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Even Edge - Graceful Labeling of $T_{t,n,m}$

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Abstract: ***An even- edge - graceful labeling of a (p, q)- graph G(V, E) is an injection f from E to {1, 2, 3,...,2q} such that the induced mapping f^+ defined on V by $f^+(x) = \left(\sum f(xy)\right)(mod\ 2k)$ over all edges xy are distinct and even, where k=max{p,q}. A graph G that admits an even- edge – graceful labeling is called an even edge - graceful graph. $T_{t,n,m}$ is a graph obtained by joining the centers of $K_{1,n}$ and $K_{1,m}$ by a path P_t. In this paper, we obtain even - edge - graceful labeling of $T_{t,n,m}$.***

Keywords: Even-edge graceful labeling, Even-edge graceful graphs

Subject classification: 05C78

1. Introduction

By a graph G, we mean a finite, undirected simple graph. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or edge labeling). Graph labelings were first introduced by Rosa in the late 1960's. Labeled graphs serve as useful models for a broad range of applications in [2&3]. For good account on graph labeling we refer Gallian survey[5]. In 1985, Lo introduced the concept of edge - graceful graphs which is a dual notion of graceful . We have introduced even- edge - graceful labeling of graphs and obtained several results in [4]. An ***even- edge - graceful labeling*** of a *(p, q)*- graph *G(V, E)* is an injection *f* from *E* to *{1, 2, 3,...,2q}* such that the induced mapping f^+ defined on *V* by $f^+(x) = \left(\sum f(xy)\right)(mod\ 2k)$ over all edges *xy* are distinct and even, where *k=max{p,q}*. A graph *G* that admits an even- edge – graceful labeling is called an ***even edge - graceful graph***. In this paper, we obtain even - edge - graceful labeling of $T_{t,n,m}$.

2. Definition

$T_{t,n,m}$ is a graph obtained by joining the centers of $K_{1,n}$ and $K_{1,m}$ by a path P_t. It consists of $t + n + m$ vertices and $t + n + m - 1$ edges.

2.1 Remark

If the graph is a tree then even edge graceful labeling and edge graceful labeling coincide.

2.2 Theorem [4]

If G is edge - graceful then $q(q+1) \equiv \frac{(p+1)p}{2} (mod\ p)$.

3. Main Results

3.1 Theorem

The graph $T_{t,n,m}$ of odd order is even edge - graceful.

Proof

We note that the graph $T_{t,n,m}$ is of odd order if and only if either *t, n, m* are all odd numbers or any one of *t, n, m* is an odd number.

Case (1): *t, n, m* are odd numbers.

Let $\{w_1, w_2,\dots,w_t, v_1, v_2,\dots,v_n,v_{n+1},v_{n+2},\dots,v_{n+m}\}$ be the vertices of $T_{t,n,m}$ and edges e_i, e_i' (see fig:3.1) are defined as follows.

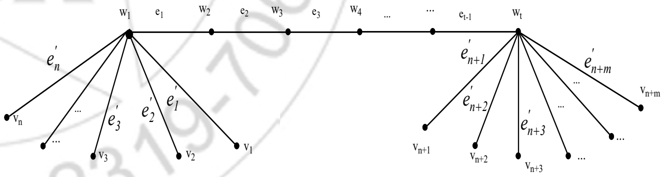

Figure 3.1: $T_{t,n,m}$ with ordinary labeling

$$e_i = (w_i, w_{i+1}) \quad for \quad 1 \le i \le t-1;$$

$$e_i' = (w_1, v_i) \quad for \quad 1 \le i \le n \quad \text{and}$$

$$e_i' = (w_t, v_i) \quad for \quad n+1 \le i \le n+m$$

Consider the Diophantine equation,
$x_1 + x_2 = 2k$ ……… (1); where $k = max\ \{p,q\} = t+n+m$
The even pair of solutions of equation (1) is of the form *(s, 2k-s)* where *s* is an even number in $2 \le s \le k-1$.

There will be $\frac{t+n+m-1}{2}$ even pair of solutions. We now label the edges as follows

$f(e_i) = i+1 \quad for \quad 1 \le i \le t-2$, when *i* is odd

$f(e_i) = 2(n+m)+t-1+i \quad for \quad 2 \le i \le t-1$ when *i* is even

$f(e_1') = 2(n+m)+t-1$ & $f(e_{n+1}') = t+1$

These labels covers $\frac{t+1}{2}$ pair of solutions. The remaining pair of solutions are $\frac{n+m}{2} - 1 = \frac{n-1}{2} + \frac{m-1}{2}$. Out of these choose any $\frac{n-1}{2}$ pair of solutions and label the edges of $K_{1,n}$ by the co-ordinates of the pair. The other $\frac{m-1}{2}$ pairs are used to label the edges of $K_{1,m}$. Then the induced vertex labels are

$$f^+(w_i) = 2(n+m)+t-1+2i \quad for \quad 1 \le i \le \frac{t-1}{2}$$

$$f^+(w_i) = 2i\text{-}t\text{-}1 \quad for \quad \frac{t+1}{2} \le i \le t$$

$$f^+(v_1) = 2(n+m)+t-1; \; f^+(v_{n+1}) = t+1$$

Now the remaining pendant vertices will have the labels of the edges with which they are incident and are distinct & even. Hence the graph $T_{t,n,m}$ is an even edge - graceful graph.

Illustration

Consider the graph $T_{7,5,7}$. Here $t=7; n=5; m=7;$ $k = max\ \{p,q\} = 19; 2k = 38.$

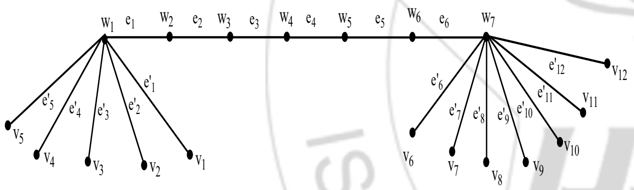

Figure 3.2: $T_{7,5,7}$ with ordinary labeling

Consider the Diophantine equation $x_1+x_2 = 38$ ....(1)

The even pair of solutions of (1) are; *(s, 2k-s)* where *s* is even number in [2, 19]. That is (2, 36), (4, 34),(6, 32), (8, 30), (10, 28), (12, 26), (14, 24), (16, 22), (18, 20). The even edge - graceful labeling of $T_{7,5,7}$ is given in fig.3.3

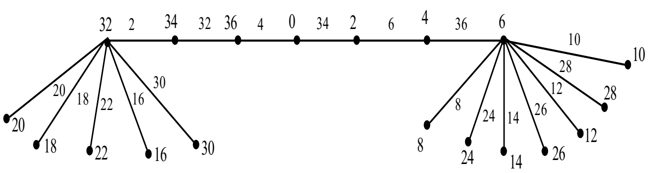

Figure 3.3: Even edge - graceful labeling of $T_{7,5,7}$

Case (2): *t* is odd number, *n* & *m* are both even numbers.

Let the vertices and edges are defined as in case (1) (see fig. 3.1.)

We now label the edges as follows;

$f(e_i) = i+1 \quad for \quad 1 \le i \le t-2$, when *i* is odd

$f(e_i) = 2(n+m)+t-1+i \quad for \quad 2 \le i \le t-1$ when *i* is even

$f(e_1') = t+1; \quad f(e_2') = 2(n+m-1);$

$f(e_{n+1}') = 2(n+m)+t\text{-}1; \quad f(e_{n+2}') = 2(t+1);$

As we have labeled the edges e_1', e_2', e_{n+1}' and e_{n+2}' there will be even pair of edges incident with each w_1 and w_t. We label these edges as was done in case (1).

There will be $\frac{n+m}{2} - 2 = \frac{n-2}{2} + \frac{m-2}{2}$ even pair of solutions left out. Out of these, choose any $\frac{n-2}{2}$ pair of solutions and label the edges of $K_{1,n}$ by the co-ordinates of the pair. The other $\frac{m-2}{2}$ pairs are used to label the edges of $K_{1,m}$.

Then the induced vertex labels are

$$f^+(w_i) = 2(n+m)+t-1+2i \quad for \quad 1 \le i \le \frac{t-1}{2}$$

$$f^+(w_i) = 2i\text{-}t\text{-}1 \quad for \quad \frac{t+1}{2} \le i \le t$$

$$f^+(v_1) = t+1; \quad f^+(v_2) = 2(n+m-1)$$

$$f^+(v_{n+1}) = 2(n+m)+t-1; \quad f^+(v_{n+2}) = 2(t+1)$$

The pendant vertices will have the labels of the edges with which they are incident. They are distinct and even. Hence the graph $T_{t,n,m}$ is an even edge - graceful graph.

Illustration

Consider the graph $T_{9,8,10}$. Here $t = 9;\ n = 8;\ m = 10;$ $k = max\ \{p,q\} = 27;\ 2k = 54$

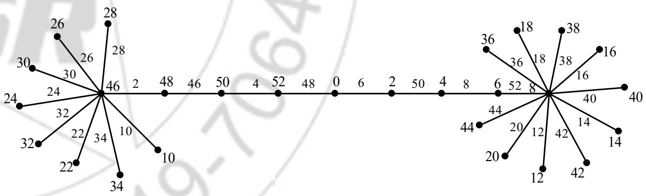

Figure 3.4: Even edge – graceful labeling of $T_{9,8,10}$

Consider the Diophantine equation $x_1+x_2 = 54,$

The even pairs of solutions are: *(s, 2k-s)* where *s* is even number in [2, 27] ie, (2, 52), (4, 50), (6, 48), (8, 46), (10, 44), (12, 42), (14,40), (16, 38), (18, 36) (20, 34), (22, 32),(24, 30),(26, 28). The even edge - graceful labeling of $T_{9,8,10}$ is given in fig.3.4.

Case (3): *t, m* are even and *n* is odd.

Let the vertices and edges are defined as in case (1) (see fig. 3.1).

We now label the edges as follows;

$f(e_i) = i+1 \quad for \quad 1 \le i \le t-1$, when *i* is odd

$f(e_i) = 2(n+m)+t+i \quad for \quad 2 \le i \le t-2$ when *i* is even

$f(e_1') = 2(n+m)+t$.

As we have labeled the edge e_1' there will be even pair of edges incident with each w_1 and w_t. We label these edges as was done in case (1).

There will be $\frac{n+m-1}{2}=\frac{n-1}{2}+\frac{m}{2}$ even pair of solutions left out. Out of these, choose any $\frac{n-1}{2}$ pair of solutions and label the edges of $K_{1,n}$ by the co-ordinates of the pair. The other $\frac{m}{2}$ pairs are used to label the edges of $K_{1,m}$.

Then the induced vertex labels are

$$f^{+}(w_i)=2(n+m)+t+2i \quad for \quad 1\le i\le \frac{t-2}{2}$$

$$f^{+}(w_i)=2i-t \quad for \quad \frac{t}{2}\le i\le t$$

$$f^{+}(v_i)=2(n+m)+t$$

The pendant vertices will have the labels of the edges with which they are incident. They are distinct and even. Hence the graph $T_{t,n,m}$ is an even edge - graceful graph.

Illustration

Consider the graph $T_{8,9,10.}$ Here $t = 8;\ n = 9;\ m = 10;$
$k = max\ \{p,q\}=27;\ 2k=54$

Consider the Diophantine equation $x_1+x_2 = 54,$

The even pairs of solutions are: *(s, 2k-s)* where *s* is even number in [2, 27] ie, (2, 52), (4, 50), (6, 48), (8, 46), (10, 44), (12, 42), (14,40), (16, 38), (18, 36) (20, 34), (22, 32),(24, 30),(26, 28).

The even edge - graceful labeling of $T_{8,9,10}$ is given in fig.3.5.

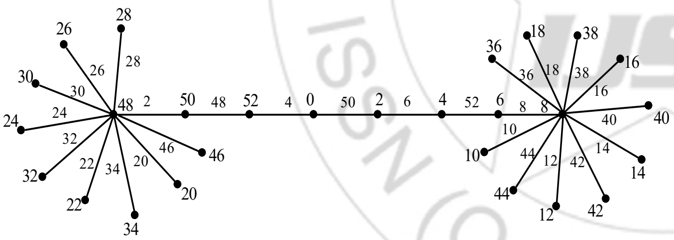

Figure 3.5: Even edge - graceful labeling of $T_{8,9,10}$

Case (4) : t, n are even and m is odd.

Let the vertices and edges are defined as in case (1) (see fig. 3.1).

We now label the edges as follows;

$f(e_i)=2(n+m)+t+i-1 \quad for \quad 1\le i\le t-1$ when i is odd

$f(e_i)=i \quad for \quad 2\le i\le t-2$, when i is even

$f(e'_{n+1})=t$

These label covers $\frac{t}{2}+1$ pair of solution. There will be $\frac{t+n+m-1}{2}-1-\frac{t}{2}=\frac{n+m-3}{2}$ pair of solutions left out. Among these assign $\frac{n-2}{2}$ pairs to the edges of $K_{1,n}$ in any order and $\frac{m-1}{2}$ pairs to the edges of $K_{1,m}$.

Then the induced vertex labels are

$$f^{+}(w_i)=2(n+m)+t+2(i-1) \quad for \quad 1\le i\le \frac{t}{2}$$

$$f^{+}(w_i)=2(i-1)-t \quad for \quad \frac{t+2}{2}\le i\le t$$

$$f^{+}(v_{n+1})=t$$

The pendant vertices will have the labels of the edges with which they are incident. They are distinct and even. Hence the graph $T_{t,n,m}$ is an even edge - graceful graph.

Illustration

Consider the graph $T_{10,6,9.}$ Here $t=10;\ n=6;\ m=9;$ $k = max\ \{p,q\} = 25;\ 2k =50.$

Consider the Diophantine equation $x_1 + x_2 = 50,$

The even pairs of solutions are: *(s, 2k-s)* where *s* is even number in [2, 25] ie, (2, 48), (4, 46), (6, 44), (8, 42), (10, 40), (12, 38), (14,36), (16, 34), (18, 32) (20, 30), (22, 28), (24, 26). These pairs are labeled in $T_{10,6,9}$ graph.

3.2 Theorem

The graph $T_{t,n,m}$ of even order is not even edge - graceful graph.

Proof

Here $p = t+ n +m$ and p is even if either *t, n, m* are all even or any two of t, *n, m* are odd. Since Any tree of even order is not an even edge - graceful graph by Theorem 2.2 and Remark 2.1. Hence $T_{t,n,m}$ of even order is not even edge - graceful graph.

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