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Risk, Return and Portfolio Theory – A Contextual Note

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Abstract: ***In investment, particularly in the portfolio management, the risk and returns are two crucial measures in making investment decisions. This paper attempts to provide a brief theoretical explanation with illustrations on determining the returns and associated risk of shares, and of the portfolio of the shares. The illustrations of tables and figures can significantly contribute to the understanding of a reader in relation to portfolio management of risk and returns. The illustrative table and figures are the significance of this paper and it is believed that the reader of this paper would gain reasonable knowledge in portfolio management.***

Keywords: risk, return, shares, portfolio, standard deviation, minimum variance

JEL code: G1, G10, G11

1. Introduction

The risk and return are two basic determinants of investments in shares and bonds for adding values to an investor's wealth. Risk can be referred to as the chance of loss. When an asset has greater chances of loss, the asset can be considered as a risky asset. Return is a measure resulting from the total gain or loss experienced by the owner with respect to an asset (share/bond), over a given period of time.

Because of the complexity in understanding and handling of risk and return, specifically in portfolio management, this paper provides brief explanations on them with illustrations and related tables and figures. It is believed that this paper can contribute to make the reader to understand the relationship of returns and risk, especially in handling of shares in a portfolio to reduce the risk with diversification effects.

2. Return and Risk

Return can be referred to as the measure of total gain or loss from an investment over a given time period with respect to both changes in market value and cash distributions. Normally, the return is said to be a percentage;

$$\text{i.e., } \boldsymbol{E(r)} = \left(\frac{C_t}{P_{t-1}}\right) + \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)$$

where E(r) = Expected return
C_t = Cash flow during period t
P_t = Price at time t
P_{t-1} = Price at time t-1

When C_t = dividend, then
$\left(\frac{C_t}{P_{t-1}}\right) = \textit{dividend yield}$ and $\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) = \textit{Capital gain}$

Therefore, E(r) = Dividend yield + Capital gaining.

There are various sources of risk that affect both firms and their stakeholders.

a) Firm's specific risk, such as business risk and financial risk
b) Shareholder specific risks, such as interest rate risk, liquidity risk, and market risk.
c) Firm and shareholder risk, such as event risk, purchasing power risk, and tax risk.

It is also possible to indicate that there are different behavioural attitudes of investors towards risk. They are: risk (neutral) indifferent, risk averse, and risk seeking. The risk (neutral) indifferent investors have concern over the returns they expect, not about the risk level they face from an asset. However, the risk averse-investors always expect reasonable return to the risk they face from an asset (expect an increase in return for a given increase in risk) and the risk seeking investors have concern over how they face the risk over an asset, irrespective of the return they can gain (potentially a decrease in return for a given increase in risk). Generally, most investors are risk averse, i.e., for a given increase in risk, they accordingly acquire increase in return. Indicatively, if future returns were known with certainty, there would be no risk.

While investing, an investor often estimates pessimistic, most likely, and optimistic returns based on the risk associated with the assets. In this context, probabilities are in consideration as possible mechanism for more accurately assessing the risk involved in an asset. The probability that an event will occur may be viewed as the percentage chance of its occurrence.

The expected (or mean) value of return in a probability distribution is indicative of the most likely income, with respect to the event to occur. The expected value of return with probability can be computed as:

$$\boldsymbol{E(r) = \bar{r} = \textstyle\sum_{i=1}^{n} r_i \,.\, P(r_i)},$$

Where $\sum_{i=1}^{n} P(r_i) = 1$
$E(r) = \bar{r}$ = Expected value of return
r = Return
P = Probability operator (Probability of . . .)
n = number (types) of outcomes (returns)

Standard deviation, σ (that measures dispersion around the expected value or mean of the return), is used as the most common measure of risk of an asset. It is given by:

$$\sigma_r = \sqrt{\sum_{i=1}^{n} (r_i - \bar{r})^2 . P(r_i)}$$

If returns of an asset have a normal probability distribution, there is about 68.26% probability that a return should be within one standard deviation (1σ) of the mean return, similarly about 95.44% within 2σ of the mean and 99.76% within 3σ of the mean.

The coefficient of variation (CV) allows assessing the relative riskiness of assets within differing expected values. This is calculated as:

$$CV = \frac{\sigma_r}{\bar{r}} = \frac{\sigma_r}{E(r)}$$

Now, to understand the measures of return and risk, we can consider the following Illustrations.

Exhibit 1

Harris, a financial analyst of a cement manufacturing firm, wishes to estimate the rate of return for two similar risk investments – InA and InB. His best data are the performances of the past two years.

At the beginning of the year, InA had a market value of \$10000 and InB had a market value of \$23000. During the year, InA paid cash return of \$2000, while InB paid \$3000. The current market values of InA and InB are \$ 11000 and 25000, respectively.

Required to answer:

a) Calculate the actual rates of return on the two investments for the past period, and arithmetic mean return of both investments; and
b) Assuming that the coming year will equal the returns of previous year, and that the two investments are equally risky, which would Harris prefer, and why?

Answer:

a) The expected return

$$E(r) = \bar{r} = Dividend\ yield + Capital\ gain$$
$$= \left(\frac{C_t}{P_{t-1}}\right) + \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)$$

For InA, $\bar{r}_A = \left(\frac{2000}{10000}\right) + \left(\frac{11000-10000}{10000}\right)$

$= (0.2 + 0.1) = 30\%$

For InB, $\bar{r}_B = \left(\frac{3000}{23000}\right) + \left(\frac{25000-23000}{23000}\right)$

$= (0.1304 + 0.087) = 21.74\%$

The arithmetic mean of both returns
= (0.3 + 0.2174)/2 = 25.87%

b) Based on the rates of returns, Harris should prefer InA, because it has a higher rate of return for the same given level of (equal) risk, than InB.

Exhibit 2

Martin as an individual investor must evaluate three investments A, B and C. He earns 14% on his investments, which has a risk index measure of 10%. The investments under consideration are:

Investment	*E(r) in %*	*E(risk) index in %*
A	21	16
B	14	8
C	10	17

Required to know:

a) Which investment would martin prefer, if he was risk-indifferent? Why?
b) Which investment would martin select, if he was risk-averse? Why?
c) Which investment would martin select, if he was risk-seeker? Why?
d) Which investment would martin select, if he was a common typical investor?

Answer:

a) The risk indifferent person would select investment A, because it has a higher rate of return (21%) → nothing to worry about the level of risk.

b) The risk-averse manager would probably select investment B, because it offers the highest return for a given level of risk – note for this selection, Martin will consider and compare with current return (14%) and the level of risk (10%)

c) The risk seeking person would prefer and select investment C, since it offers the high risk (17%).

d) Most typical investors are risk-averse and would prefer and select investment B.

Exhibit 3

An investor attempts to evaluate the risk of each of two assets. He has made pessimistic, most likely and optimistic estimates of annual returns as follow.

Annual rate of return	*Asset A*	*Asset B*
Pessimistic	0.14	0.08
Most likely	0.16	0.16
Optimistic	0.18	0.24

Required:

a) Determine the range using estimates
b) Using range, state the risk of the assets
c) If the probabilities of the 3 estimates are 25%, 60% and 15%, respectively, what is the expected return of each asset?

Answer:

a) The range is the difference between the highest and minimum returns of each asset.

Status	Asset A	Asset B
Optimistic	0.18	0.24
(-) Pessimistic	0.14	0.08
Range	0.04	0.16

b) Since asset A has a range of only 0.04, while asset B has a range of 0.16 (high volatility of return), asset B is more risky

c) The average return would be calculated with the probabilistic nature of event as

$$E(r) = \bar{r} = \sum_{i=1}^{n} r_i . P(r_i)$$

$$\overline{r_A} = 0.14(0.25) + 0.16(0.60) + 0.18(0.15) = 0.158 = 15.8\%$$
$$\overline{r_B} = 0.08(0.25) + 0.16(0.60) + 0.24(0.15) = 0.152 = 15.2\%$$

Asset A is more preferred than asset B.

Exhibit 4

Use the table below and perform the following.

a) Determine the standard deviation of each asset
b) Calculate the coefficient of variance for each asset
c) Compare and contrast the risk of the two projects, using the findings from (a) and (b) above.

Asset R		Asset Q	
Outcome	Probability	Outcome	Probability
0.06	0.10	5%	20%
0.09	0.20	11%	30%
0.12	0.40	13%	30%
0.15	0.20	19%	20%
0.18	0.10		

Answer:

a) $\overline{r_R} = 0.06(0.1) + 0.09(0.2) + 0.12(0.4) + 0.15(0.2) + 0.18(0.1) = 0.12 = 12\%$

$$\sigma_r = \sqrt{\sum_{i=1}^{n} (r_i - \bar{r}) . P(r_i)}$$

$$\sigma_R = \sqrt{(0.06 - 0.12)^2.(0.1) + (0.09 - 0.12)^2.(0.2) + (0.12 - 0.12)^2.(0.4) + (0.15 - 0.12)^2.(0.2) + (0.18 - 0.12)^2.(0.1)}$$

$$\sigma_R = \sqrt{(0.00036) + (0.00018) + (0.0000) + (0.00018) + (0.00036)}$$
$$\sigma_R = \sqrt{(0.000108)} = 0.0329 \text{ or } 3.29\%$$

$$\overline{r_Q} = 0.05(0.2) + 0.11(0.3) + 0.13(0.3) + 0.19(0.2) = 0.12 = 12\%$$

$$\sigma_Q = \sqrt{(0.05 - 0.12)^2.(0.2) + (0.11 - 0.12)^2.(0.3) + (0.13 - 0.12)^2.(0.3) + (0.19 - 0.12)^2.(0.2)}$$

$$\sigma_Q = \sqrt{(0.00098) + (0.0003) + (0.0003) + (0.00098)}$$

$$\sigma_Q = \sqrt{(0.000202)} = 0.0449 \text{ or } 4.49\%$$

b) **Coefficient of Variation**

$$CV = \frac{\sigma_r}{\bar{r}} = \frac{\sigma_r}{E(r)}$$

$$CV_R = \frac{0.0329}{0.12} = 0.2742 \text{ and } CV_Q = \frac{0.0449}{0.12} = 0.3742$$

c) Asset R has less risk, measured by (a) the lower standard deviation (σ_R = 0.0329) and (b) lower coefficient of variation (CV_R = 0.2742).

Exhibit 5

Suppose a financial analyst believe that there are four likely state of the economy: Depression, Recession, Normal and Boom times. The returns of the Supertech Company are expected to follow the economy closely, while the returns of Slowpok Company are not. The return predictions are as follow.

	Supertech = r_A	Slowpok = r_B
Depression	-20%	5%
Recession	10%	20%
Normal	30%	-12%
Boom	50%	9%

Required to determine:

a) Expected return of each stock
b) Variance of each stock
c) Standard deviations of the stocks
d) Coefficient of variance for each stock
e) Covariance of the stocks
f) Correlation coefficient of the stocks

Answer:

a) Expected return

$E(r_A)$ = (-0.2 + 0.2 + 0.3 + 0.5)/4 = 0.175 = 17.5%
$E(r_B)$ = (0.05 + 0.2 – 0.12 + 0.09)/4 = 0.055 = 5.5%

b) Variance of the stocks

$$\sigma_A^2 = \frac{(-0.2-0.175)^2+(0.2-0.175)^2+(0.3-0.175)^2+(0.5-0.175)^2}{4} = 0.066875$$
$$\sigma_B^2 = \frac{(0.05-0.055)^2+(0.2-0.055)^2+(-0.12-0.055)^2+(0.09-0.055)^2}{4} = 0.013225$$

c) Standard Deviations of the stocks = σ

$$\sigma_A = \sqrt{\sigma_A^2} = \sqrt{0.066875} = 0.2586 = 25.86\% \text{ and } \sigma_B = \sqrt{\sigma_B^2} = \sqrt{0.13225} = 0.115 = 11.5\%$$

d) Coefficient of Variance = (σ/Expected return)

$$CV_A = \frac{0.2586}{0.175} = 1.478 \text{ and } CV_B = \frac{0.115}{0.055} = 2.091$$

e) Covariance of the stocks = σ_{AB}

$$\sigma_{AB} = \frac{[(-0.2-0.175)(0.05-0.055)+(0.1-0.175)(0.2-0.055)+(0.3-0.175)(-0.12-0.055)+(0.5-0.175)(0.09-0.055)]}{4}$$
$$\sigma_{AB} = \frac{-0.0195}{4} = -0.004875$$

f) Correlation Coefficient of the Stock

$$\rho_{AB} = \frac{Covariance\ of\ A\ and\ B}{\sigma_A\ \sigma_B} = \frac{\sigma_{AB}}{\sigma_A\ \sigma_B} = \frac{-0.004875}{\sqrt{(0.066875)(0.013225)}} = -0.1639$$

Alternatively, the above results can also be presented in with formulating tables with appropriate columns. In this context, some primary workings are important as considered above and given below. Also note that all possible economic statuses are considered with equal opportunities, i.e., 25% of occurrence; and hence the results are produced with probabilistic applications too.

Share A						
Economy (E)	P(E)	r_A	P(E) * r_A	r_A – E(r_A)	[r_A – E(r_A)]^2 *P(E)	Co-variance σ_{AB}
Depression	0.25	-20%	-0.05	-0.3750	0.03515625	0.00046875
Recession	0.25	10%	0.025	-0.0750	0.00140625	-0.00271875
Normal	0.25	30%	0.075	0.1250	0.00390625	-0.00546875
Boom	0.25	50%	0.125	0.3250	0.02640625	0.00284375
			0.175 = E(r_A)		0.066875 $= \sigma_A^2$	-0.004875 $= \sigma_{AB}$
Share B						
Economy	P(E)	r_B	P(E) * r_B	r_B – E(r_B)	[r_B – E(r_B)]^2 *P(E)	Co-variance σ_{AB}
Depression	0.25	5%	0.0125	-0.0050	0.00000625	0.00046875
Recession	0.25	20%	0.05	0.1450	0.00525625	-0.00271875
Normal	0.25	-12%	-0.03	-0.1750	0.00765625	-0.00546875
Boom	0.25	9%	0.0225	0.0350	0.00030625	0.00284375
			0.055 = E(r_B)		0.013225 $= \sigma_B^2$	-0.004875 $= \sigma_{AB}$

Now it is easier to answer from the above table at a glance.

a) Expected return:

$E(r_A) = 0.175 = 17.5\%$ and $E(r_B) = 0.055 = 5.5\%$

b) Variance of the stocks:

$\sigma_A^2 = 0.066875$ and $\sigma_B^2 = 0.013225$

c) Standard Deviations of the stocks = σ

$$\sigma_A = \sqrt{\sigma_A^2} = \sqrt{0.066875} = 0.2586 = 25.86\% \quad \text{and}$$

$$\sigma_B = \sqrt{\sigma_B^2} = \sqrt{0.13225} = 0.115 = 11.5\%$$

d) Covariance of the stocks $\sigma_{AB} = -0.004875$
e) Correlation Coefficient of the Stock

$$\rho_{AB} = \frac{Covariance\ of\ A\ and\ B}{\sigma_A\,\sigma_B} = \frac{\sigma_{AB}}{\sigma_A\,\sigma_B} = \frac{-0.004875}{\sqrt{(0.066875)(0.013225)}} = -0.163$$

3. Relationship between Risk and Return

The relationship between the risk and required return is normally positive with respect to a risk-averse investor, i.e., higher the risk leads to higher the expected return from an asset. Hence, the expected return can be expressed as the function of risk free return (r_f) and specific risk premium, i.e.,

Expected Return
= Risk free rate of return + Risk Premium
= r_f *+ Risk Premium*

Risk free rate of return refers to the return available on a security with certainty (no risk of default and the promised interest on the principal).Generally, the risk free return can be referred to the rate of interest that one can earn from the government securities or bank on his/her savings.

Risk free return can be regarded as:

r_f*= Real interest rate + Expected inflation premium*
(Purchasing power less premium)

Similarly, the Risk Premium is a potential reward that an investor expects to receive from market for the specific risk, when making a risky investment. Hence, as the investors are generally considered *Risk Averse*, they expect that on average, the risk, which they face in investment projects, should be compensated with reasonable rewards. Thus, over the long term, expected return and required return from securities will tend to be equal (note: this is why the terms expected returns and required returns are interchangeably used).

4. General Types of Risk

The risk is the potentiality of failure associated with an investment project. Generally, the risk can be categorized into: ***Systematic Risk*** and ***Unsystematic Risk***.

Systematic Risk refers to that portion of return variability is caused by the factors affecting the security market as a whole, such as a change in general economic outlook. The ***systematic risk*** is non-diversifiable. Some of the sources of systematic risk that can cause the returns to vary more or less from all securities together include the following.

- Interest changes
- Changes in purchasing power (inflation)
- Changes in investor expectations about overall performance of the economy, etc.

Since the diversification cannot eliminate systematic risk, this type of risk is the predominant determinant of individual security risk premium.

Unsystematic Risk refers to the portion of the variability of an individual security's return caused by factors unique to that security. The ***unsystematic risk*** can be greatly reduced or even totally eliminated by investors who hold a broad (diversified) collection (portfolio) of securities. Some of the sources of unsystematic risk are:

- Management capabilities and decisions
- Strikes
- Availability of raw material
- The unique effect of government regulations (like pollution control)
- The effects of foreign competition
- The particular level of financial and operating leverage that the firm employs
- Etc.

5. The Return and Risk for a Portfolio

In investment, it is understood that different types shares/securities, which form a portfolio can generally diversify (reduce) the risk, if they have no similar qualities of producing returns in the similar scenarios. Particularly, if the combined results of the returns from two stocks in a portfolio has similar trend within the same scenarios, their returns can possibly be highly positively (or perfectly) correlated. In this context, the combined risk of such securities as in the portfolio is very high, as seems not having risk diversification effect. Hence, the correlation coefficient (reflecting association of returns of securities) is highly important, while investing in various shares/securities in forming a portfolio.

To determine the return on a portfolio, it is important to be certain about how much of total investment is put into a particular stock/security. This is in percentage known as the weight of the investment on the particular stock/security. Hence, the return on a portfolio ($\boldsymbol{r_p}$) is the weighted average of the returns on the individual assets, i.e.:

$$r_p = \sum_{j=1}^{n} w_j . r_j$$

Where, W = weight on a particular stock, r = return on a stock, and generally,

$$\sum_{j=1}^{n} w_j = 1$$

In a portfolio investment, the assets selected must be those that best diversify (reduce) the risk, while generating an acceptable return. This is the main objective of setting portfolios in investment approaches. The process of setting meaningful portfolios with risk diversification is highly facilitated with the correlation coefficients of the returns of shares in the portfolio.

Therefore, the correlation, a measure of statistical relationship between series of numbers (returns in a portfolio), must be understood in order to create efficient portfolios that achieve a maximum return for a given amount of risk. The correlation coefficient has a range from +1 to -1. A series of numbers (returns) with positive correlation coefficient move together, while negative correlation move in opposite direction. To diversify and reduce risk, and to create an efficient portfolio, the assets that are best combined with existing assets are those that have negative (or low positive) correlation.

With respect to correlation coefficient between two series of share returns, the following can be notable:

a) Combining less than perfectly positive correlated assets
   - Overall variability of risk can be reduced.
b) Combining two perfectly positively (+) correlated assets (r = +1)
   - Cannot lower the portfolio risk in investment.
c) Combining two perfectly negatively (–) correlated assets (r = -1)
   - Can possibly result in zero risk in investments.
d) For each pair of assets, there is a combination of assets (stocks/securities) that will result in the lowest possible risk.
e) The best strategy is to diversify a firm's (or shareholder's) risk across assets in order to either: (1) maximise the return for a given level of risk or (2) minimise the risk for a given level of return – where *the portfolio, which meets these objectives is referred to as an efficient portfolio*.

Considering individually the investment weightages and risk (standard deviations) of the assets in a portfolio, the standard deviation (σ_p) of a portfolio (of two assets) can be calculated as:

$$\sigma_p = \sqrt{w_1^2 . \sigma_1^2 + w_2^2 . \sigma_2^2 + 2w_1 . w_2 . \sigma_{12}}$$

$$\sigma_{12} = Covariance$$

Where $\boldsymbol{Covariance\ \sigma_{12} = \rho_{12} . \sigma_1 . \sigma_2}$

$$\sigma_p = \sqrt{w_1^2 . \sigma_1^2 + w_2^2 . \sigma_2^2 + 2w_1 . w_2 . (\rho_{12} . \sigma_1 . \sigma_2)}$$

For three assets:

$$\sigma_p = \sqrt{\begin{matrix} w_1^2 . \sigma_1^2 + w_2^2 . \sigma_2^2 + w_3^2 . \sigma_3^2 + 2w_1 . w_2 . \sigma_{12} \\ +2w_1 . w_3 . \sigma_{13} + 2w_2 . w_3 . \sigma_{23} \end{matrix}}$$

Similarly, this can be modified for replacing the measures of covariance with respective correlation coefficients and standard deviations.

$$\sigma_p = \sqrt{\begin{array}{l} w_1^2.\sigma_1^2 + w_2^2.\sigma_2^2 + w_3^2.\sigma_3^2 + 2w_1.w_2.(\rho_{12}.\sigma_1.\sigma_2) \\ +2w_1.w_3.(\rho_{13}.\sigma_1.\sigma_3) + 2w_2.w_3.(\rho_{23}.\sigma_2.\sigma_3) \end{array}}$$

Considering the above formulas, the returns and risk of a portfolio with two (or three) assets can be determines. For easy application of these formulas, the following illustration is devised with two assets for understanding the same.

Exhibit 6

Consider ***Exhibit 5*** above: returns of Supertech R_A = 17.5% and Slowpok R_B = 5.5%. An investor having $10,000 invests $6,000 on Supertech shares and $4,000 in Slowpok shares. In addition, consider the standard deviations and correlation of the shares are same as in Exhibit 4, i.e., σ_A = 25.86%, σ_B = 11.5% and σ_{AB} = -0.004875, respectively. Required to find: (a) Expected return and (b) Variance (or SD) of the portfolio.

Answer:

(a) Expected return on portfolio $R_p = x_A.R_A + x_B.R_B$ (Where x = percentage of total investment in a particular share).

Assume Supertech shares are A and Slowpok shares are B.

The investment % in asset A = X_A = **(6000/10000) = 0.6** and The investment % in asset B = X_B = **(4000/10000) = 0.4**

As R_A = 17.5% and R_B = 5.5%, the expected return on the portfolio:

$$R_p = (0.6).(0.175) + (0.4).(0.055) = 0.127 = 12.7\%$$

(b) Variance of the portfolio with two assets:

$$\sigma_p^2 = x_A^2.\sigma_A^2 + x_B^2.\sigma_B^2 + 2x_A.x_B.\sigma_{AB}$$

Where

X_A = 0.6, **X_B = 0.4,**
σ_A = 25.86%, **σ_B = 11.5%** and
σ_{AB} = -0.004875.

$$\sigma_p^2 = (0.6^2).(0.2586^2) + (0.4^2).(0.115^2) + 2.(0.6).(0.4).(-0.004875)$$

$$\sigma_p^2 = 0.023851$$

$$\rightarrow \textit{the SD of the portfolio } \sigma_p = \sqrt{0.023851} = 0.1544 = 15.44\%$$

The Matrix Approach	Supertech (A) X_A = 0.6, σ_A = 25.86%, σ_{AB} = -0.004875	Slowpok (B) X_B = 0.4, σ_B = 11.5%, σ_{AB} = -0.004875
Supertech (A) **X_A = 0.6, σ_A = 25.86%,** **σ_{AB} = -0.004875**	(A^2) → $X_A.X_A.\ \sigma_A.\sigma_A$ = $(0.6)^2.(0.2586)^2$ **= 0.024075**	(AB) → $X_A.X_B.\ \sigma_{AB}$ = (0.6).(0.4).(-0.004875) **= -0.00117**
Slowpok (B) **X_B = 0.4, σ_B = 11.5%,** **σ_{AB} = -0.004875**	(AB) → $X_A.X_B.\ \sigma_{AB}$ = (0.6).(0.4).(-0.004875) **= -0.00117**	(B^2) → $X_B.X_B.\ \sigma_B.\sigma_B$ = $(0.4)^2.(0.115)^2$ **= 0.002116**
Total $\sigma_p^2 = 0.023851$	**= 0.022905**	**(+) 0.0000946**

Alternatively, the variance of a portfolio can be determined with correlation coefficient of the as shares in the portfolio as:

$$\sigma_p^2 = w_A^2.\sigma_A^2 + w_B^2.\sigma_B^2 + 2w_A.w_A.(\rho_{AB}.\sigma_A.\sigma_B)$$

$$\sigma_p^2 = (0.06)^2.(0.2586)^2 + (0.04)^2.(0.115)^2 + 2.(0.6).(0.4).(-0.1639).(0.2586).(0.115)$$

$$\sigma_p^2 = 0.023851 \rightarrow \textit{the SD of the portfolio } \sigma_p = \sqrt{0.023851} = 0.1544 = 15.44\%$$

Note that **σ_p = 20.12% = 0.2012**, when Correlation Coefficient $\boldsymbol{\rho_{AB}} = \mathbf{1}$. You can calculate and check it now.

6. The Diversification Effect

This is to compare the standard deviation of the portfolio $(\sigma_p = 0.1544)$ with the weighted average standard deviation **(WASD)** of the portfolio.

Where, $WASD = x_A.\sigma_A + x_B.\sigma_B$

WASD = (0.06).(0.2586) + (0.4).(0.115) = 0.2012

The difference between σ_p and **WASD** is notable $(\sigma_p < WASD)$.There is a different result for standard deviation of portfolio, while applying the method for expected return on a portfolio. It is generally argued that the result for the standard deviation of a portfolio is due to diversification (i.e., influenced by the correlation coefficient between the two stocks$(\rho_{AB} = -0.1639)$.

In our illustration, the above stocks are negatively correlated $(\rho_{AB} = -0.1639)$.This implies that one or the other above and below the average return. Note that the variance and hence the standard deviation of a portfolio's return (σ_p) must fall, as the correlation drops below 1, i.e., **As long as**$\rho_{AB} < 1$**, the** $(\sigma_p < WASD)$.This indicates that the diversification effect applies as long as there is less than perfect positive correlation $(\rho_{AB} < +1)$. In this illustration, $\sigma_p = 0.1544$ indicates lower risk as a diversification effect than **WASD = 0.2012**.

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7. Graphical Representation of Diversification Effect

The above information can be shown in a table (**Table 1**) and diagram (**Figure 1**) given. Referring to the diagram, some of the important points can be noted are:

a) Though the straight line **(AB)** and the curved line are both represented in the graph, they do not simultaneously exist in the world.

Though an investor can choose between different points on the curve if $\rho_{AB} = -0.1639$ ***, the investor cannot choose between points on the curve and points on the straight line.***

b) The point **MV** represents the minimum variance portfolio, which is the portfolio with the lowest possible standard deviation (**SD**).

Notably, minimum σ_p ***would actually be better***

c) An investor can achieve any point on the curve by selecting the appropriate mix between two securities. He cannot select any point above the curve, because he cannot: (1) increase the return on the individual security, (2) decrease the SDof the securities, or (3) decrease the correlation between the two securities.

On the other hand, as a rational investor, he cannot lower the returns on securities (below the line), increase the SD, or increase the correlation.

d) The curve is ***backward bending between B and MV***. This indicates that for a portion of the feasible set, SD actually decreases as one increases expected return.

This is due to the diversification effect. Actually, backward bending always occurs, when $\rho_{AB} < 0$***.It may or may not occur, when*** $\rho_{AB} > 0$***.The curve bends backward only for a portion of its length.***

e) No investor would want to hold a portfolio with an expected return below the minimum variance (between B and MV on the curve), because any point of portfolio between B and MV has the expected return higher for the same SD (return at point H is lower than return at point I for the same level of SD).

Though the curve B to A is called the feasible set, the investors only consider that the curve from point MV to point A is the efficient set or the efficient frontier.

Note: Table 1 consists of information required to draw lines in Figure 1 and Figure 2, where Figure 2 additionally provides how various combination of investments in these two shares with different correlation coefficient of them can be drawn with respect to their portfolio returns and standard deviations.

Portfolio Risk Diversification

Share	Type	Depression	Recession	Normal	Boom
Superdeck	A	-0.2	0.1	0.3	0.5
Slowpork	B	0.05	0.2	-0.12	0.09

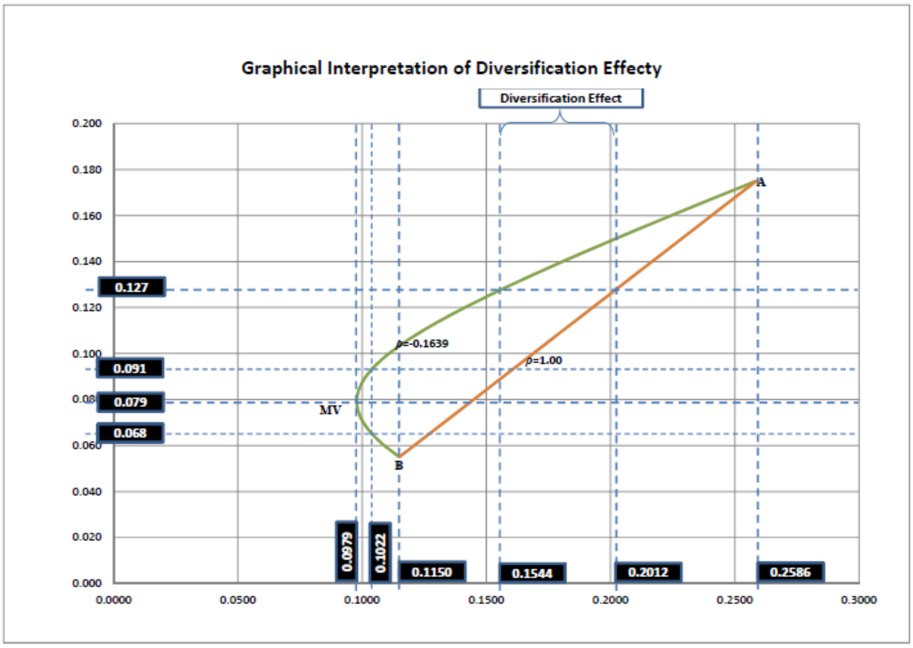
$$\overline{R_p} = W_a R_a + W_b R_b$$

$$\sigma_p = \sqrt{W_a^2 \sigma_a^2 + W_b^2 \sigma_b^2 + 2 W_a W_b \rho_{ab} \sigma_a \sigma_b}$$

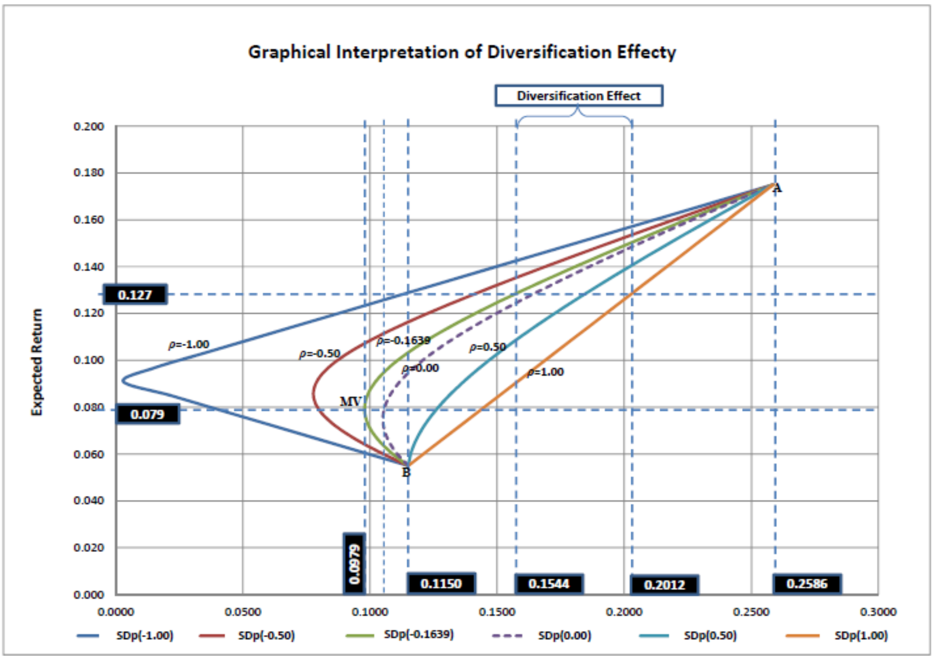
R_A = 0.175	SD_A = 0.258602	V_A = 0.066875	Covariance = -0.004875
R_B = 0.055	SD_B = 0.115	V_B = 0.013225	Correlation = -0.163924883

	Correlation		1.00	0.75	0.50	0.25	0.00	-0.1639	-0.25	-0.50	-0.75	-1.00	
W_A	W_B	Rp	SDp(1.00)	SDp(0.75)	SDp(0.50)	SDp(0.25)	SDp(0)	SDp(-0.1639)	SDp(-0.25)	SDp(-0.50)	SDp(-0.75)	SDp(-1.00)	Rp
0.00	1.00	0.055	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.055
0.05	0.95	0.061	0.1222	0.1193	0.1163	0.1132	0.1100	0.1079	0.1068	0.1034	0.0999	0.0963	0.061
0.10	0.90	0.067	0.1294	0.1241	0.1186	0.1128	0.1067	0.1025	0.1002	0.0933	0.0858	0.0776	0.067
0.15	0.85	0.073	0.1365	0.1294	0.1219	0.1138	0.1052	0.0991	0.0957	0.0853	0.0733	0.0590	0.073
0.20	0.80	**0.079**	0.1437	0.1352	0.1261	0.1163	0.1055	**0.0979**	0.0936	0.0799	0.0633	0.0403	0.079
0.25	0.75	0.085	0.1509	0.1414	0.1311	0.1200	0.1078	0.0989	0.0940	0.0777	0.0570	0.0216	0.085
0.30	0.70	0.091	0.1581	0.1479	0.1369	0.1250	0.1118	0.1022	0.0968	0.0791	0.0560	0.0029	0.091
0.35	0.65	0.097	0.1653	0.1547	0.1433	0.1310	0.1174	0.1075	0.1020	0.0838	0.0603	0.0158	0.097
0.40	0.60	0.103	0.1724	0.1618	0.1503	0.1379	0.1243	0.1145	0.1091	0.0912	0.0690	0.0344	0.103
0.45	0.55	0.109	0.1796	0.1691	0.1578	0.1457	0.1324	0.1230	0.1177	0.1009	0.0806	0.0531	0.109
0.50	0.50	0.115	0.1868	0.1766	0.1657	0.1541	0.1415	0.1326	0.1277	0.1122	0.0942	0.0718	0.115
0.55	0.45	0.121	0.1940	0.1843	0.1740	0.1631	0.1514	0.1432	0.1387	0.1247	0.1089	0.0905	0.121
0.60	0.40	0.127	0.2012	0.1921	0.1826	0.1725	0.1618	**0.1544**	0.1504	0.1380	0.1244	0.1092	0.127
0.65	0.35	0.133	0.2083	0.2001	0.1914	0.1824	0.1728	0.1663	0.1628	0.1520	0.1405	0.1278	0.133
0.70	0.30	0.139	0.2155	0.2082	0.2005	0.1926	0.1843	0.1786	0.1756	0.1665	0.1568	0.1465	0.139
0.75	0.25	0.145	0.2227	0.2164	0.2098	0.2031	0.1961	0.1914	0.1888	0.1813	0.1734	0.1652	0.145
0.80	0.20	0.151	0.2299	0.2246	0.2193	0.2138	0.2082	0.2044	0.2024	0.1964	0.1902	0.1839	0.151
0.85	0.15	0.157	0.2371	0.2330	0.2289	0.2247	0.2205	0.2177	0.2161	0.2117	0.2072	0.2026	0.157
0.90	0.10	0.163	0.2442	0.2415	0.2387	0.2359	0.2330	0.2311	0.2301	0.2272	0.2242	0.2212	0.163
0.95	0.05	0.169	0.2514	0.2500	0.2486	0.2472	0.2457	0.2448	0.2443	0.2428	0.2414	0.2399	0.169
1.00	0.00	0.175	0.2586	0.2586	0.2586	0.2586	0.2586	0.2586	0.2586	0.2586	0.2586	0.2586	0.175

Minimum Variance Portfolio $Weight_B = \frac{(\sigma_A^2 - \sigma_A \sigma_B \rho_{AB})}{(\sigma_A^2 + \sigma_B^2 - 2\sigma_A \sigma_B \rho_{AB})} = 0.80$ **Therefore Weight of stock "A" = 0.20**



MV = Minimum Variance



MV = Minimum Variance

8. Optimum Investment Portfolio

The optimum portfolio refers to the investment made in securities that gives return at a minimum standard deviation. Alternatively, the percentage of portfolio fund that is invested in each security provides returns at a minimised risk (standard deviation).Hence, the determination of the percentage the portfolio fund to be invested in each security is vital for determining the Optimum Investment Portfolio (OIP) or Minimum Variance (Risk) Portfolio (MVP or MRP).

The determination of the fund allocation to be made in two (2) securities can be determined as follow. The total variance of a portfolio can be given as:

$$\sigma_p^2 = w_1^2.\sigma_1^2 + w_2^2.\sigma_2^2 + 2w_1.w_1.\sigma_{12},$$
$$\text{where, } (w_1 + w_2 = 1)$$

$$\sigma_p^2 = w_1^2.\sigma_1^2 + w_2^2.\sigma_2^2 + 2w_1.w_2.(\rho.\sigma_1.\sigma_2)$$

$$\sigma_p^2 = w_1^2.\sigma_1^2 + (1-w_1)^2.\sigma_2^2 + 2w_1.(1-w_1).(\rho.\sigma_1.\sigma_2)$$

Differentiating the function of variance with respect to w_1 results in:

$$\frac{d(\sigma_p^2)}{dw_1} = 2w_1.\sigma_1^2 - 2(1-w_1).\sigma_2^2 + 2(\rho.\sigma_1.\sigma_2).(1-2w_1)$$

For minimum (or maximum) of variance for a given value of w_1,

$$\frac{d(\sigma_p^2)}{dw_1} = 0,$$

i.e., $$2w_1.\sigma_1^2 - 2(1-w_1).\sigma_2^2 + 2(\rho.\sigma_1.\sigma_2).(1-2w_1) = 0$$

Hence,

$$2w_1.\sigma_1^2 - 2.\sigma_2^2 + 2.w_1.\sigma_2^2 + 2(\rho.\sigma_1.\sigma_2) - 4w_1(\rho.\sigma_1.\sigma_2) = 0$$

Dividing by 2,

$$w_1.\sigma_1^2 - \sigma_2^2 + w_1.\sigma_2^2 + (\rho.\sigma_1.\sigma_2) - 2w_1(\rho.\sigma_1.\sigma_2) = 0$$

Re-arranging

$$w_1(\sigma_1^2 + \sigma_2^2 - 2.\rho.\sigma_1.\sigma_2) - \sigma_2^2 + (\rho.\sigma_1.\sigma_2) = 0$$

$$w_1(\sigma_1^2 + \sigma_2^2 - 2.\rho.\sigma_1.\sigma_2) = \sigma_2^2 - (\rho.\sigma_1.\sigma_2)$$

$$w_1 = \frac{\sigma_2^2 - (\rho.\sigma_1.\sigma_2)}{(\sigma_1^2 + \sigma_2^2 - 2.\rho.\sigma_1.\sigma_2)}$$

Note: ***It is important at this point to confirm that the second derivative of the total variance function of portfolio provides greater that zero (>0) for any value of w_1, if the function is subject to minimum.***

As $(\mathbf{w_1 + w_2 = 1}) \rightarrow (\mathbf{w_1 = 1 - w_2})$, substituting the results of w_1 above results in:

$$w_2 = \frac{\sigma_1^2 - (\rho.\sigma_1.\sigma_2)}{(\sigma_1^2 + \sigma_2^2 - 2.\rho.\sigma_1.\sigma_2)}$$

Now consider the whole set of information that we previously notified for the two shares: Supertech and Slowpok. The information of share returns:

	Supertech = r_A	Slowpok = r_B
Depression	-20%	5%
Recession	10%	20%
Normal	30%	-12%
Boom	50%	9%

This information confirms the following:

		Supertech = r_A	Slowpok = r_B
Expected return	**E(r)**	**17.5%**	**5.5%**
Variance	σ^2	**0.066875**	**0.013225**
St. Deviation	σ	**25.86%**	**11.5%**
Covariance	σ_{AB}	**-0.004875**	**-0.004875**
Corr. Coefficient	ρ_{AB}	**-0.1639**	**-0.1639**
Capital sharing	**$**	**6000**	**4000**
	w	**0.6**	**0.4**

This information reveals:

Portfolio return r_p= 12.7%

St. Dev. σ_p = 15.44%

As the weighted average standard deviation **WASD = 20.12%** (where $\rho_{AB} = 1$), the diversification effect as the reduction in risk is **4.48%** for the same level of portfolio return **12.7%**. This diversification (risk reduction) effect is the result of negative correlation coefficient between the stocks (ρ_{AB} = **-0.1639**).All these details are presented in Table 1 and the diagrams (Figure 1 and Figure 2.

Though the illustrations given for the same information on returns and relative standard deviations(with respect to various possible correlation coefficients and investment weights between the shares A and B), the minimum (variance) risk portfolio investment for the correlation coefficient of ρ_{AB} = **-0.1639** results in as:

The capital weight on shares w_A = 0.20 and w_B = 0.80; and

The portfolio return r_p = 0.079 with the risk σ_p = 0.0979.

The above can be checked with the capital weightage formulas for the minimum variance (risk).Substituting relevant values in the capital weightage formula, you can determine the investment weights in each share that gives you minimum variance (risk) return.

As shown above,

$$w_A = \frac{\sigma_B^2 - (\rho.\sigma_A.\sigma_B)}{(\sigma_A^2 + \sigma_B^2 - 2.\rho.\sigma_A.\sigma_B)}$$

Substituting

$$\sigma_A^2 = 0.066875, \sigma_B^2 = 0.013225,$$
$$\rho = -0.1639, \sigma_A = 0.2586 \text{ and } \sigma_B = 0.115,$$

it is possible to derive:

$$w_A = \frac{0.013225 - (-0.1639).(0.2586).(0.115)}{0.066875 + 0.013225 - 2.(-0.1639).(0.2586).(0.115)}$$

$$= \frac{0.0180992221}{0.0898484442}$$

$$w_A = 0.20 = 20\%$$

and therefore, $w_B = 1 - w_A = 1 - 0.20 = 0.8 = 80\%$

9. Concluding Remarks

This paper provides a brief explanation of the relationship between risk and returns of shares. Further explanations provide how the portfolio of shares can result in returns and risks at various levels of investment weights and the correlation coefficient of share returns, relatively. This paper mainly targets tertiary education (undergraduate) students and introductory market players to make them educated in the theory of portfolio management.

This paper consists of illustrations and exhibits appropriately to make the reader to understand how a portfolio of two shares can be handled to invest in them and how the minimum risk (variance/standard deviation) portfolio can be set out for a given capital/investment amount. The illustrations of tables and figures signify the importance of this paper and it is believed that the reader of this paper would gain substantial knowledge in portfolio management.

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