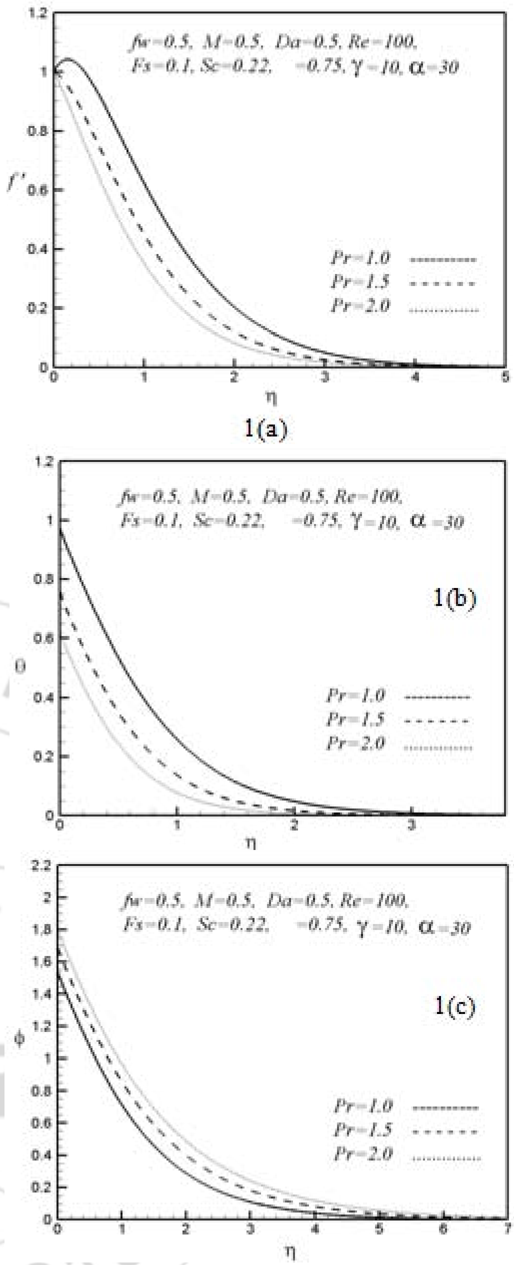
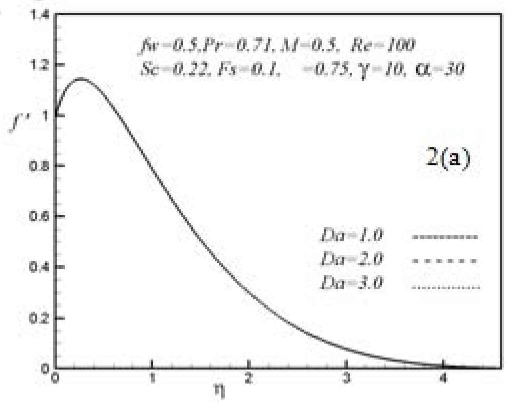


Figure 1: (a) velocity (b) temperature (c) concentration profiles for different values of Pr



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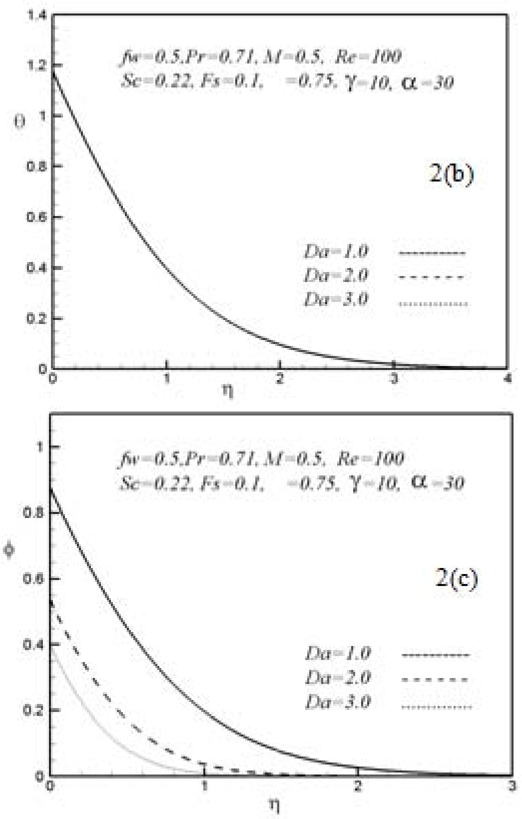
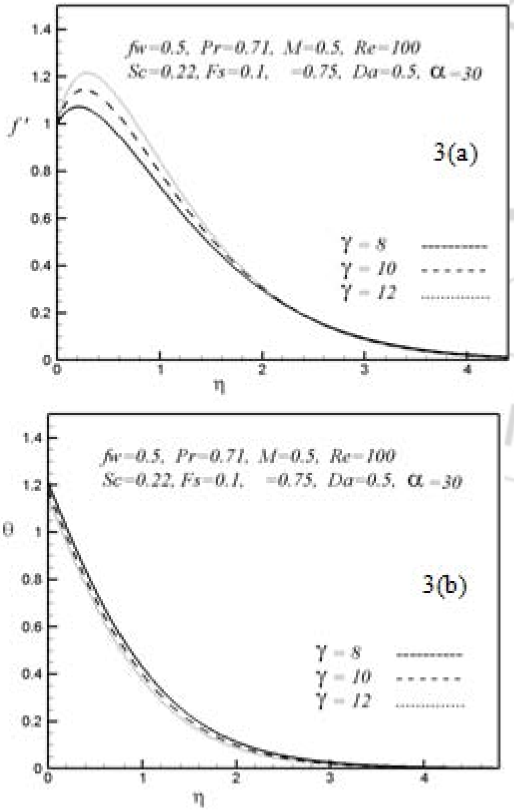


Figure 2: (a) velocity (b) temperature (c) concentration profiles for different values of *Da*



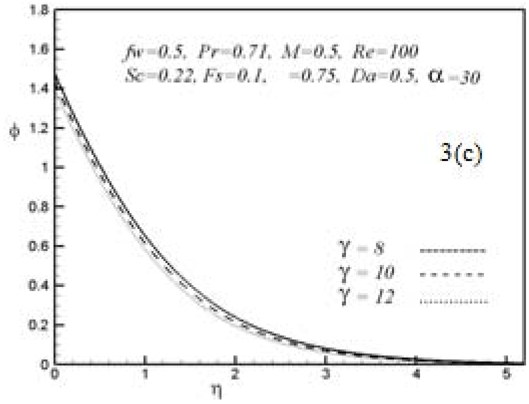


Figure 3: (a) velocity (b) temperature (c) concentration profiles for different values of γ

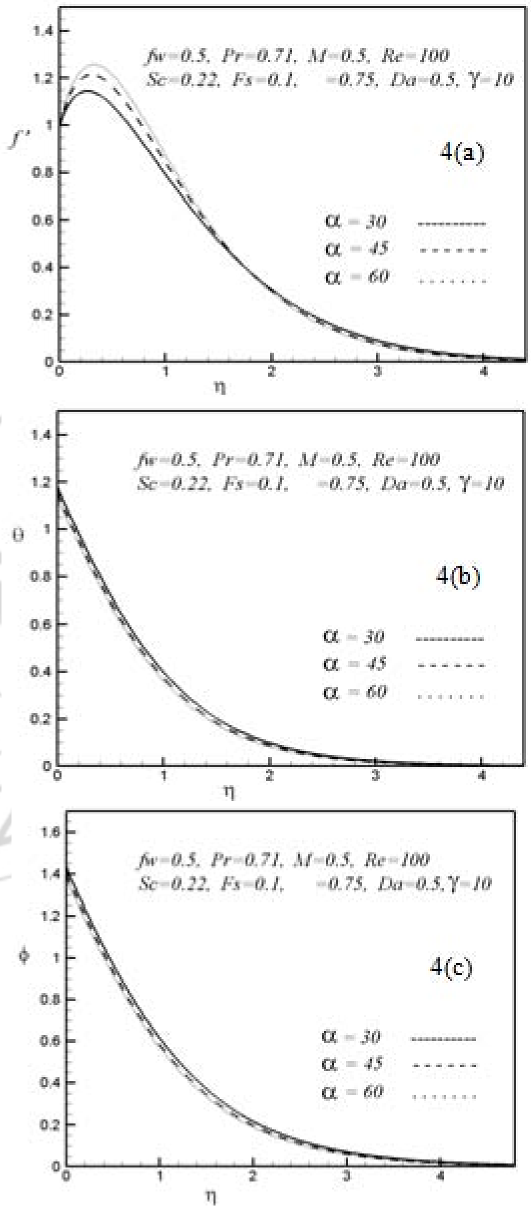


Figure 4: (a) velocity (b) temperature (c) concentration profiles for different values of α

is observed to decrease the velocity and temperature profiles from Figure 1(*a*) and 1(*b*) whereas it is seen that as the parameter *Pr* increases, the concentration profile also increases as shown in Figure 1(*c*).

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Figure 2 illustrates the effect of Darcy parameter in velocity, temperature and concentration distributions taken as Da = 1.0, 2.0 3.0. It is found that all of the velocity, temperature and concentration distributions are decreased with the rising values of Darcy number though in case of velocity and temperature profiles are almost identical but concentration profile shows a noticeable variation for different values of Da.

The influence of the buoyancy parameter on the velocity, temperature and concentration profiles are presented in Figure 3. By analyzing these Figures, it is clearly revealed that the effect of buoyancy parameter is to increase the velocity and decrease both temperature and concentration distributions. Figure 4 displays the results for velocity, temperature and concentration distributions as a function of η, for the three different inclination of porous surface. It is noteworthy, from Figure 4(a) that as the value of α increases the velocity profiles increases in one section whereas it decreases in other section. But it shows that the temperature and concentration profiles decrease smoothly everywhere with an increase of leaning parameter α.

Table 2: Effects of Pr on the local skin-friction coefficient (Cf_x), local Nusselt number (Nu_x) and local Sherwood number (Sh_x) for $Da = 0.5, f_w = 0.5, M = 0.5, Re = 100, F_s = 0.1, \gamma = 10, Sc = 0.22, Q = 0.75$ and $\alpha = 30^0$

Pr	Cf_x	Nu_x	Sh_x
0.71	0.7937	2.5068	1.6273
1.0	0.6239	3.8624	1.3997
1.5	0.4518	7.2833	1.1652
2.0	0.3522	12.8205	1.0342
2.5	0.2915	21.7391	0.9529

Table 2 focuses an idea about the numerical values of local skin-friction coefficient, local Nusselt number and local sherwood number for some chosen values of the Prandtl number Pr that indicates different types of fluids. It is apparent from this table that for rising values of Pr, the value of Cf_x and Sh_x decreases. On the other hand it is observed that Nu_x increases while Pr increases.

5. Conclusion

Coupled heat and mass transfer by natural convection over an inclined porous plate in a porous medium is analyzed in this paper. The results of this work show that the Prandtl number plays a significant role on the velocity, temperature, and concentration fields. In addition, both the share stress and mass transfer rate tend to decrease as the value of Pr enhances, while heat transfer rate increases with Pr. The influence of Darcy number is not remarkable in velocity and temperature distributions but noteworthy for concentration field. The conclusions of the study can be summarized as follows:

1) It is found that the velocity decreases with increasing Pr and Da but increases with increasing γ and α.
2) The temperature decreases with increasing all of the considered parameters in the present investigation.
3) The concentration increases with increasing Pr and decreases with higher values of the rest three parameters as Da, γ and α.

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Nomenclature

b Stretching rate	Sh Sherwood number
B_0 Magnetic field intensity	T Fluid temperature
C Concentration of the fluid	T_w Fluid temperature at the surface
C_w Fluid concentration at the surface	T_∞ Fluid temperature in free stream
C_∞ Fluid concentration in the free stream	u, v Fluid velocity components in the x and y-direction respectively
C_p Specific heat at constant pressure	x, y Cartesian coordinates along the plate and normal
Da Darcy number	to it respectively
D_m Molecular diffusivity	α Thermal diffusivity
F Magnetic force	β Coefficient of thermal expansion
Fs Local Forchheimer number	γ Temperature buoyancy parameter
f_w Dimensionless suction velocity	φ Dimensionless concentration
g Acceleration due to gravity	λ Thermal conductivity of fluid
k Darcy permeability constant	η ελβαιραϖ ψτιραλιμιΣ
Gr Grashof number	ν Kinematic viscosity
M Magnetic field parameter	θ Dimensionless temperature
Nu Nusselt number	ρ Density of the fluid
Pr Prandtl number	σ Electrical conductivity
Q_0 Volumetric rate of heat generation	ζ σσαΜ ρετεμαραπ ψχναψουβ
Q Heat generating parameter	w Condition at surface
Re Reynolds number	∞ ψτινιφνι τα νοιτιδνοΧ
Sc Schmidt number	

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