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K-Even Sequential Harmonious Labeling of Some Cycle Related Graphs

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Abstract: ***Graham and Sloane [6] introduced the harmonious graphs and Singh & Varkey [7] introduced the odd sequential graphs. Gayathri & Hemalatha [2] introduced even sequential harmonious labeling of graphs. We studied even sequential harmonious labeling of trees in [3]. In [4] we have extended this notion to k-even sequential harmonious labeling graphs. It is further studied in [5]. Here, we investigate the k-even sequential harmonious labeling of some cycle related graphs.***

Keywords: *k*-ESHL, *k*-ESHG.

AMS Subject Classification: 05C78.

1. Introduction

All graphs in this paper are finite, simple and undirected. The symbols V(G) and E(G) denote the vertex set and the edge set of a graph G.

The cardinality of the vertex set is called the order of G. The cardinality of the edge set is called the size of G. A graph with p vertices and q edges is called a (p, q) graph.

A **graph labeling** is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices or edges then the labeling is called **vertex or edge labeling**.

Graph labeling was first introduced in late 1960's. In the recent years, dozens of graph labeling techniques have been studied in over 1200 papers.

Labeled graphs serve as useful models for a broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network addressing etc. [7 Gallian]

We say that a labeling is an k-even sequential harmonious labeling if there exists an injection f from the vertex set V to $\{k-1, k+1, ..., k+2q-1\}$ such that the induced mapping f^* from the edge set E to $\{2k, 2k+2, 2k+4, ..., 2k+2q-2\}$ defined by $f^*(uv) = \begin{cases} f(u)+f(v), \text{ if } f(u)+f(v) \text{ is even} \\ f(u)+f(v)+1, \text{ if } f(u)+f(v) \text{ is odd} \end{cases}$ are distinct.

A graph G is said to be an k-even sequential harmonious graph if it admits an k-even sequential harmonious labeling. In this paper, we investigate some results on k-even sequential harmonious labeling of some cycle related graphs. Throughout this paper, k denote any positive integer ≥ 1. For brevity, we use k-ESHL for k-even sequential harmonious labeling.

2. Main Results

Theorem 3.2
The cycle C_m is a k-even sequential harmonious graph for $m \geq 3$.

Proof
Let $\{v_1, v_2, \ldots, v_m\}$ be the vertices and $\{e_1, e_2, \ldots, e_m\}$ be the edges of C_m which are denoted as in Fig. 3.1.

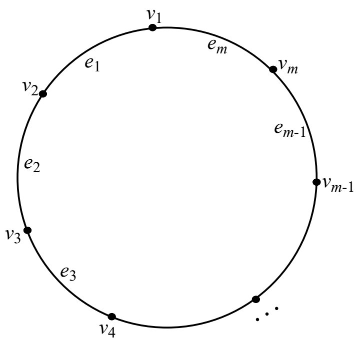


Figure 3.1: C_n with ordinary labeling

First we label the vertices as follows:
Define $f: V \to \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(v_i) = \begin{cases} k+2i-3 & 1 \leq i \leq \frac{m+1}{2}, \text{if } m \text{ is odd} \\ & 1 \leq i \leq \frac{m+2}{2}, \text{if } m \text{ is even} \\ k+2m-2(i-1) & \frac{m+3}{2} \leq i \leq m, \text{if } m \text{ is odd} \\ & \frac{m+4}{2} \leq i \leq m, \text{if } m \text{ is even} \end{cases}$$

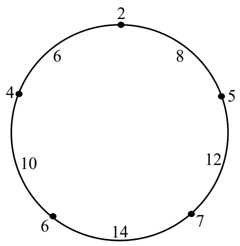
Then the induced edge labels are:

$$f^{+}(e_i)=\begin{cases} 2k+4(i-1) & 1\le i\le \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & 1\le i\le \frac{m}{2}, \text{ if } m \text{ is even} \\ 2k+4(m-i)+2 & \frac{m+3}{2}\le i\le m, \text{ if } m \text{ is odd} \\ & \frac{m+2}{2}\le i\le m, \text{ if } m \text{ is even} \end{cases}$$

Therefore, $f^{+}(E)$ = {2k, 2k + 2, ..., 2k + 2q – 2}. So, *f* is a *k*-even sequential harmonious labeling and hence, the graph C_m ($m \ge 3$) is a *k*-even sequential harmonious graph for any *k*.

Illustration 3.3

3-*ESHL* of C_5 and 6-*ESHL* of C_6 are shown in Fig. 3.2(a) and Fig. 3.2(b) respectively.



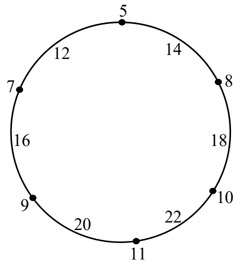


Figure 3.2(a): 3-*ESHL* of C_5 **Figure 3.2(b):** 6-*ESHL* of C_6

Theorem 3.4

The triangular snake T_n ($n \ge 2$) is a *k*-even sequential harmonious graph for any *k*.

Proof

Let $\{v_1, v_2, ..., v_{n+1}, u_1, u_2, ..., u_n\}$ be the vertices and $\{e_1, e_2, ..., e_n, e_1^{'}, e_2^{'}, ..., e_{2n}^{'}\}$ be the vertices of T_n which are denoted as in Fig. 3.3.

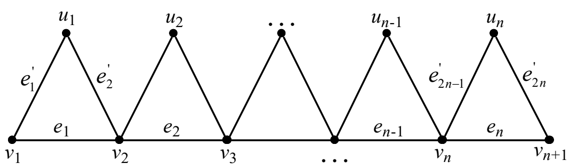


Figure 3.3: T_n with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

For $1 \le i \le n+1$,

$$f(v_i) = k+i-2$$

For $1 \le i \le n$,

$$f(u_i) = k+2n+3i-2$$

Then the induced edge labels are as follows

For $1 \le i \le n$,

$$f^{+}(e_i) = 2k+2(i-1)$$

For $1 \le i \le 2n$,

$$f^{+}(e_i^{'}) = 2k+2n+2(i-1)$$

Therefore, $f^{+}(E)$ = {2k, 2k + 2, ..., 2k + 2q – 2}. So, *f* is a *k*-even sequential harmonious labeling and hence, the graph T_n is a *k*-even sequential harmonious graph for any *k*.

Illustration 3.5

(a) 1-*ESHL* of T_5 is shown in Fig. 3.4(a).

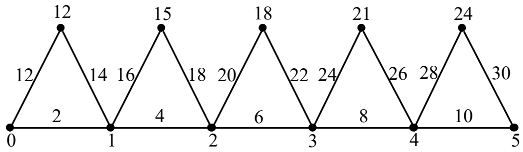


Figure 3.4(a): 1-*ESHL* of T_5

(b) 4-*ESHL* of T_6 is shown in Fig. 3.4(b).

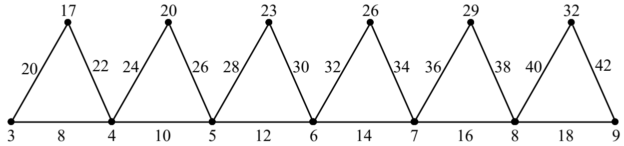


Figure 3.4(b): 4-*ESHL* of T_6

Theorem 3.6

The graph C_m @ P_n ($m \ge 3$, $n \ge 1$) is a *k*-even sequential harmonious graph for any *k*.

Proof

Let $\{v_i, u_j; 1 \le i \le m, 1 \le j \le n\}$ be the vertices and $\{e_i; 1 \le i \le m+n\}$ be the edges where $\left\{e_{m+1}=(v_i u_1), i=\frac{m+3}{2}, \text{if } m \text{ is odd}, i=\frac{m+2}{2}, \text{if } m \text{ is even}\right\}$ which are denoted as in Fig. 3.5.

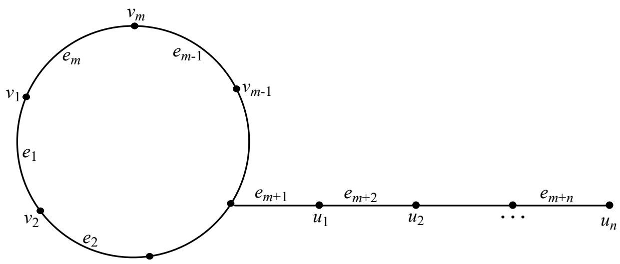


Figure 3.5: C_m @ P_n with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(v_i)=\begin{cases} k+2i-3 & 1\le i\le \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & 1\le i\le \frac{m+2}{2}, \text{ if } m \text{ is even} \\ k+2m-2(i-1) & \frac{m+3}{2}\le i\le m, \text{ if } m \text{ is odd} \\ & \frac{m+4}{2}\le i\le m, \text{ if } m \text{ is even} \end{cases}$$

For $1 \le j \le n$,

$$f(u_j) = \begin{cases} m+k+j-2, & \text{if } j \text{ is even} \\ m+j+k, & \text{if } j \text{ is odd} \end{cases}$$

Then the induced edge labels are:

$$f^+(e_i)=\begin{cases} 2k+4(i-1) & 1\le i\le \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & 1\le i\le \frac{m}{2}, \text{ if } m \text{ is even} \\ 2k+4(m-i)+2 & \frac{m+3}{2}\le i\le m, \text{ if } m \text{ is odd} \\ & \frac{m+2}{2}\le i\le m, \text{ if } m \text{ is even}\end{cases}$$

For $m + 1 \le i \le m + n$,

$$f^+(e_i) = 2i + 2k - 2$$

Therefore, $f^+(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k-even sequential harmonious labeling and hence, the graph $C_m @ P_n$ $(m \ge 3, n \ge 1)$ is a k-even sequential harmonious graph for any k.

Illustration 3.7

(a) 1-*ESHL* of $C_8 @ P_3$ is shown in Fig. 3.6(a).

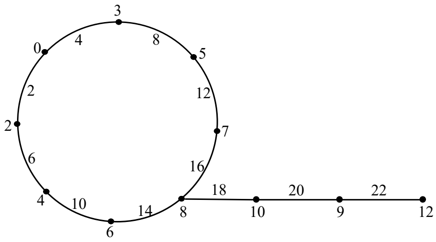


Figure 3.6(a): 1-*ESHL* of $C_8 @ P_3$

(b) 3-*ESHL* of $C_{10} @ P_4$ is shown in Fig. 3.6(b).

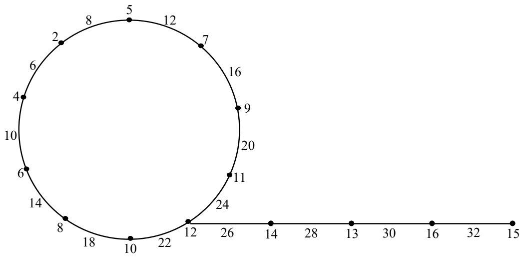


Figure 3.6(b): 3-*ESHL* of $C_{10} @ P_4$

Theorem 3.8

The crown $C_m \odot K_1$ $(m \ge 3)$ is a k-even sequential harmonious graph for any k.

Proof

Let $\{v_i, u_i, 1 \le i \le m\}$ be the vertices and $\{e_i, e_i^{'}; 1 \le i \le m\}$ be the edges which are denoted as in Fig. 3.7.

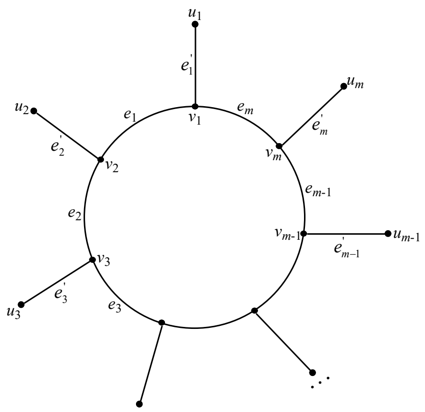


Figure 3.7: $C_m \odot K_1$ with ordinary labeling

First we label the vertices as follows:

Define $f: V \rightarrow \{k - 1, k, k + 1, ..., k + 2q - 1\}$ by

$$f(v_i)=\begin{cases} k+2i-3 & 1\le i\le \frac{m+2}{2}, \text{when } m \text{ is even} \\ & 1\le i\le \frac{m+1}{2}, \text{when } m \text{ is odd} \\ k+2m-2i+2 & \frac{m+4}{2}\le i\le m, \text{ when } m \text{ is even} \\ & \frac{m+3}{2}\le i\le m, \text{ when } m \text{ is odd}\end{cases}$$

$$f(u_1) = k + 2m$$

$$f(u_i)=\begin{cases} k+2m+2i-3 & 2\le i\le \frac{m+2}{2}, \text{when } m \text{ is even} \\ & 2\le i\le \frac{m+1}{2}, \text{when } m \text{ is odd} \\ k+4m-2i+2 & \frac{m+4}{2}\le i\le m, \text{ when } m \text{ is even} \\ & \frac{m+3}{2}\le i\le m, \text{ when } m \text{ is odd}\end{cases}$$

Then the induced edge labels are:

$$f^+(e_i) = \begin{cases} 2k+4(i-1) & 1\le i\le \frac{m}{2}, \text{when } m \text{ is even} \\ & 1\le i\le \frac{m+1}{2}, \text{when } m \text{ is odd} \\ 2k+4m-4i+2 & \frac{m+2}{2}\le i\le m, \text{ when } m \text{ is even} \\ & \frac{m+3}{2}\le i\le m, \text{ when } m \text{ is odd}\end{cases}$$

$$f^+(e_i^{'}) = 2k + 2m$$

$$f^+(e_i^{'}) = \begin{cases} 2k+2m+4i-6 & 2\le i\le \frac{m+2}{2}, \text{when } m \text{ is even} \\ & 2\le i\le \frac{m+1}{2}, \text{when } m \text{ is odd} \\ 2k+6m-4i+4 & \frac{m+4}{2}\le i\le m, \text{ when } m \text{ is even} \\ & \frac{m+3}{2}\le i\le m, \text{ when } m \text{ is odd}\end{cases}$$

Therefore, $f^+(E) = \{2k, 2k + 2, ..., 2k + 2q - 2\}$. So, f is a k-even sequential harmonious labeling and hence, the graph $C_m \odot K_1$ $(m \ge 3)$ is a k-even sequential harmonious graph for any k.

Illustration 3.9

(a) 1-*ESHL* of $C_3 \odot K_1$ is shown in Fig. 3.8(a).

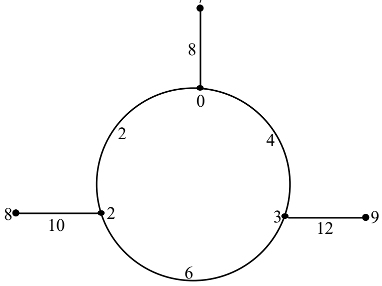


Figure 3.8(a): 1-*ESHL* of $C_3 \odot K_1$

(b) 1-*ESHL* of $C_6 \odot K_1$ is shown in Fig. 3.8(b).

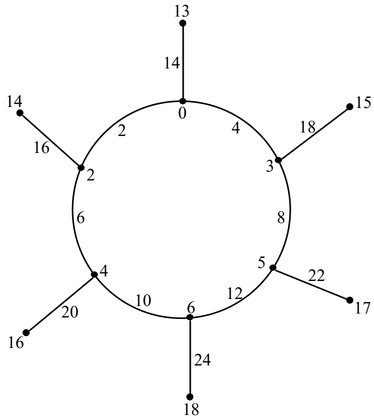


Figure 3.8(b): 1-*ESHL* of $C_6 \odot K_1$

Theorem 3.10
The sunflower graph $SF(n)$ ($n \geq 3$) is a k-even sequential harmonious graph for any k.

Proof
Let $\{v_i, w_i, 1 \leq i \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq n, e_i^{'}, 1 \leq i \leq 2n\}$ be the edges which are denoted as in Fig. 3.9.

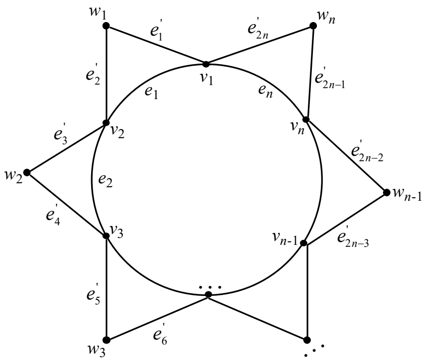


Figure 3.9: Ordinary labeling of $SF(n)$

First we label the vertices as follows:
Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

Case (i): *n* is odd

For $1 \leq i \leq \frac{n+1}{2}$,

$$f(v_i) = k + 2i - 3$$

For $\frac{n+3}{2} \leq i \leq n$,

$$f(v_i) = k + 2n - 2i + 2$$

For $1 \leq i \leq \frac{n+1}{2}$,

$$f(w_i) = k + 2n + 2i - 1$$

For $\frac{n+3}{2} \leq i \leq n$,

$$f(w_i) = k + 6i - 4$$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n+1}{2}$,

$$f^+(e_i) = 2k + 4i - 4$$

For $\frac{n+3}{2} \leq i \leq n$,

$$f^+(e_i) = 2k + 4n - 4i + 2$$

For $1 \leq i \leq n+1$,

$$f^+(e_i^{'}) = 2k + 2n + 2(i-1)$$

For $n+2 \leq i \leq 2n$,

$$f^+(e_i^{'}) = \begin{cases} 2k+2n+2i & i \text{ odd} \\ 2k+2n+2i-4 & i \text{ even} \end{cases}$$

Case (ii): *n* is even

For $1 \leq i \leq \frac{n+2}{2}$,

$$f(v_i) = k + 2i - 3$$

For $\frac{n+4}{2} \leq i \leq n$,

$$f(v_i) = k + 2n - 2i + 2$$

For $1 \leq i \leq \frac{n}{2}$,

$$f(w_i) = k + 2n + 2i - 1$$

For $\frac{n+2}{2} \leq i \leq n$,

$$f(w_i) = k + 6i - 4$$

Then the induced edge labels are:

For $1 \leq i \leq \frac{n}{2}$,

$$f^+(e_i) = 2k + 4i - 4$$

For $\frac{n+2}{2} \leq i \leq n$,

$$f^+(e_i) = 2k + 4n - 4i + 2$$

For $1 \leq i \leq n$,

$$f^+(e_i^{'}) = 2k + 2n + 2(i-1)$$

For $n+1 \leq i \leq 2n$,

$$f^+(e_i^{'}) = \begin{cases} 2k+2n+2i & i \text{ odd} \\ 2k+2n+2i-4 & i \text{ even} \end{cases}$$

Therefore, $f^+(E) = \{2k, 2k+2, ..., 2k+2q-2\}$. So, f is a k-even sequential harmonious labeling and hence, the sunflower graph $SF(n)$ is a k-even sequential harmonious graph for any k.

Illustration 3.11

(a) 2-*ESHL* of $SF(6)$ is shown in Fig. 3.10(a).

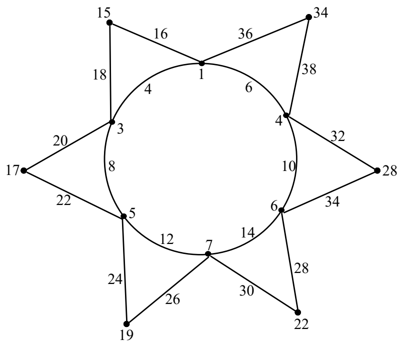


Figure 3.10(a): 2-*ESHL* of $SF(6)$

(b) 4-*ESHL* of *SF*(9) is shown in Fig. 3.10(b).

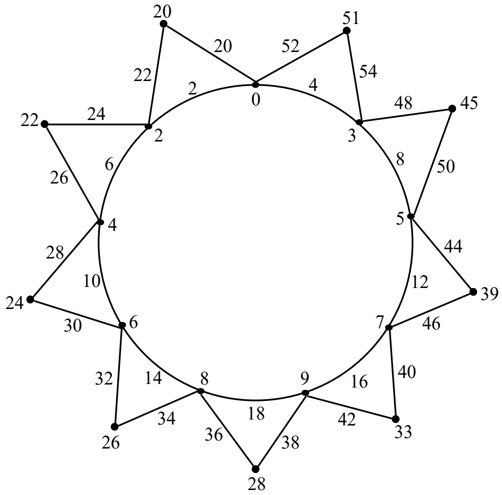


Figure 3.10(b): 1-*ESHL* of *SF*(9)

Theorem 3.12

The graph PC_n $(n \geq 3)$ is a k-even sequential harmonious graph for any k when n is even.

Proof

Let $\{v_0, v_1, ..., v_{n-1}\}$ be the vertices and $\left\{e_i,\ 1 \leq i \leq n,\ e_i^{'},\ 1 \leq i \leq \frac{n-2}{2}\right\}$ be the edges which are denoted as in Fig. 3.11.

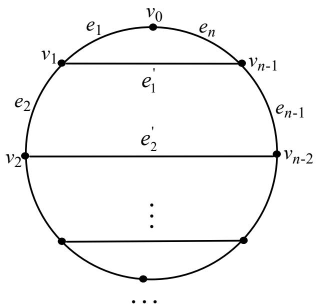


Figure 3.11: PC_n with ordinary labeling

First we label the vertices as follows:

Define $f: V \to \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(v_i) = \begin{cases} k-1 & i = 0 \\ 3i+k-2 & 1 \leq i \leq \frac{n-2}{2} \\ k+3n-3i-1 & \frac{n}{2} < i \leq n-1 \end{cases}$$

Then the induced edge labels are:

$$f^{+}(e_i) = \begin{cases} 6i+2k-6 & 1 \leq i \leq \frac{n}{2} \\ 2k+6n-6i+2 & \frac{n+2}{2} \leq i \leq n \end{cases}$$

$$f^{+}(e_i^{'}) = 2k+6i-2 \qquad 1 \leq i \leq \frac{n-2}{2}$$

Therefore, $f^{+}(E) = \{2k, 2k+2, ..., 2k+2q-2\}$. So, f is a k-even sequential harmonious labeling and hence, the graph PC_n $(n \geq 3)$ is a k-even sequential harmonious graph for any k.

Illustration 3.13

(a) 1-*ESHL* of PC_8 is shown in Fig. 3.12(a).

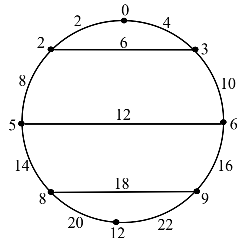


Figure 3.12(a): 1-*ESHL* of PC_8

(b) 4-*ESHL* of PC_{10} is shown in Fig. 3.12(b).

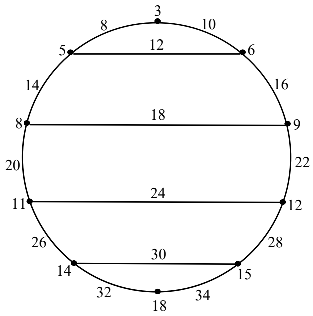


Figure 3.12(b): 4-*ESHL* of PC_{10}

Theorem 3.16

The graph CH_4^n $(n \geq 3)$ is a k-even sequential harmonious graph for any k.

Proof

Let $\{v_{i1}, v_{i2}, v_{i3}, v_{i4}, 1 \leq i \leq n\}$ be the vertices and $\{e_{i1}, e_{i2}, e_{i3}, e_{i4}, 1 \leq i \leq n, a_{i1}, a_{i2}, a_{i3}, a_{i4}, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Fig. 3.15.

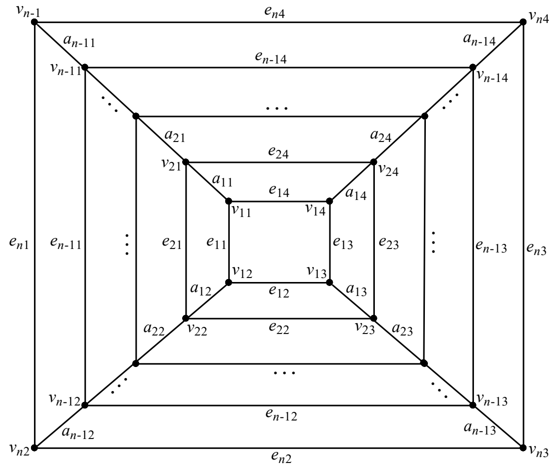


Figure 3.15: Ordinary labeling of CH_4^n

First we label the vertices as follows:

Define $f: V \to \{k-1, k, k+1, ..., k+2q-1\}$ by

For $1 \leq i \leq n$,

$$f(v_{i1}) = \begin{cases} 8(i-1)+k-1 & i \text{ odd} \\ 8i-6+k-1 & i \text{ even} \end{cases}$$

$$f(v_{i3}) = \begin{cases} 8i-4+k-1 & i \text{ odd} \\ 8i-5+k-1 & i \text{ even} \end{cases}$$

$$f(v_{i4}) = \begin{cases} 8i-5+k-1 & i \text{ odd} \\ 8(i-1)+k-1 & i \text{ even} \end{cases}$$

$$f(v_{12}) = k+1$$

$$f(v_{32}) = k + 16$$
$$f(v_{22}) = k + 11$$

For $3 \le i \le n$,

$$f(v_{i2}) = \begin{cases} 8i - 4 + k - 1 & i \text{ even} \\ 8i - 6 + k - 1 & i \text{ odd} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le n$,

$$f^+(e_{i1}) = \begin{cases} 16i - 14 + 2k - 2 & i \text{ odd} \\ 16i - 10 + 2k - 2 & i \text{ even} \end{cases}$$

$$f^+(e_{i2}) = \begin{cases} 16i - 10 + 2k - 2 & i \text{ odd} \\ 16i - 8 + 2k - 2 & i \text{ even} \end{cases}$$

$$f^+(e_{i3}) = \begin{cases} 16i - 8 + 2k - 2 & i \text{ odd} \\ 16i - 12 + 2k - 2 & i \text{ even} \end{cases}$$

$$f^+(e_{i4}) = \begin{cases} 16i - 12 + 2k - 2 & i \text{ odd} \\ 16i - 14 + 2k - 2 & i \text{ even} \end{cases}$$

For $1 \le i \le n - 1$,

$$f^+(a_{i1}) = 16i + 2k - 8$$
$$f^+(a_{i2}) = 16i + 2k - 4$$
$$f^+(a_{i3}) = 16i + 2k - 2$$
$$f^+(a_{i4}) = 16i + 2k - 6$$

Therefore, $f^+(E) = \{2k, 2k + 2, 2k + 4, ..., 2k + 2q - 2\}$.

So, f is a k-even sequential harmonious labeling and hence, the graph CH_n ($n \ge 3$) is a k-even sequential harmonious graph for any k.

Illustration 3.17

(a) 1-*ESHL* of CH_4^3 is shown in Fig. 3.16(a).

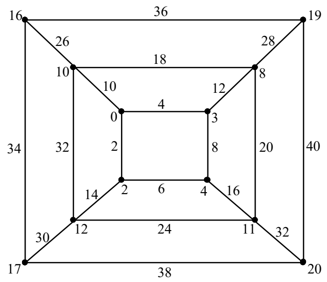


Figure 3.16(a): 1-*ESHL* of CH_4^3

(b) 1-*ESHL* of CH_4^4 is shown in Fig. 3.16(b).

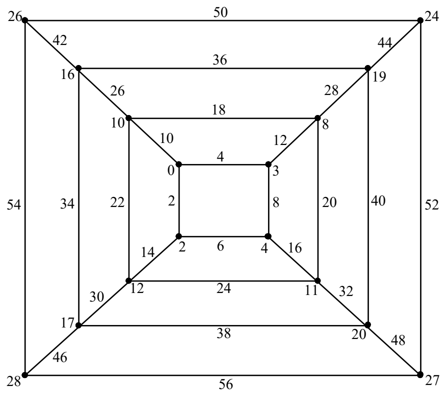


Figure 3.16(b): 1-*ESHL* of CH_4^4

Theorem 3.18

The graph Flag Fl_m ($m \ge 3$) is a k-even sequential harmonious graph for any k.

Proof

Let $\{v_i, w, 1 \le i \le m\}$ be the vertices and $\{e_i, 1 \le i \le m + 1, e_{m+1} = (v_1w)\}$ be the edges which are denoted as in Fig. 3.17.

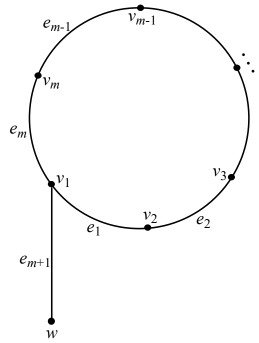


Figure 3.17: Flag Fl_m with ordinary labeling

First we label the vertices as follows:

Define $f: V \to \{k - 1, k, k + 1, ..., k + 2q - 1\}$ by

$$f(v_i) = \begin{cases} 2(i-1) + k - 1 & 1 \le i \le \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & 1 \le i \le \frac{m+2}{2}, \text{ if } m \text{ is even} \\ k + 2m - 2(i-1) & \frac{m+3}{2} \le i \le m, \text{ if } m \text{ is odd} \\ & \frac{m+4}{2} \le i \le m, \text{ if } m \text{ is even} \end{cases}$$

$$f(w) = 2m + k$$

Then the induced edge labels are:

$$f^+(e_i) = \begin{cases} 4i + 2k - 4 & 1 \le i \le \frac{m+1}{2}, \text{ if } m \text{ is odd} \\ & 1 \le i \le \frac{m}{2}, \text{ if } m \text{ is even} \\ 2k + 4m - 4i + 2 & \frac{m+3}{2} \le i \le m, \text{ if } m \text{ is odd} \\ & \frac{m+2}{2} \le i \le m, \text{ if } m \text{ is even} \end{cases}$$

$$f^+(v_1w) = 2(m + 1) + 2k - 2$$

Therefore, $f^+(E) = \{2k, 2k + 2, 2k + 4, ..., 2k + 2q - 2\}$. So, f is a k-even sequential harmonious labeling and hence, the graph flag Fl_m ($m \ge 3$) is an k-even sequential harmonious graph for any k.

Illustration 3.19

(a) 2-*ESHL* of Fl_8 is shown in Fig. 3.18(a).

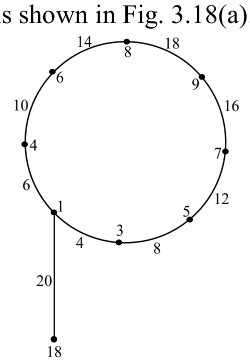


Figure 3.18(a): 2-*ESHL* of Fl_8

(b) 3-*ESHL* of Fl_5 is shown in Fig. 3.18(b).

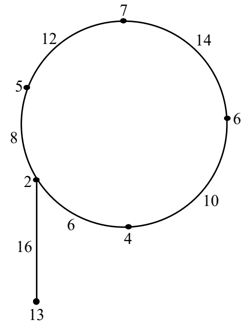


Figure 3.18(b): 3-*ESHL* of Fl_5

Theorem 3.20
The star $K_{1,n}$ ($n \geq 3$) is a *k*-even sequential harmonious graph for any *k*.

Proof
Let $\{u_i, 1 \leq i \leq n, u\}$ be the vertices and $\{e_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Fig. 3.19.

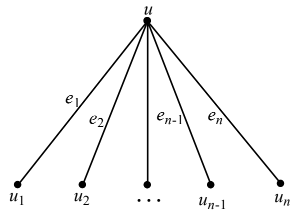


Figure 3.19: $K_{1,n}$ with ordinary labeling

First we label the vertices as follows:
Define $f: V \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by
For $1 \leq i \leq n$,

$$f(u_i) = k + 2i - 2$$
$$f(u) = k - 1$$

Then the induced edge labels are:
For $1 \leq i \leq n$,

$$f^+(e_i) = 2k + 2(i-1)$$

Therefore, $f^+(E) = \{2k, 2k+2, ..., 2k+2q-2\}$. So, f is a *k*-even sequential harmonious labeling and hence, the star graph $K_{1,n}$ ($n \geq 3$) is a *k*-even sequential harmonious graph for any *k*.

Illustration 3.21
(a) 4-*ESHL* of $K_{1,5}$ is shown in Fig. 3.20(a).

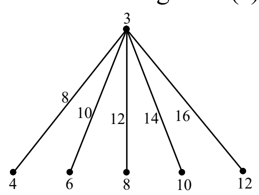


Figure 3.20(a): 4-*ESHL* of $K_{1,5}$

(b) 1-*ESHL* of $K_{1,7}$ is shown in Fig. 3.20(b).

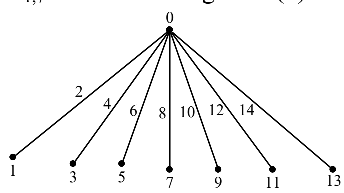


Figure 3.20(b): 1-*ESHL* of $K_{1,7}$

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