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A New Approach to Obtain an Optimal Solution for the Assignment Problem

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Abstract: ***As a method to bring efficiency in assignment problem herewith we propose a new technique namely SS method of maximization/ minimization of assignment problem. Here each row is discussed with 1's assignment method with the systematic procedure. The proposed method has the systematic procedure, easy to apply and less calculation time. An example using matrix 1's assignment methods is discussed and the result is compared with Hungarian method. In order to give a better understanding of this method, we have provided with some of the illustrated examples by end of this research paper.***

Keywords: Assignment for a salesman, SS method, cost matrix, maximization, profit, minimization, optimization

1. Introduction

To assign a number of origins to an equal number of destinations or based on the skills to assign the work for workers in such a condition to allocate one job to one worker is called assignment problem. The deftness with which this method is employed is such that it maximizes the profit or minimizes the cost/ time.

The optimized way of solution is obtained by assignment problem with the variables being efficiently used to assign "n" resources to "m" activities. The ideal purpose of using assignment problem is to optimize/ reduce the total cost involved and efficiently use the time/ man hours or to maximize the profit in sales.

These assignment problems can be applied in the following cases (but not limited to it):

1) Allocation of salesman to respective sales territory(s)
2) Arrival / departure of flights in respective Gates of terminal
3) Supply of midday meals from centralized kitchen to various government schools of the State on or before 12 noon

There are numerous researches done here and articles available with varied methods of application towards this objective, the Hungarian method amongst these is most popular but it seems to be tedious compared the Iterative method. This iterative method involves finding a maximum (minimum) element in every row and divide that row using the maximum(minimum) element so as to create some 1's in the given cost matrix, after which derive a complete assignment in terms of allocating 1's based on its position. The utility of this method is to determine the optimum allocation for origins to its destinations. A unique method herewith adopted on finding an approach for solving assignment problem which differs from the already existing ones.

The arrangement of the research paper is as follows; in section II we present the mathematical form of assignment models. In section III algorithms has been discussed and in section IV some of the numerical examples have been discussed and in section V deals with the conclusion and brief discussion of the results.

2. Mathematical form of Assignment problem

For n origins (rows) and n destinations (columns), we need to assign origins to an equal number of destinations so as to be paired singularly.

The data matrix for this

District

$$
\begin{array}{ll|cccc|c}
 & & 1 & 2 & \ldots\ldots & n & \\
\hline
 & 1 & K_{11} & K_{12} & \ldots\ldots & k_{1n} & 1 \\
\text{Salesman} & 2 & k_{21} & k_{22} & \ldots\ldots & k_{2n} & 1 \\
 & . & . & . & . & . & \\
 & . & . & . & . & . & \\
 & n & k_{n1} & k_{n2} & \ldots\ldots & k_{nn} & 1 \\
\hline
 & & 1 & 1 & \ldots\ldots & 1 &
\end{array}
$$

Let P_{ij} denote the assignment of salesman i to district j such that

$$P_{ij} = \begin{cases} 1 \text{ , if the } i^{th} \text{ salesman is assigned to the } j^{th} \text{ district} \\ 0 \text{ , if the } i^{th} \text{ salesman is not assigned to the } j^{th} \text{ district} \end{cases}$$

The assignment problem can be mathematically defined as the objective function is to,

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij} \; P_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} P_{ij} = 1 \quad \text{For all i}$$

$$\sum_{j=1}^{n} P_{ij} = 1 \quad \text{For all j}$$

Where $P_{ij} = 0$, if the i^{th} resource is not assigned to the j^{th} activity.

Where $P_{ij} = 1$ if the i^{th} resource is assigned to the j^{th} activity. And K_{ij} represents the cost of assignment of worker i to job j.

3. Algorithm

Step 1

Construct a matrix for the assignment problem. Ideally, the matrix should be a square matrix, if not; we bring it to be a square matrix.

Step 2

For each row in assignment problem, the maximum or minimum value (say m_i) from the row is picked depending upon the nature of the problem . And the chosen element (say m_i) is divided in each row resulting in a unity (1), at least once.

Step 3

Each row is to be discussed

Consider the 1's of $(i , j)^{th}$ position and consider the distinct position of the matrix in the column. The assignment is given for that distinct position. Delete the corresponding rows and columns.

The remaining table is then discussed. The process is continued to the remaining table until the completion of the assignment.

Step 4

In the contrary for the above condition, if there is identical column for more than one row, selection of the column employs the calculation of the difference between two unit costs, one being the largest unit cost and the other one being the penultimate largest unit cost, for maximization, and similarly for minimization the smallest and penultimate smallest unit costs are considered while calculating the difference.

From the outcome of the above calculation, the column with the maximum difference gets assigned. And the corresponding rows & column are deleted.

Step 5

For the difference value to be a tie, the calculation employs unit costs which are largest and antepenultimate largest value (smallest and antepenultimate smallest value) for the column. Column with the maximum difference value gets assigned by canceling the corresponding row and column.

Step 6

A unique assignment of a row is obtained by iterating steps 2 to step 5 until all rows get assigned.

Step 7

Final step involves the calculation of total cost as below:

$$\text{Total cost} = \sum_{i=1}^{n} \sum_{j=1}^{n} K_{ij}\ P_{ij}$$

4. Numerical Examples

Example.4.1 (minimization problem)

A work manger has to allocate four different drivers to four schools to supply the lunch for students. Depending on the efficiency and the time taken by the individual differ by the capacity as shown in the table.

Drivers	Schools			
	A	B	C	D
D_1	10	20	18	14
D_2	15	25	9	25
D_3	30	9	17	12
D_4	19	24	20	10

How should the drivers be assigned to school so as to minimize the total man-hours?

Step 1

	A	B	C	D	Min value (m_i)
D_1	10	20	18	14	10
D_2	15	25	9	25	9
D_3	30	9	17	12	9
D_4	19	24	20	10	10

	A	B	C	D
D_1	1	$\frac{20}{10}$	$\frac{18}{10}$	$\frac{14}{10}$
D_2	$\frac{15}{9}$	$\frac{25}{9}$	1	$\frac{25}{9}$
D_3	$\frac{30}{9}$	1	$\frac{17}{9}$	$\frac{12}{9}$
D_4	$\frac{19}{10}$	$\frac{24}{10}$	$\frac{20}{10}$	1

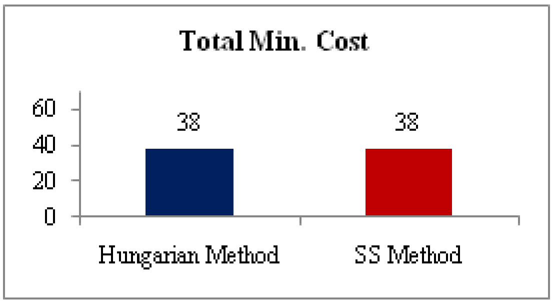
Locate the position of 1's

Drivers	**Schools**
D_1	**A**
D_2	**C**
D_3	**B**
D_4	**D**

Assign D_1→ A, D_2→ C, D_3 →B, D_4→ D

Optimal solution= 10 + 9 + 9 + 10 = 38

Comparison of Result



Example.4.2 (minimization problem)

A departmental head has four subordinates, and four jobs to be performed. The subordinates differ in efficiency, and the jobs differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below.

Jobs	Men			
	A	B	C	D
J_1	18	26	17	11
J_2	13	28	14	26
J_3	38	19	18	15
J_4	19	26	24	10

How should the jobs be allocated so as to minimize the total man-hours?

Solution

Step 1

	A	B	C	D	Min value (m_i)
J_1	18	26	17	11	11
J_2	13	28	14	26	13
J_3	38	19	18	15	15
J_4	19	26	24	10	10

	A	B	C	D
J_1	$\frac{18}{11}$	$\frac{26}{11}$	$\frac{17}{11}$	1
J_2	1	$\frac{28}{13}$	$\frac{14}{13}$	$\frac{26}{13}$
J_3	$\frac{38}{15}$	$\frac{19}{15}$	$\frac{18}{15}$	1
J_4	$\frac{19}{10}$	$\frac{26}{10}$	$\frac{24}{10}$	1

Locate the position of 1's

Job	**Men**
J_1	D
$\boldsymbol{J_2}$	**A**
J_3	D
J_4	D

Assign $\boldsymbol{J_2}\rightarrow$ A,

Delete the assigned rows and column.

Step 2

	B	C	D	Min Value (m_i)
J_1	26	17	11	11
J_3	19	18	15	15
J_4	26	24	10	10

	B	C	D
J_1	$\frac{26}{11}$	$\frac{17}{11}$	1
J_3	$\frac{19}{15}$	$\frac{18}{15}$	1
J_4	$\frac{26}{10}$	$\frac{24}{10}$	1

Locate the position of 1's

Job	**Men**	**Difference**
J_1	D	6
J_3	D	3
J_4	**D**	**14***

Assign $\boldsymbol{J_4}\rightarrow$**D**

Delete the assigned rows and column.

Step 3

	B	C	Min value (m_i)
J_1	26	17	17
J_3	19	18	18

	B	C	Min value(m_i)
J_1	$\frac{26}{17}$	1	17
J_3	$\frac{19}{18}$	1	18

Locate the position of 1's

Task	**Men**	**Difference**
$\boldsymbol{J_1}$	**C**	**9***
J_3	C	1

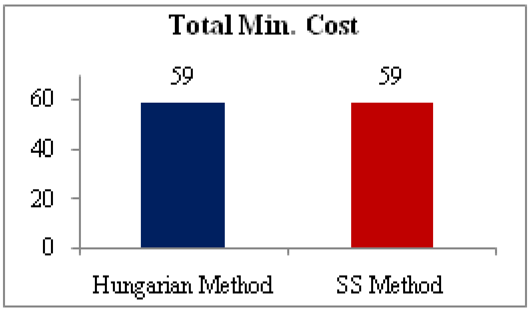
Assign $\boldsymbol{J_1} \rightarrow \boldsymbol{C}$ **and** $J_3 \rightarrow$ **B**

Delete the assigned rows and column.

Hence assign $\boldsymbol{J_1}\rightarrow$ **C**, $\boldsymbol{J_2}\rightarrow$**A**, $\boldsymbol{J_3}\rightarrow$**B**, $\boldsymbol{J_4}\rightarrow$ **D**

And the optimal solution is **17 + 13 +19 +10 = 59**

Comparison of Result



Example.4.3 (Maximization problem)

A Marketing manager has five salesman five sales districts considering the capability of the salesman and the nature of districts, the marketing manager estimates that sales per month for each district would be as follows

		District				
		A	B	C	D	E
Salesman	S_1	32	38	40	28	40
	S_2	40	24	28	21	36
	S_3	41	27	23	30	37
	S_4	22	38	41	36	36
	S_5	29	33	40	35	39

How should the salesman be allocated so as to maximize the profit?

Solution

Step 1

Find the maximum value in each row and write it in the right-hand side and divide by the value

Salesman		District					Max. Value (m_i)
		A	B	C	D	E	
	S_1	32	38	40	28	40	40
	S_2	40	24	28	21	36	40
	S_3	41	27	23	30	37	41
	S_4	22	38	41	36	36	41
	S_5	29	33	40	35	39	40

Salesman		District				
		A	B	C	D	E
	S_1	$\frac{32}{40}$	$\frac{38}{40}$	1	$\frac{28}{40}$	1
	S_2	1	$\frac{24}{40}$	$\frac{28}{40}$	$\frac{21}{40}$	$\frac{36}{40}$
	S_3	1	$\frac{27}{40}$	$\frac{23}{41}$	$\frac{30}{41}$	$\frac{37}{41}$
	S_4	$\frac{22}{40}$	$\frac{38}{40}$	1	$\frac{36}{41}$	$\frac{36}{41}$
	S_5	$\frac{29}{40}$	$\frac{33}{40}$	1	$\frac{35}{40}$	$\frac{39}{40}$

Locate the position of 1's

Salesman	**District**	**Difference**
S_1	C, E	
$\boldsymbol{S_2}$	**A**	**(4) (12)***
S_3	A	(4) (11)
S_4	C	
S_5	C	

Assign $S_2 \rightarrow$ A and delete the corresponding row and column

Step 2

Salesman		District				Max. Value (m_i)
		B	C	D	E	
	S_1	38	40	28	40	40
	S_3	27	23	30	37	37
	S_4	38	41	36	36	41
	S_5	33	40	35	39	40

Salesman		District			
		B	C	D	E
	S_1	$\frac{38}{40}$	1	$\frac{28}{40}$	1
	S_3	$\frac{27}{37}$	$\frac{23}{37}$	$\frac{30}{37}$	1
	S_4	$\frac{38}{40}$	1	$\frac{36}{41}$	$\frac{36}{41}$
	S_5	$\frac{33}{40}$	1	$\frac{35}{40}$	$\frac{39}{40}$

Locate the position of 1's

Salesman	**District**	**Difference**
S_1	C, E	
S_3	E	
$\boldsymbol{S_4}$	**C**	**3***
S_5	C	1

Assign $S_4 \rightarrow$ C and delete the corresponding row and column

Step 3

Salesman		District			Max. Value (m_i)
		B	D	E	
	S_1	38	28	40	40
	S_3	27	30	37	37
	S_5	33	35	39	39

Salesman		District			Max. Value (m_i)
		B	D	E	
	S_1	$\frac{38}{40}$	$\frac{28}{40}$	1	40
	S_3	$\frac{27}{37}$	$\frac{30}{37}$	1	37
	S_5	$\frac{33}{39}$	$\frac{35}{39}$	1	39

Locate the position of 1's

Salesman	*District*	*Difference*
S_1	E	2
S_3	E	7 *
S_5	E	4

Assign $S_3 \rightarrow$ E .

Delete the corresponding row and column

Step 4

Salesman		District		Max. Value (m_i)
		B	D	
	S_1	38	28	38
	S_5	33	35	35

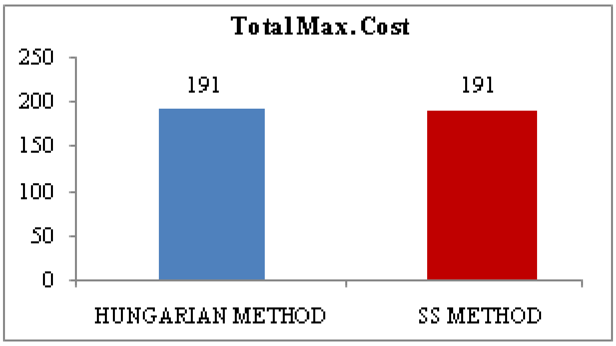
Salesman		District	
		B	D
	S_1	1	$\frac{28}{38}$
	S_5	$\frac{33}{35}$	1

Assign $S_1 \rightarrow$ B, $S_5 \rightarrow$ D

Hence assign **$S_1 \rightarrow$ B, $S_2 \rightarrow$A, $S_3 \rightarrow$E, $S_4 \rightarrow$C,$S_5 \rightarrow$ D**

Maximum sales =38 + 40 + 37 + 41 +35 = 191

Comparison of Result



5. Conclusion

Herewith bringing in a new method namely SS method for addressing assignment problems. All kinds of assignment problems can be addressed using this method. A systematic and easy way of approach is inherent in this method. To draw a conclusion based on this research paper, it provides to be an optimal solution with few direct steps involved by assigning the position of 1's for the assignment problem. With its aim of providing optimal solutions with lesser steps involved, this method proves to be a real boon for the decision makers for its applicability. In coherence to results obtained optimally as through Hungarian method, this SS method comes in play to propose a new way of addressing assignment problem with its unique approach not applied/ employed in the preceding methods.

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