Comparison between Two-Sample Adaptive Tests and Traditional Tests in Location Problem

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Abstract: ***The present paper considers power comparison between two sample Adaptive tests and traditional non parametric tests in the field of location problem. In the proposed approach it is shown by Monte- Carlo simulation study, that the Adaptive tests are superior to the other existing competitors in terms of both robustness of size and power.***

Keywords: Adaptive tests, Gastwirth test, Wilcoxon test, HFR test, long tailed test, t Test , Monte Carlo simulation

1. Introduction

One of the fundamental problems of statistics, often encountered in applications, is the two-sample location problem. In the two-sample location problem the application of the t-test depends on very restrictive assumptions such as normality and equal variances of the two random variables X_1 and X_2. If the assumptions of the t-test are not satisfied it is more appropriate to apply a robust version of the t-test, like the Welch test or the trimmed t-test, or a nonparametric test, like the Wilcoxon. But usually we have no information about the underlying distribution of the data. Therefore, an adaptive test should be applied. It would be desirable, therefore to use data itself to determine the nature of F(.), and on the basis of that information, we could choose an appropriate set of scores. We would then use that same data to perform the test. Such two-stage analyses are termed as adaptive test.

In the past seven decades many important distribution free tests for differences in location between samples had been developed. In the mid 1940s the Wilcoxon -Mann- Whitney test was introduced for testing differences in location between two samples and it was developed by Wilcoxon and extended by Mann and Whitney. But further, it turns out that there exist simple adaptive rank tests that can discover differences between distributions more easily than WMW tests. These adaptive non parametric procedures display significant improvements in power over the parametric t-test in samples of large and moderate sizes.

The purpose of this chapter is two folds , first to introduce the selector statistics , secondly compare the t-test with adaptive distribution-free test like Wilcoxon test, test based on scores under normality and under different models of nonnormality, like heavy tailed or asymmetric distributions Adaptive tests are important in applications because the practicing statistician usually has no information about the underlying distribution . The adaptive testing procedures that are truly nonparametric distribution-free. That is, the two stages of the inference process are constructed in such a way that it control the overall α -level . Monte-Carlo simulations are used for comparison of the tests with respect to level α and power β.

2. Selector statistics for selection of test

We apply the concept of Hogg(1974) that is based on following lemma:

(i) Let F denote the class of distributions under consideration. Suppose that each of k tests $T_1, T_2, \ldots, T_k$ is distribution –free over F , that is $Pr_{Ho}(T_i \in C_i) = \alpha$ for each $F \in F$, $h = 1, \ldots, k$.

(ii) Let S be some statistic (called a selector statistic) that is, under Ho, independent of $T_1, \ldots, T_k$ for each $F \in F$. Suppose we use S to decide which test T_h to conduct. Specially, let M_s denote the set of all values of S with the following decomposition: $M_s = D_1 \cup D_2 \cup \ldots \cup D_h, D_i \cap D_j = \emptyset \; for \; i \neq j$. So that $S \in D_h$ corresponds to the decision to use test T_h.

The overall testing procedure is then defined by: If $S \in D_h$ then reject Ho if $T_h \in C_h$. This two-staged adaptive test is distribution-free under Ho over the class F ,i.e. it maintains the level α for each $F \in F$.

The proof of this lemma is given by Randle and Wolfe(1979).Using the lemma, as a selector statistic, we use a function of order statistics of combined sample. We choose the selector statistic as

$$S = (\widehat{Q_1}, \widehat{Q_2})$$

Table 1.1: Theoretical values of Q_1 and Q_2 for selected distributions

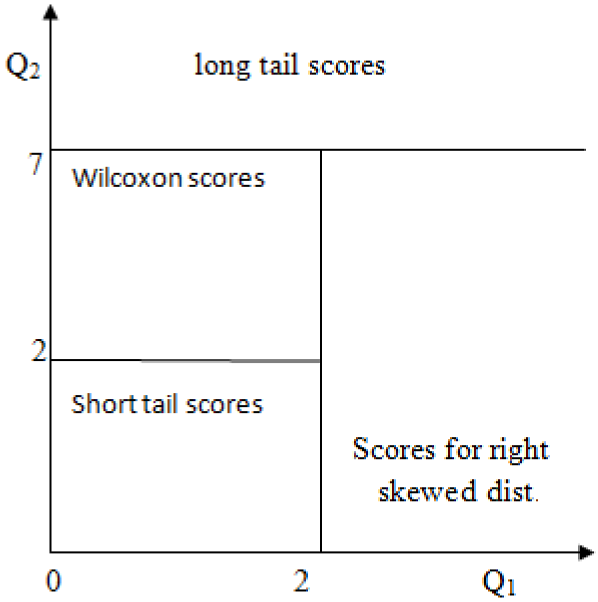
Distribution	Q_1	Q_2
Uniform(0,1)	1	1.9
Normal	1	2.585
Exponential(with λ=1)	4.569	2.864

Where $\widehat{Q_1} = \frac{\hat{U}_{0.05} - \hat{M}_{0.5}}{\hat{M}_{0.5} - \hat{L}_{0.05}}$

and $\hat{Q}_2 = \frac{\hat{U}_{0.05} - \hat{L}_{0.05}}{\hat{U}_{0.5} - \hat{L}_{0.5}}$

And Hogg's(1974) measures for skewness and tailweight, and $\hat{L}_\gamma$, $\hat{M}_\gamma$ and $\hat{U}_\gamma$ denote the average of the smallest , middle and largest γN order statistics, respectively, in the combined sample; fractional items are used when γN is not an integer. Obviously $\hat{Q}_1 = 1$ if the data are symmetric and $\hat{Q}_1 < 1$ (>1) if the data are skewed to the left(right) . The longer the tails the greater is $\hat{Q}_2$. Table 1 shows the theoretical measures of Q_1 and Q_2 for selected distributions.

As in Buning (1996) , we define the adaptive test as follows (Fig2..1)



If $\hat{Q}_1 \leq 2, \hat{Q}_2 \leq 2$ perform the Gastwirth test,
If $\hat{Q}_1 \leq 2, 2 < \hat{Q}_2 \leq 3$ perform Wilcoxon test,
If $\hat{Q}_1 > 2, 2 < \hat{Q}_2 \leq 3$ perform HFR test, and
If $\hat{Q}_2 > 3$ perform the LT test.

However, sometimes a larger critical value for $\hat{Q}_2$ was used to differentiate between tests(Hogg1975). Therefore, we define a second adaptive test as follows:
If $\hat{Q}_1 \leq 2, \hat{Q}_2 \leq 2$ perform the Gastwirth test,
If $\hat{Q}_1 \leq 2, 2 < \hat{Q}_2 \leq 5$ perform Wilcoxon test,
If $\hat{Q}_1 > 2, 2 < \hat{Q}_2 \leq 5$ perform HFR test, and
If $\hat{Q}_2 > 5$ perform the LT test.

3. Test Procedures

Let $x_{i1}, x_{i2}, \ldots, x_{in_i}$, i = 1,2 be independent random samples from parent populations with continuous distribution function $F[(x-\mu_i)]$. Let μ_i represent the location parameters and σ_i the scale parameters of the populations assumed to be same. The problem is to test Ho: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$. In our case we have considered only the alternative $\mu_1 > \mu_2$. For testing this hypothesis the procedures are

3.1 Student's t- test

$$\mathbf{t} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (2.1)$$

where $s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$; and $\bar{x}_i$'s and s_i^2 's are the means and variances of the two samples. The statistic t follows Student's t distribution with $n_1 + n_2 - 2$ degrees of freedom.

3.2 Wilcoxon Test

$\mathbf{W} = \sum_{i=1}^{N} \boldsymbol{iZ_i}$, $\mathbf{N = n_1 + n_2}$
Where Z_i is a indicator variable. It take the value 1, if the ith observation from first sample and zero ,otherwise.

3.3 Two sample tests based on some scores

$T = \sum_{i=1}^{N} g(i) V_i$
where g(i) are real valued scores, and V_i = 1 when the ith smallest of the $N = n_1 + n_2$ observation s is from the first sample and V_i = 0 otherwise. Two-sample tests on T are distribution –free , under Ho , we have
$E(T) = \frac{n_1}{N} \sum_{i=1}^{N} g(i)$
$Var(T) = \frac{n_1 n_2}{N^2(N-1)} [N \sum_{i=1}^{N} g^2(i) - (\sum_{i=1}^{N} g(i)^2]$
And the standardized statistic
$\frac{T - E(T)}{\sqrt{Var\,(T)}}$

Follows asymptotically a standard normal distribution (Hajek et al. 1999).When some condition about the scores g(i) are fulfilled, T can be asymptotically normal under an alternative, too (Chernoff and Savage,1958). However, in general, the rejection probability under the alternative depends on the distribution of the data. Therefore, different choices for scores $g(i)$ were proposed.

Now we will discuss some scores on which adaptive tests A(s) based such as short tailed test , medium tests, long tailed tests and right skewed tail tests.

Gastwirth test (short tails)

$$g(i) = \begin{cases} i - \frac{N+1}{4} \; for \; i \leq \frac{N+1}{4} \\ 0 \; for \; \frac{N+1}{4} \leq i \leq \frac{3(N+1}{4} \\ i - \frac{3(N+1)}{4} \; for \; i \geq \frac{3(N+1)}{4} \end{cases}$$

Wilcoxon test (median tails): $g(i) = i$

Long tails Test (long tails):

$$g(i) = \begin{cases} -\left(\left[\frac{N}{4}\right] + 1\right) \; for \; i < \left[\frac{N}{4}\right] + 1 \\ i - \frac{N+1}{2} \; for \; \left[\frac{N}{4}\right] + 1 \leq i \leq \left[\frac{3(N+1)}{4}\right] \\ \left[\frac{N}{4}\right] + 1 \; for \; i > \left[\frac{3(N+1)}{4}\right] \end{cases}$$

Hogg-Fisher- Randles(HFR) test (right skewed):

$$g(i) = \begin{cases} i - \frac{N+1}{2} \; for \; i \leq \frac{N+1}{2} \\ 0 \; for \; i > \frac{N+1}{2} \end{cases}$$

4. The Monte Carlo Study

For the simulation study of the t- test , Wicoxon test, Gastwirth test, Long-tail test and short-tail test(HFR) six families of distributions are selected. These are – the Normal, the Cauchy, the Double exponential, the Logistic , the Lognormal and the Exponential.

The study was conducted on computer at the Department of Statistics, Dibrugarh University. To generate the standard normal deviate, the method described in Monte Carlo Method by Hammersly and Handscomb(1964) were used and deviate from the other distributions were generated by using the inverse distribution function on uniform deviates.

In studying the significant levels, we first considered distributions with location parameter equal to zero and with equal scale parameters. Specifically, we considered the distribution functions F(x $-\mu_i$) , where μ_i were the location parameters. For each set of sample N = $\sum_i n_i$, i =1,2 , the experiment was repeated 5,000 times and proportion of rejection of the true null hypothesis was recorded and presented in table 1.2 to 1.13.

For the power study of the tests, random deviates were generated as above for each group and added to μ_i . Proportion of rejections based on 5000 replications at the levels .05 and .01 for different combinations of μ_i were recorded and presented in the table 1.2 to table 1.13.

Table 1.2: Empirical Level and power of tests under Normal distribution for equal sample sizes:

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 10	0 0	.0547	.0032	.0542	.0096	.0551	.0082	.0540	.0106	.0559	.0097
	0 0.5	.1609	.0190	.1738	.0534	.1847	.2952	.1951	.0631	.1947	.0558
	0 1.0	.4588	.1152	.5110	.2544	.5206	.3802	.5675	.3059	.5523	.2732
	0 1.5	,7625	.3595	.8388	.6126	.8122	.5089	.8874	.6789	.8758	.6380
	0 2.0	.9301	.6579	.9695	.8838	.9742	.7007	.9874	.9288	.9828	.9056
30 30	0 0	.0517	.0090	.0504	.0096	.0492	.0088	.0495	.0100	.0508	.0094
	0 0.5	.4087	.1811	.4095	.1940	.4099	.9960	.4741	.2365	.4545	.2230
	0 1.0	.9366	.7867	.9349	.8067	.9200	.9992	.9662	.8775	.9583	.855
	0 1.5	.9992	.9932	.9996	.9961	.9900	.9812	1.000	.9991	1.000	.9990
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50 50	0 0	.0530	.0088	.0542	.0102	.0488	.0102	.0480	.0110	.0530	.0098
	0 0.5	.6284	.3774	.6208	.3702	.5818	.3312	.7160	.4840	.6776	.4272
	0 1.0	.9964	.9714	.9956	.9740	.9920	.9542	1.000	.9970	.9974	.9876
	0 1.5	1.000	1.000	1.000	1.000	1.000	.9998	1.000	1.000	1.000	1.000
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 1.3: Empirical Level and power of tests under Normal distribution for unequal sample sizes:

Sample sizes n_i	Location parameter μ_i	G 5%	G %	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0506	.0066	.0544	.0080	.0482	.0092	.0614	.0142	.0506	.0066
	0 0.5	.2142	.0524	.2018	.0686	.2208	.0760	.2634	.1182	.2238	.0706
	0 1.0	.6144	.2984	.6232	.3394	.6278	.3726	.7268	.5042	.6742	.3858
	0 1.5	.9094	.6802	.9228	.7520	.9288	.7762	.9688	.8916	.9500	.8132
	0 2.0	.9918	.9204	.9946	.9630	.9962	.9724	.9994	.9928	.9972	.9798
25 30	0 0	.0534	.0086	.0532	.0100	.0526	.0098	.0612	.0122	.0560	.0088
	0 0.5	.3870	.1556	.3794	.1738	.3634	.1596	.4608	.2446	.4282	.1974
	0 1.0	.9098	.7202	.9076	.7540	.8858	.7138	.9516	.8598	.9398	.8032
	0 1.5	.9990	.9842	.9984	.9902	.9960	.9794	1.000	.9986	.9996	.9954
	0 2.0	1.000	.9998	1.000	1.000	1.000	.9980	1.000	1.000	1.000	1.000
35 50	0 0	.0528	.0088	.0534	.0128	.0548	.0086	.0570	.0138	.0538	.0114
	0 0.5	.5510	.2948	.5432	.2954	.5258	.2798	.6288	.3920	.5956	.3406
	0 1.0	.9836	.9302	.9826	.9296	.9778	.9116	.9942	.9734	.9918	.9600
	0 1.5	1.000	.9998	1.000	1.000	1.000	.9998	1.000	1.000	1.000	1.000
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

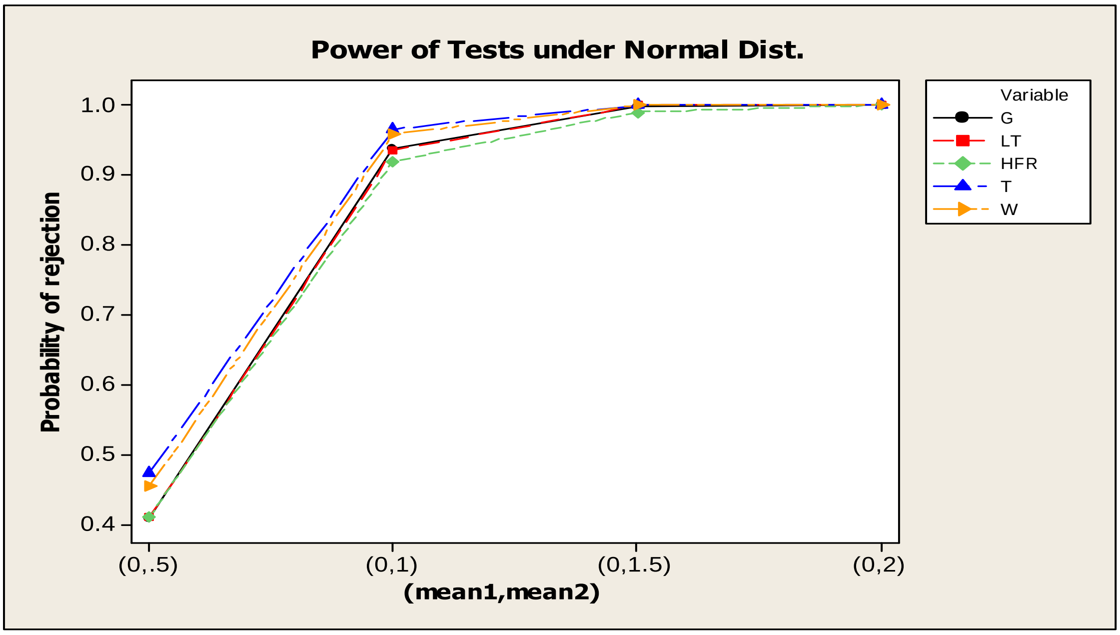


Figure 1.2: Empirical power of tests under Normal distribution for n_1=n_2=30 at 5% Level

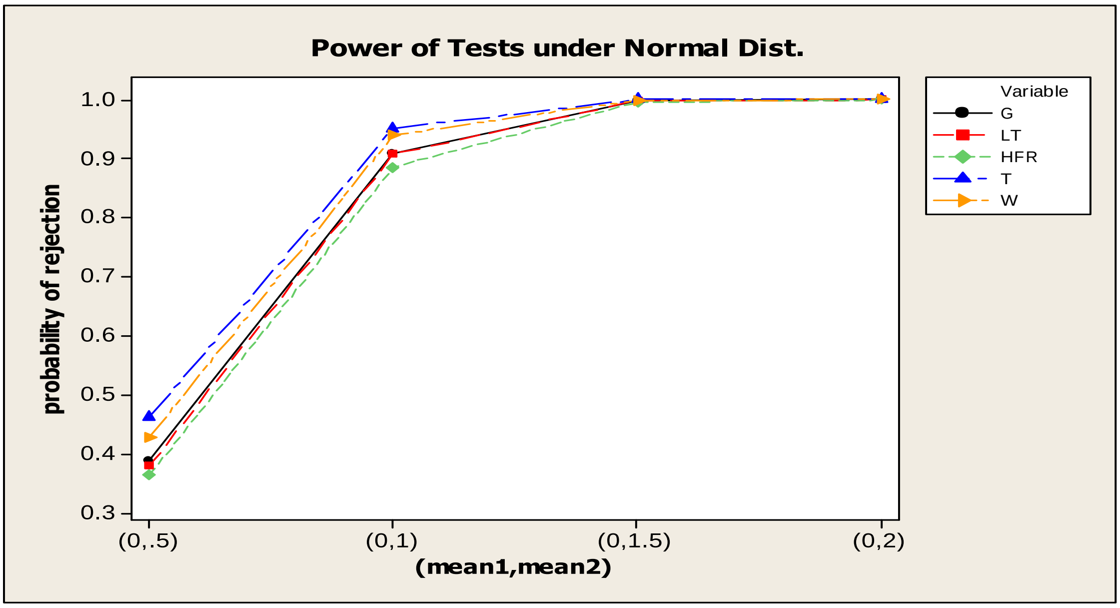


Figure 1.3: Empirical power of tests under Normal distribution for n_1=25, n_2=30 at 5% level

Table 1.4: Empirical level and power of tests under Cauchy distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0510	.0027	.0450	.0094	.0501	.0079	.0197	.0013	.0495	.0087
	0 0.5	.0638	.0063	.0960	.0240	.0852	.0179	.0313	.0036	.0924	.0212
	0 1.0	.0967	.0146	.2310	.0778	.1783	.0532	.0675	.0161	.2031	.0658
	0 1.5	.1355	.0356	.4045	.1821	.2961	.1153	.1227	.0408	.3483	.1454
	0 2.0	.1770	.0639	.5725	.3084	.4243	.1900	.1904	.0763	.4916	.2394
30 30	0 0	.0459	.0093	.0513	.0112	.0503	.0098	.0224	.0022	.0517	.0103
	0 0.5	.0867	.0190	.2117	.0770	.1498	.0465	.0361	.0053	.1736	.0591
	0 1.0	.1867	.0629	.6123	.3607	.4176	.2003	.0772	.0181	.4974	.2644
	0 1.5	.3200	.1369	.8844	.7091	.6751	.4464	.1360	.0499	.7889	.5633
	0 2.0	.4640	.2349	.9740	.9041	.8441	.6540	.2065	.0948	.9242	.7945
50 50	0 0	.0442	.0072	.0440	.0080	.0450	.0064	.0226	.0010	. .046(	5 .0068
	0 0.5	.1050	.0286	.3374	.1426	.2260	.0830	.0352	.0068	.2604	.1044
	0 1.0	.2714	.1076	.8362	.6358	.6126	.3824	.0820	.0248	.7164	.4884
	0 1.5	.4830	.2520	.9078	.9368	.8812	.7154	.1448	.0578	.9472	.8492
	0 2.0	.6710	.4226	.9992	.9950	.9698	.9030	.2226	.1070	.9930	.9674

Table 1.5: Empirical level and power of tests under Cauchy distribution for unequal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0482	.0068	.0468	.0098	.0504	.0082	.0388	.0054	.0472	.0076
	0 0.5	.0702	.0120	.1086	.0272	.1074	.0274	.0516	.0112	.0996	.0250
	0 1.0	.1126	.0284	.2996	.1134	.2472	.0952	.0890	.0270	.2458	.0866
	0 1.5	.1850	.0578	.5314	.2632	.4478	.2176	.1418	.0584	.4406	.2018
	0 2.0	.2660	.0902	.7090	.4436	.6318	.3750	.2062	.1012	.6130	.3474
25 30	0 0	.0538	.0094	.0496	.0100	.0478	.0092	.0310	.0024	.0536	.0090
	0 0.5	.0876	.0198	.1872	.0650	.1546	.0474	.0432	.0080	.1578	.0542
	0 1.0	.1788	.0518	.5572	.3096	.3994	.2028	.0820	.0248	.4592	.2242
	0 1.5	.2978	.1158	.8504	.6454	.6712	.4356	.1428	.0586	.7466	.5014
	0 2.0	.4260	.1984	.9628	.8688	.8422	.6540	.2160	.1060	.9048	.7334
35 50	0 0	.0430	.0072	.0492	.0088	.0490	.0094	.0256	.0016	.0466	.0100
	0 0.5	.0916	.0234	.2844	.1186	.2026	.0728	.0400	.0060	.2288	.0856
	0 1.0	.2344	.0836	.7534	.5160	.5730	.3426	.0796	.0238	.6302	.3836
	0 1.5	.4182	.2004	.9664	.8786	.8606	.6796	.1444	.0544	.9064	.7464
	0 2.0	.6008	.3472	.9964	.9794	.9646	.8868	.2114	.1006	.9822	.9272

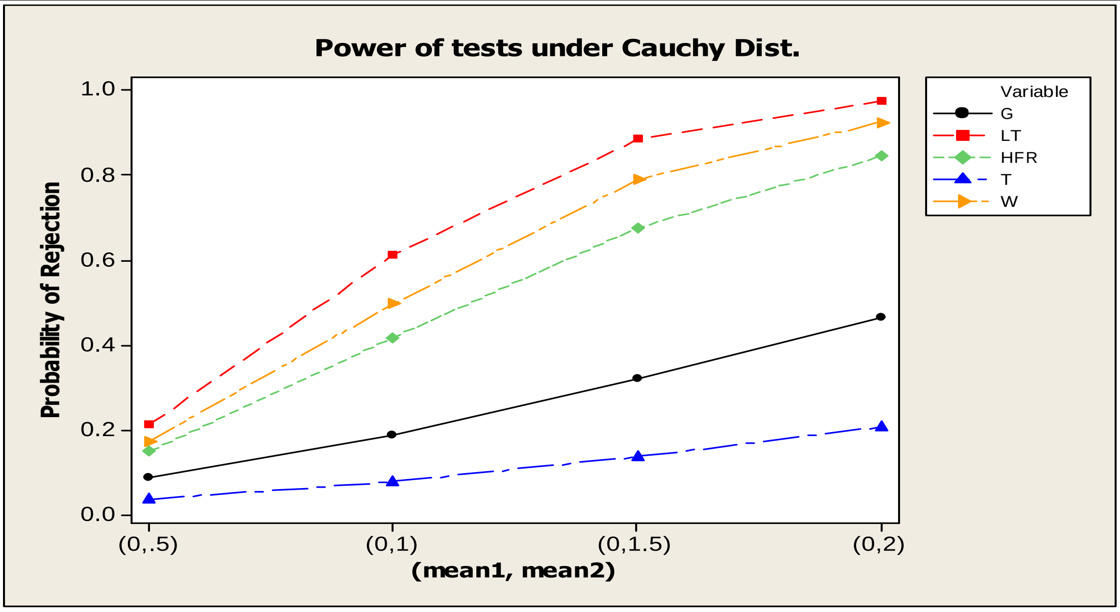


Figure 1.4: Empirical power of tests under Cauchy distribution for n_1=n_2=30 at 5% Level

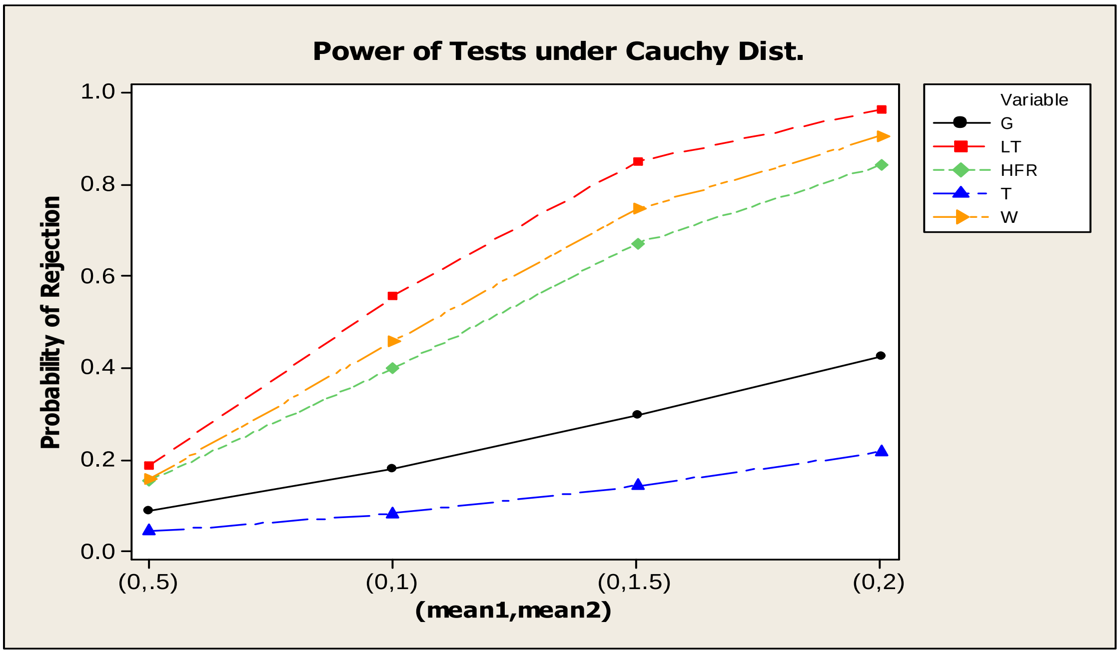


Figure 1.5: Empirical power of tests under Cauchy distribution for n_1=25, n_2=30 at 5% level

Table 1.6: Empirical level and power of tests under Logistic distribution for equal sample sizes:

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 10	0 0	.0546	.0022	.0505	.0086	.0510	.0073	.0465	.0075	.0533	.0081
	0 0.5	.0842	.0066	.0913	.0230	.0902	.0159	.0908	.0222	.0985	.0222
	0 1.0	.1771	.0253	.2282	.0773	.2057	.0562	.2302	.0772	.2404	.0756
	0 1.5	.3184	.0736	.4283	.1941	.3719	.1436	.4352	.2060	.4509	.1968
	0 2.0	.5866	.1669	.6415	.3757	.6675	.2748	.6529	.3945	.6678	.3811
30 30	0 0	.0481	.0085	.0483	.0082	.0483	.0073	.0499	.0083	.0498	.0079
	0 0.5	.1592	.0427	.1863	.0637	.1640	.0503	.1873	.0610	.1934	.0646
	0 1.0	.4739	.2233	.5646	.3192	.4879	.2531	.5622	.3169	.5880	.3354
	0 1.5	.7939	.5470	.8849	.7141	.8223	.6012	.8777	.7034	.8978	.7373
	0 2.0	.9572	.8327	.9880	.9424	.9658	.8753	.9867	.9370	.9912	.9521
50 50	0 0	.0460	.0088	.0542	.0114	.0504	.1040	.0438	.0084	.0512	.0118
	0 0.5	.2374	.0858	.2910	.1176	.2502	.0954	.2822	.1266	.3024	.1212
	0 1.0	.6968	.4424	.7922	.5684	.7172	.4682	.7148	.5222	.8132	.5920
	0 1.5	.9588	.8480	.9858	.9394	.9698	.8868	.9184	.8366	.9886	.9534
	0 2.0	.9980	.9864	.9996	.9964	.9984	.9926	.9754	.9466	.9994	.9978

Table 1.7: Empirical level and power of tests under Logistic distribution for unequal sample sizes:

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0474	.0074	.0518	.0084	.0484	.0050	.0590	.0144	.0498	.0072
	0 0.5	.0932	.0188	.1020	.0252	.1098	.0312	.1198	.0408	.1046	.0264
	0 1.0	.2280	.0666	.2802	.0988	.2810	.1136	.3166	.1454	.2922	.1050
	0 1.5	.4484	.1810	.5366	.2684	.5294	.2826	.5736	.3584	.5502	.2856
	0 2.0	.6622	.3448	.7650	.5070	.7572	.5228	.8002	.6080	.7882	.5264
25 30	0 0	.0496	.0064	.0508	.0104	.0468	.0112	.0558	.0124	.0500	.0096
	0 0.5	.1516	.0372	.1820	.0578	.1646	.0526	.1946	.0722	.1894	.0598
	0 1.0	.4504	.1978	.5190	.2872	.4732	.2496	.5506	.3270	.5496	.3020
	0 1.5	.7614	.4954	.8534	.6454	.7982	.5774	.8566	.6888	.8736	.6744
	0 2.0	.9334	.7784	.9804	.9132	.9576	.8558	.9804	.9256	.9864	.9242
35 50	0 0	.0436	.0086	.0480	.0106	.0548	.0108	.0508	.0128	.0474	.0114
	0 0.5	.1903	.0676	.2410	.0920	.2170	.0904	.2462	.0976	.2484	.0940
	0 1.0	.6000	.3514	.7086	.4616	.6492	.4062	.7052	.4816	.7264	.4884
	0 1.5	.9088	.7430	.9642	.8664	.9386	.8214	.9578	.8754	.9690	.8840
	0 2.0	.9916	.9514	.9978	.9898	.9948	.9794	.9976	.9898	.9986	.9926

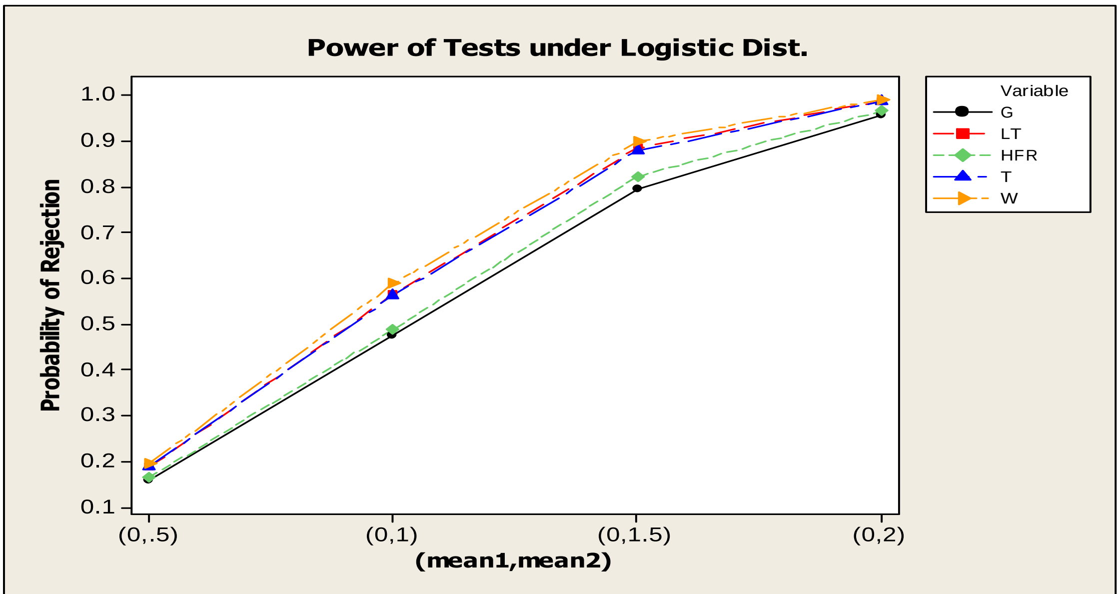


Figure 1.6: Empirical power of tests under Logistic distribution for n_1=n_2=30 at 5% Level

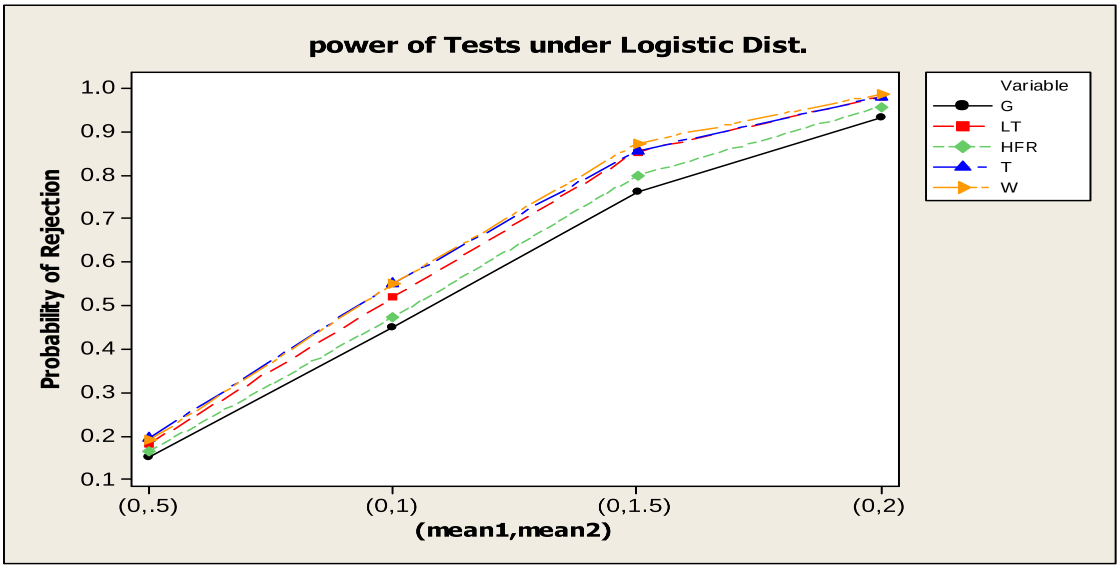


Figure 1.7: Empirical power of tests under Logistic distribution for n_1=25 n_2=30 at 5% level

Table 1.8: Empirical level and power of tests under Lognormal distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 10	0 0	.0547	.0032	.0542	.0096	.0551	.0082	.0270	.0030	.0559	.0097
	0 0.5	.2069	.0319	.2113	.0785	.3663	.1198	.1239	.0243	.2422	.0791
	0 1.0	.3457	.1001	.5395	.3118	.7665	.4664	.3804	.1152	.5619	.2862
	0 1.5	.4402	.1833	.7717	.5740	.9305	.7508	.6581	.2973	.7743	.5135
	0 2.0	.5245	.2759	.8922	.7589	.9807	.8942	.8225	.4854	.8838	.6823
30 30	0 0	.0517	.0090	.0504	.0096	.0492	.0088	.0422	.0056	.0508	.0094
	0 0.5	.6549	.3679	.4670	.2464	.8117	.5914	.3280	.1157	.5703	.3248
	0 1.0	.9329	.7260	.9236	.8035	.9974	.9850	.8453	.5880	.9505	.8479
	0 1.5	.9795	.8584	.9943	.9782	1.000	.9999	.9799	.8977	.9970	.9805
	0 2.0	.9932	.9274	.9998	.9980	1.000	1.000	.9927	.9578	.9999	.9980
50 50	0 0	.0530	.0088	.0542	.0102	.0488	.0102	.0438	.0084	.0530	.0098
	0 0.5	.8854	.6856	.6750	.4376	.9612	.8720	.2822	.1266	.7934	.5826
	0 1.0	.9982	.9764	.9916	.9658	1.000	.9994	.7148	.5222	.9978	.9818
	0 1.5	1.000	.9960	1.000	.9994	1.000	1.000	.9184	.8366	1.000	.9998
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	.9754	.9466	1.000	1.000

Table 1.9: Empirical level and power of tests under Lognormal distribution for unequal sample sizes:

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0506	.0066	.0544	.0080	.0482	.0092	.0466	.0108	.0506	.0066
	0 0.5	.4216	.1852	.2332	.0830	.4918	.2584	.1420	.0402	.3156	.1222
	0 1.0	.7450	.4608	.5498	.3330	.8668	.6964	.4904	.2102	.6628	.4166
	0 1.5	.8830	.6256	.7750	.5870	.9732	.9090	.7298	.5434	.8544	.6692
	0 2.0	.9338	.7224	.8906	.7666	.9948	.9736	.8890	.7454	.9358	.8198
25 30	0 0	.0534	.0086	.0532	.0100	.0526	.0098	.0474	.0074	.0560	.0088
	0 0.5	.6528	.3568	.4196	.2126	.7712	.5536	.3010	.0768	.5384	.2882
	0 1.0	.9344	.7434	.8770	.7310	.9924	.9672	.7400	.3428	.9272	.8008
	0 1.5	.9860	.8714	.9870	.9528	1.000	1.000	.9008	.6452	.9926	.9674
	0 2.0	.9996	.9302	.9990	.9944	1.000	1.000	.9902	.8428	1.000	.9964
35 50	0 0	.0528	.0088	.0534	.0128	.0548	.0086	.0500	.0090	.0538	.0114
	0 0.5	.8380	.6380	.5644	.3328	.9074	.7762	.6574	.2046	.7102	.4816
	0 1.0	.9944	.9714	.9602	.8842	.9996	.9974	.8612	.7616	.9848	.9420
	0 1.5	.9998	.9952	.9976	.9904	1.000	1.000	.9862	.8778	.9990	.9964
	0 2.0	1.000	.9996	1.000	.9998	1.000	1.000	.9906	.9906	1.000	.9998

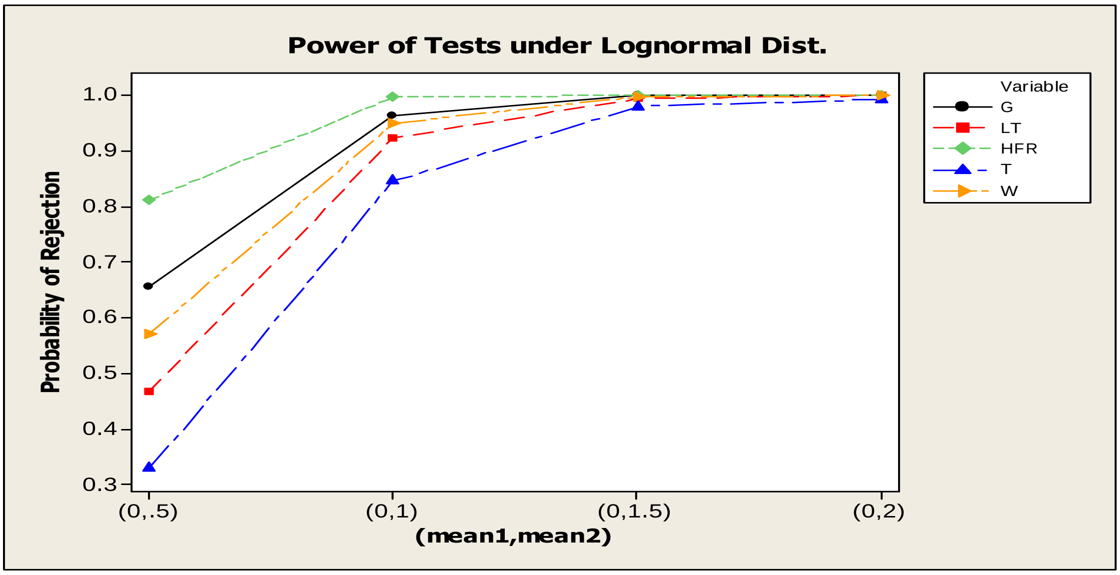


Figure 1.8: Empirical power of tests under Lognormal distribution for $n_1=n_2=30$ at 5% level

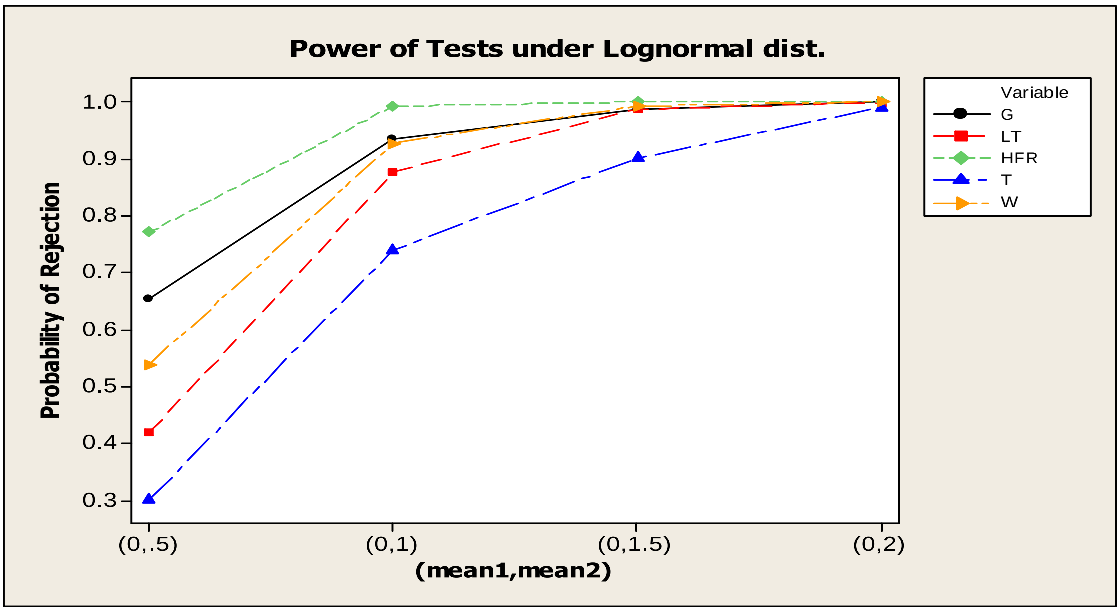


Figure 1.9: Empirical power of tests under Lognormal distribution for $n_1=25$, $n_2=30$ at 5% level

Table 1.10: Empirical level and power of tests under Exponential distribution for equal sample sizes:

Sample sizes n_i	Location parameter μ_i	G		LT		HFR		T		W	
		5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
10 10	0 0	.0546	.0022	.0505	.0086	.0510	.0073	.0421	.0070	.0533	.0081
	0 0.5	.2883	.0528	.2953	.1224	.4855	.1852	.2280	.0777	.3397	.1241
	0 1.0	.5006	.1883	.7146	.4861	.8862	.6479	.6205	.3813	.7412	.4679
	0 1.5	.6590	.3640	.9247	.7950	.9846	.9098	.8768	.7178	.9266	.7629
	0 2.0	.7829	.5398	.9852	.9356	.9986	.9854	.9700	.9034	.9836	.9088
30 30	0 0	.0481	.0085	.0483	.0082	.0483	.0073	.0483	.0087	.0498	.0079
	0 0.5	.8464	.5982	.6474	.4202	.9235	.7844	.5051	.2771	.7633	.5373
	0 1.0	.9907	.9181	.9903	.9610	.9997	.9988	.9540	.8680	.9942	.9752
	0 1.5	.9993	.9873	1.000	.9991	1.000	1.000	.9991	.9951	1.000	.9996
	0 2.0	1.000	.9981	1.000	1.000	1.000	1.000	1.000	.9999	1.000	1.000
50 50	0 0	.0504	.0104	.0542	.0114	.0504	.0104	.0564	.0090	.0512	.0118
	0 0.5	.9942	.9690	.8550	.6746	.9942	.9690	.7156	.5038	.9372	.8158
	0 1.0	1.000	1.000	.9998	.9980	1.000	1.000	.9972	.9862	.9998	.9992
	0 1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 1.11 : Empirical level and power of tests under Exponential distribution for unequal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
10 20	0 0	.0474	.0074	.0518	.0084	.0484	.0050	.0524	.0126	.0498	.0072
	0 0.5	.5678	.2926	.3208	.1282	.6164	.3790	.3086	.1362	.4324	.2008
	0 1.0	.8906	.6616	.7280	.5224	.9450	.8412	.7436	.5680	.8208	.6186
	0 1.5	.9730	.8340	.9206	.8092	.9948	.9732	.9480	.8728	.9628	.8662
	0 2.0	.9906	.9144	.9838	.9414	.9998	.9970	.9906	.9760	.9940	.9646
25 30	0 0	.0496	.0064	.0508	.0104	.0468	.0112	.0488	.0104	.0500	.0096
	0 0.5	.8370	.5912	.5908	.3628	.8910	.7330	.5038	.2860	.7252	.4796
	0 1.0	.9914	.9246	.9764	.9204	.9998	.9966	.9466	.8582	.9912	.9546
	0 1.5	.9998	.9860	.9996	.9982	1.000	1.000	.9988	.9924	.9998	.9994
	0 2.0	1.000	.9980	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35 50	0 0	.0436	.0086	.0480	.0106	.0548	.0108	.0554	.0080	.0474	.0114
	0 0.5	.9556	.8504	.7398	.5310	.9716	.9100	.6458	.4162	.8710	.7042
	0 1.0	1.000	.9988	.9962	.9820	1.000	.9996	.9914	.9616	.9990	.9964
	0 1.5	1.000	.9998	.9998	1.000	1.000	1.000	1.000	1.000	1.000	.9998
	0 2.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

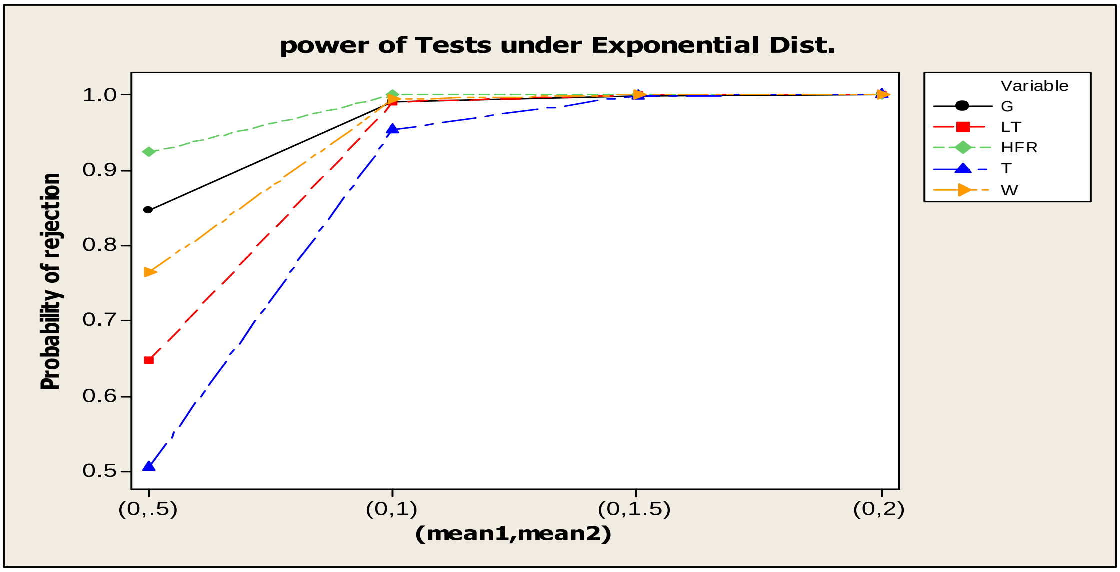


Figure 1.10: Empirical power of tests under Exponential distribution for n_1=n_2=30 at 5% level

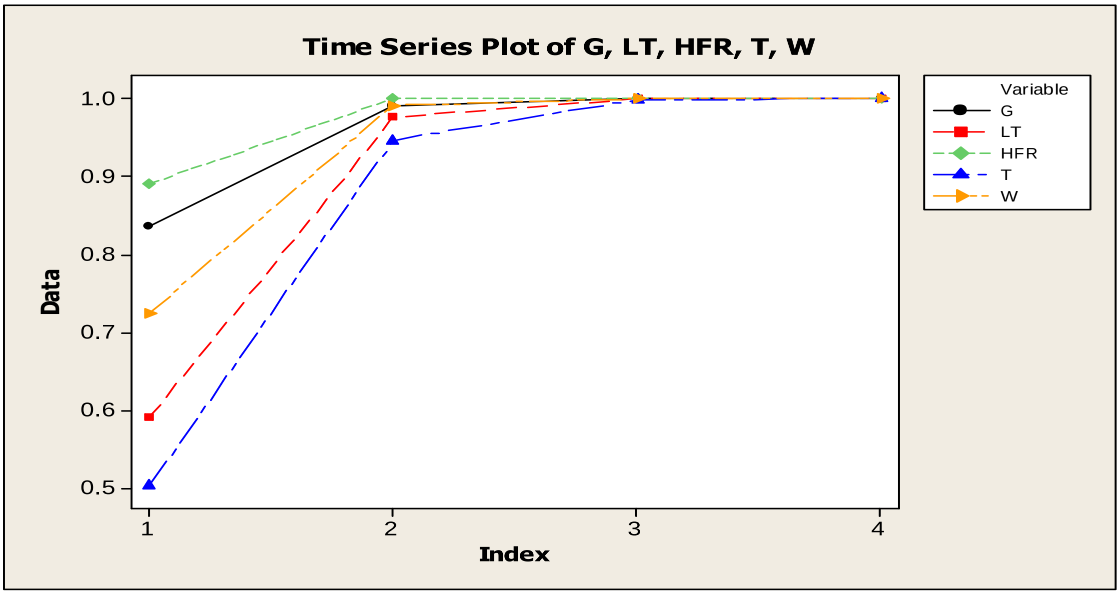


Figure 1.11: Empirical power of tests under Exponential distribution for n_1=25, n_2=30 at 5% level

Table 1.12: Empirical level and power of tests under Double Exponential Distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	t 5%	t 1%	W 5%	W 1%
10 10	0 0	.0546	.0022	.0505	.0086	.0510	.0073	.0449	.0065	.0533	.0081
	0 0.5	.1015	.0111	.1576	.0472	.1398	.0322	.1290	.0330	.1584	.0425
	0 1.0	.2339	.0543	.4307	.1995	.3544	.1363	.3597	.1583	.4168	.1845
	0 1.5	.4110	.1560	.7165	.4495	.5936	.3117	.6373	.3907	.7003	.4186
	0 2.0	.5932	.3083	.8918	.6985	.7888	.5140	.8411	.6446	.8834	.6606
30 30	0 0	.0481	.0085	.0464	.0123	.0483	.0073	.0494	.0080	.0498	.0079
	0 0.5	.2110	.0691	.3962	.1818	.3044	.1236	.2833	.1152	.3660	.1617
	0 1.0	.6114	.3448	.8966	.7339	.7783	.5549	.7700	.5513	.8667	.6885
	0 1.5	.9060	.7252	.9957	.9761	.9718	.8928	.9764	.9108	.9937	.9643
	0 2.0	.9871	.9333	1.000	.9996	.9984	.9866	.9992	.9945	1.000	1.000
50 50	0 0	.0460	.0088	.0542	.0114	.0504	.0104	.0518	.0096	.0512	.0118
	0 0.5	.3216	.1364	.5918	.3564	.4668	.2438	.4390	.2284	.5522	.3194
	0 1.0	.8362	.6262	.9882	.9512	.9514	.8414	.9404	.8318	.9834	.9294
	0 1.5	.9908	.9498	.9996	.9994	.9994	.9934	.9990	.9962	.9998	.9988
	0 2.0	.9996	.9970	1.000	1.000	1.000	1.000	1.000	.9998	1.000	.9998

Table 1.13: Empirical level and power of tests under Double Exponential Distribution for unequal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	t 5%	t 1%	W 5%	W 1%
10 20	0 0	.0474	.0074	.0518	.0084	.0484	.0050	.0586	.0126	.0498	.0072
	0 0.5	.1144	.0280	.1960	.0566	.1824	.0610	.1682	.0636	.1800	.0524
	0 1.0	.3278	.1128	.5460	.2792	.5092	.2662	.4742	.2736	.5162	.2568
	0 1.5	.5878	.2976	.8334	.5986	.8058	.5872	.7756	.5824	.8154	.5718
	0 2.0	.7924	.5066	.9510	.8342	.9482	.8302	.9322	.8374	.9460	.8156
25 30	0 0	.0496	.0064	.0508	.0104	.0468	.0112	.0560	.0114	.0500	.0096
	0 0.5	.2046	.0602	.3624	.1670	.2980	.1288	.2894	.1254	.3456	.1476
	0 1.0	.5900	.3120	.8644	.6686	.7656	.5384	.7492	.5500	.8390	.6272
	0 1.5	.8740	.6690	.9942	.9640	.9666	.8834	.9680	.8954	.9916	.9426
	0 2.0	.9780	.8928	1.000	.9992	.9976	.9840	.9992	.9928	.9998	.9984
35 50	0 0	.0436	.0086	.0480	.0106	.0548	.0108	.0502	.0112	.0474	.0114
	0 0.5	.2726	.1012	.5168	.2780	.4184	.2060	.3720	.1776	.4744	.2438
	0 1.0	.7532	.5114	.9686	.8842	.9206	.7878	.8882	.7376	.9542	.8454
	0 1.5	.9696	.8888	.9998	.9974	.9974	.9870	.9958	.9828	.9996	.9960
	0 2.0	.9986	.9904	1.000	1.000	1.000	.9994	1.000	.9998	1.000	.9998

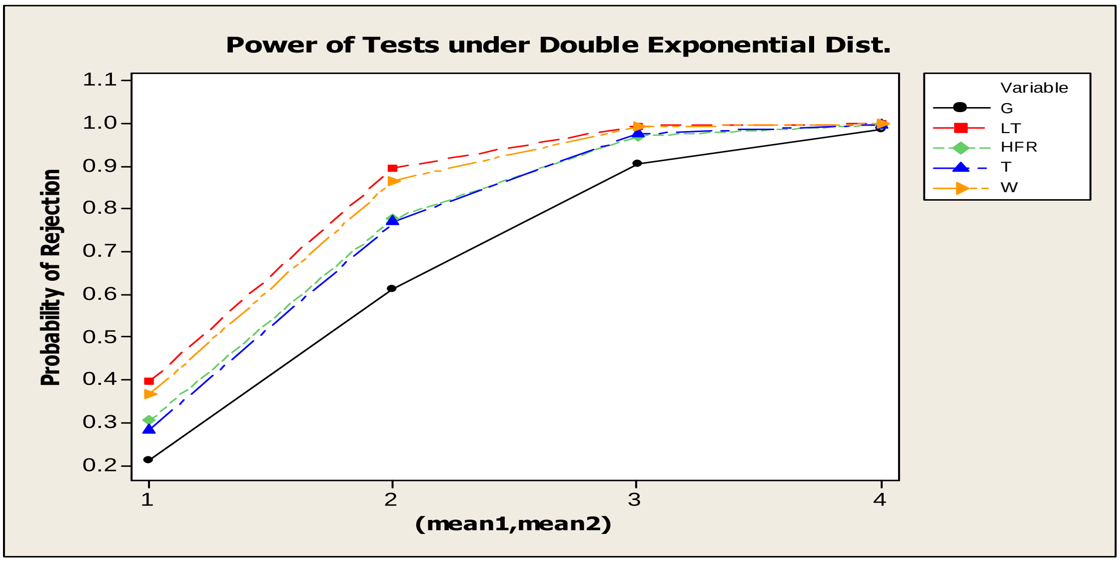


Figure 1.12: Empirical power of tests under Double Exponential distribution for $n_1=n_2=30$ at 5% level

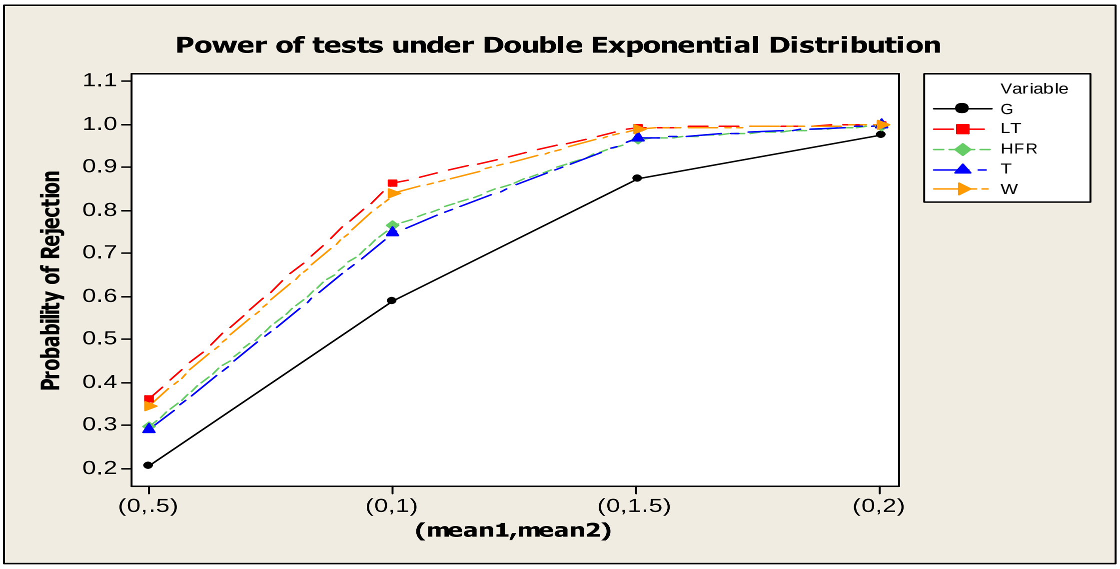


Figure 1.13: Empirical power of tests under Double Exponential distribution for n_1=25,n_2=30 at 5% level

5. Discussion

For comparison purposes we have considered various combinations of sample sizes with equal and unequal sample sizes. We have also considered different sets of μ_i's for the study.

From Tables 1.2 - 1.13 it is observed that the parametric t-test maintain the nominal level except the Cauchy and skew distribution lognormal. In these cases t-test seems to be conservatives. Wilcoxon test and all other score based test are found to be robust against the distributions in terms of the level concerned.

Table 1.2 and table 1.3 shows the power of tests under normal distribution for equal and unequal sample sizes respectively. We have seen that power of t- test is higher than the other tests in this distribution in presence of various combinations of location parameters and sample sizes. Power of Wilcoxon test is found to be slightly less than the t-test but more than other score base tests in both the two cases.

Table 1.4 and Table 1.5 gives the power of tests statistics under Cauchy distribution for equal and unequal sample sizes respectively. Here ,we observe that LT test is more powerful than other tests both in case of equal and unequal sample sizes at 5% and 1% level of significance. Second highest power is shown by the Wilcoxon test.

Table 1.6 and Table 1.7 displays the power of tests under logistic distribution. We have seen that Wilcoxon , t- test and LT test are more powerful than other two tests with both equal and unequal sample sizes and at 5% and 1% level. However, power of W test is more than all tests.

Table 1.8 and Table 1.9 shows the power of tests under lognormal distribution. Here ,HFR test i.e. right tailed test (RT) is more powerful than the all other tests both in case of equal and unequal sample sizes at 5% and 1% level.However for large sample sizes and for large shift in mean power W-test and G-test come nearer and near to the HFR test.

Table 1.10 and Table 1.11 shows the empirical power of tests under exponential distribution. Here we obtained similar results as like the lognormal distribution. That is, power of HFR is the highest of all followed by Wicoxon, G and others.

Table 1.12 and Table 1.13 shows the empirical power of tests under double exponential distribution. Here it is seen that long tail test (LT) and Wilcoxon test are more powerful than the other tests both in case of equal and unequal sample sizes and at 5% and 1% level. Power of t- test found to be less than these two tests but more than other score based tests.

6. Conclusion

In case of normal distribution , under equal variances t-test is most the preferable test, as it maintain levels and shows more power than other tests. In case of other distribution s rank test or score based test may be more preferable. The choice of a suitable rank test or score based test which is more efficient than t-test depends on the underlying distribution of data. Because the practicing statistician usually has no clear idea about the distribution, an adaptive test should be applied which takes into account the given data set.

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