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On Generalized Semi-I-Open Sets

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Abstract: ***In this paper we have introduced the notion of g-semi-I-open sets in generalized ideal topological spaces and obtained its significant properties. We have also investigated the concept of the generalized closure operator c_g^* and observed useful results***

Keywords: Ideal topological space, Generalized ideal topological space, Local function, g-semi-open set.

1. Introduction

The notion of generalized topology was introduced by Csaszar [1] in 2002. Jancovic and Hamlett [4] have studied the concept of local function in Ideal topological spaces. Using the concepts of local function, Hatir and Noiri [3] have introduce the notion of semi-I-open sets. Maitra and Tripathi [6] have studied the concept of local function in generalized ideal topological spaces.

In this paper we have introduced the notion of g-semi-I-open sets and obtained its several properties. Further we have introduced c_g^* operator on a subset of generalized ideal topological spaces and obtained significant results.

2. Preliminaries

First we recall definition of generalized topological space.

Definition 2.1 [1]: Let X be a non-empty set and let τ_g be a family of subsets of X. Than τ_g is said to be **generalized topology** on X if following two conditions are satisfied viz,;
(i) $\emptyset, X \in \tau_g$,
(ii) If $G_\lambda \in \tau_g$ for $\lambda \in \wedge$ then $\cup_{\lambda\in\wedge} G_\lambda \in \tau_g$.

The pair (X, τ_g) is called **generalized topological space**. The elements of family τ_g are called **g-open sets** and their complements are called **g-closed sets**.

Example 2.1: Let us consider set $X = \{x_1, x_2, x_3\}$. Then we see that $\tau_g = \{\emptyset, X, \{x_1, x_2\}\{x_2, x_3\}\}$ is a generalized topology on X.

Proposition 2.1: Let (X, τ_g) be a generalized topological space. Then the following conditions are satisfied:
(i) ϕ *and* X are g-closed sets in X.
(ii) Arbitrary intersection of g-closed sets is a g-closed set in X.

Remark: We note that union of two g-closed sets in X may not be a g-closed set in X.

Definition 2.2 [1]: Let X be a generalized topological space and $A \subseteq X$. Then the **g-closure** of A is defined as the intersection of all g-closed sets in X containing A. The g-closure of A is denoted by $c_g(A)$.

Remark: In a generalized topological space X we note that a set A is g-closed iff $c_g(A) = A$. Further we note that $c_g(A)$ is the smallest g-closed set in X containing A.

Proposition 2.2: Let X be a generalized topological space and let $\{A_\propto\}_{\propto\in\wedge}$ be a family of subsets of X. Then
(i) $\cup_{\propto\in\wedge} c_g(A_\propto) \subseteq c_g(\cup_{\propto\in\wedge} A_\propto)$, and
(ii) $c_g(\cap_{\propto\in\wedge} A_\propto) \subseteq \cap_{\propto\in\wedge} c_g(A_\propto)$.

Definition 2.3 [1]: Let X be a generalized topological space and $A \subseteq X$. Then the **g-interior** of A is defined as the union of all g-open sets in X contained in A. The g-interior of A is denoted by $\boldsymbol{i_g(A)}$.

Remark: In a generalized topological space X we note that a set A is g-open iff $i_g(A) = A$. Further we note that $i_g(A)$ is the largest g-open set in X contained in A.

Proposition 2.3: Let X be a generalized topological space and let $\{A_\propto\}_{\propto\in\wedge}$ be a family of subsets of X. Then
(i) $\cup_{\propto\in\wedge} i_g(A_\propto) \subseteq i_g(\cup_{\propto\in\wedge} A_\propto)$, and
(ii) $i_g(\cap_{\propto\in\wedge} A_\propto) \subseteq \cap_{\propto\in\wedge} i_g(A_\propto)$.

Proposition 2.4: Let X be a generalized topological space and $A \subseteq X$. Then
(i) $i_g(X - A) = X - c_g(A)$, and
(ii) $c_g(X - A) = X - i_g(A)$.

3. Generalized Closure Operator on Subsets of X

In this section we have introduced the notion of c_g^*, the generalized closure operator on the family of all subsets of X, where X is a generalized topological space. We have obtained significant results of the c_g^* operator. We begin with the definition of Ideal on a generalized topological space.

Definition 3.1 [6]: Let (X, τ_g) be a generalized topological space and let I be a family of subsets of X. Then I is said to be an **Ideal** on X if following two conditions are satisfied viz,;
(i) Hereditary property:- If $A \in I$ and $B \subseteq A$ then $B \in I$.
(ii) Finite additivity:- If $A, B \in I$ then $A \cup B \in I$.

The triplet (X, τ_g, I) is called **generalized ideal topological space**.

Example 3.1: Let us consider set $X = \{x_1, x_2, x_3\}$ and let $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_2, x_3\}\}$ be generalized topology on X. Let $I = \{\emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$. Then I is an ideal on the generalized topological space X, and (X, τ_g, I) is a generalized ideal topological space.

The collection of Ideals on a generalized topological space X is closed under arbitrary intersection operation, viz.:

Proposition 3.1: Let X be a generalized topological space and let $\{I_\lambda\}_{\lambda \in \Lambda}$, where Λ is an index set, be any arbitrary collection of ideals on X. Then $I = \bigcap_{\lambda \in \Lambda} I_\lambda$ is an ideal on X.

Proof: Suppose $A \in I$ and $B \subseteq A$. Then $A \in I_\lambda$ for each $\lambda \in \Lambda$. Since I_λ is an ideal on X and $B \subseteq A$, we have $B \in I_\lambda$. Therefore $B \in I_\lambda$ for each $\lambda \in \Lambda$. Hence $B \in \bigcap_{\lambda \in \Lambda} I_\lambda = I$. Further suppose that $A_1, A_2 \in I$. This implies $A_1, A_2 \in I_\lambda$ for each $\lambda \in \Lambda$. As I_λ is an ideal on X, we have $A_1 \cup A_2 \in I_\lambda$ for each $\lambda \in \Lambda$. Hence $A_1 \cup A_2 \in \bigcap_{\lambda \in \Lambda} I_\lambda = I$. Thus $I = \bigcap_{\lambda \in \Lambda} I_\lambda$ is an ideal on X.

However, collection of ideals on a generalized topological space X is not closed under the operation of union. We have following Example.

Example 3.2: Let us consider generalized topological space $X = \{x_1, x_2, x_3\}$ with generalized topology $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_1, x_3\}\}$. Further let us consider ideals $I_1 = \{\emptyset, \{x_1\} \{x_2\}, \{x_1, x_2\}\}$ and $I_2 = \{\emptyset, \{x_1\} \{x_3\}, \{x_1, x_3\}\}$ on X. Then we can see that $I_1 \cup I_2 = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}$ is not an ideal on X.

Definition 3.2: Let (X, τ_g, I) be a generalized ideal topological space, and let $A \subseteq X$. Then the **local function** of A is denoted by A^* and is defined as $A^* = \{x \in X : A \cap U \notin I$ for each g−open set $U containing\ x\}$.

Definition 3.3: Let (X, τ_g, I) be a generalized ideal topological space. Then the operator $c_g^* : P(X) \longrightarrow P(X)$, where P(X) is a power set of X, defined as $c_g^*(A) = A \cup A^*$ for all $A \in P(X)$ is called **generalized closure operator**.

Example 3.3: Let $X = \{x_1, x_2, x_3\}$ be a generalized topological space with generalized topology $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_2, x_3\}\}$ and ideal $I = \{\emptyset, \{x_1\}\}$ on X. Then generalized closure operator an each subsets of X is defined as follows:

- $c_g^*(\emptyset) = \emptyset$.
- $c_g^*\{x_1\} = \{x_1\}$.
- $c_g^*\{x_2\} = X$.
- $c_g^*\{x_3\} = \{x_3\}$.
- $c_g^*\{x_1, x_2\} = X$.
- $c_g^*\{x_1, x_3\} = \{x_1, x_3\}$.
- $c_g^*\{x_2, x_3\} = X$.
- $c_g^*(X) = X$.

Proposition 3.2: Let X be a generalized ideal topological space and $A \subseteq X$. Then $c_g^*(A) \subseteq c_g(A)$.

Proof: Let X be a generalized ideal topological space and $A \subseteq X$. Let $x \in A^*$ and F be a g-closed set such that $A \subseteq F$. Then $X - F$ is a g-open set and it is disjoint from A. This implies $x \notin X - F$. i.e., $x \in F$. Hence $A^* \subseteq F$ and we have, $A^* \subseteq c_g(A)$. Since $c_g^*(A) = A \cup A^* \subseteq A \cup c_g(A)$, it follows that $c_g^*(A) \subseteq c_g(A)$.

Proposition 3.3: Let X be a generalized ideal topological space and let A, B be subsets of X. Then

(i) $c_g^*(\phi) = \phi$, $c_g^*(X) = X$.
(ii) If $A \subseteq B$ then $c_g^*(A) \subseteq c_g^*(B)$.
(iii) $c_g^*(A) \cup c_g^*(B) \subseteq c_g^*(A \cup B)$.
(iv) $c_g^*(A \cap B) \subseteq c_g^*(A) \cap c_g^*(B)$.

Proof: (i) Since $c_g^*(\phi) = \phi \cup \phi^* = \phi \cup \phi = \phi$. Thus $c_g^*(\phi) = \phi$.
Now $c_g^*(X) = X \cup X^* = X$.
(ii) Let X be a generalized ideal topological space and $A \subseteq B \subseteq X$. Then we have, $A^* \subseteq B^*$. Now $c_g^*(A) = A \cup A^* \subseteq B \cup B^* = c_g^*(B)$. Thus $c_g^*(A) \subseteq c_g^*(B)$.

(iii) Since $A \subseteq A \cup B, B \subseteq A \cup B$ from (ii) we have $c_g^*(A) \subseteq c_g^*(A \cup B)$ and $c_g^*(B) \subseteq c_g^*(A \cup B)$. This implies $c_g^*(A) \cup c_g^*(B) \subseteq c_g^*(A \cup B)$.

(iv) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ from (ii) we have $c_g^*(A \cap B) \subseteq c_g^*(A)$ and $c_g^*(A \cap B) \subseteq c_g^*(B)$. This implies $c_g^*(A \cap B) \subseteq c_g^*(A) \cap c_g^*(B)$.

Proposition 3.4: Let X be a generalized ideal topological space and let $\{A_\propto\}_{\propto \in \Lambda}$ be a family of subsets of X. Then

(i) $\bigcup_{\propto \in \Lambda} c_g^*(A_\propto) \subseteq c_g^*(\bigcup_{\propto \in \Lambda} A_\propto)$, and
(ii) $c_g^*(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} c_g^*(A_\propto)$.

Proof: (i) Let $\{A_\propto\}_{\propto \in \Lambda}$ be a collection of subsets of generalized topological space X. Then we have $A_\propto \subseteq \bigcup_{\propto \in \Lambda} A_\propto$, for each $\propto \in \Lambda$. This implies $c_g^*(A_\propto) \subseteq c_g^*(\bigcup_{\propto \in \Lambda} A_\propto)$, for each $\propto \in \Lambda$. Therefore $\bigcup_{\propto \in \Lambda} c_g^*(A_\propto) \subseteq c_g^*(\bigcup_{\propto \in \Lambda} A_\propto)$.
(ii) We have $\bigcap_{\propto \in \Lambda} A_\propto \subseteq A_\propto$, for each $\propto \in \Lambda$. This implies $c_g^*(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq c_g^*(A_\propto)$, for each $\propto \in \Lambda$. Hence we conclude that $c_g^*(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} c_g^*(A_\propto)$.

4. g-semi-I- open sets

In this section, we have studied the notion of g-semi-I- open sets in a generalized ideal topological space and observed its properties. First we recall the definition of g-semi- open set.

Definition 4.1[2]: Let (X, τ_g) be a generalized topological space and $A \subseteq X$. Then, A is said to be **g-semi-open set** if $A \subseteq c_g(i_g(A))$.

Proposition 4.1: In a generalized topological space each g-open set is g-semi-open.

Proof: X be a generalized topological space and let A be a g-open set in X. Then $A = i_g(A)$. Since A ⊆ $c_g(A)$, we have $A \subseteq c_g\ (i_g(A))$. Hence A is a g-semi-open set in X.

The converse of above result is not necessarily true. We have following example:

Example 4.1: Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\phi, X, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}$ on X. Then the family of g-closed sets is given by $\tau_g^c = \{\phi, X, \{x_4\}, \{x_3, x_4\},$ *x1,x4*. Suppose *A=x1, x2,x4*. Then *igA={x1, x2}*. Now *cg (igA= cgx1, x2=X*. Therefore *A ⊆ cg (ig(A))*. Hence A is a g-semi-open set in X, but *A* is not g-open set in *X*.

Definition 4.2: Let (X, τ_g, I) be a generalized ideal topological space and A ⊆ X. Then A is said to be **g –semi-I-open set** if $A \subseteq c_g^*(i_g\ (A))$.

Proposition 4.2: In a generalized ideal topological space (X, τ_g, I) each g-open set is g-semi-I-open.

Proof: Let (X, τ_g, I) be a generalized ideal topological space and let A be a g-open set in X. Then $A = i_g(A)$. Now $c_g^*(i_g\ (A))$=$c_g^*(A)$= $A \cup A^* \supseteq A$. Thus we have $A \subseteq c_g^*(i_g\ (A))$. Hence A is g-semi-I-open set in X.

The converse of above result is not necessarily true. We have following example:

Example 4.2: Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\phi, X, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}$ and $I = \{\phi, \{x_3\}\}$ on X. Let $A = \{x_2, x_3, x_4\}$. Then $i_g(A) = \{x_2, x_3\}$. Now $c_g^*\ (i_g(A) = c_g^*(\{x_2, x_3\}) = \{x_2, x_3\}\ \cup \{x_2, x_3\}^* = \{x_2, x_3\}\ \cup X = X$. Therefore $A \subseteq c_g^*\ (i_g(A))$. Hence A is a g-semi-I-open set in X but *A* is not g-open set in *X*.

Proposition 4.3: In a generalized ideal topological space each g-semi$-I-$open set is g-semi-open.

Proof: Let (X, τ_g, I) be a generalized ideal topological space let A be a g-semi-I-open set in X. Then we have $A \subseteq c_g^*\left(i_g\ (A)\right)$. Since $c_g^*\left(i_g\ (A)\right) \subseteq c_g\left(i_g\ (A)\right)$, it follows that $A \subseteq c_g\left(i_g\ (A)\right)$. Hence A is g-semi-open set in X.

The converse of above result is not necessarily true. We have following example:

Example 4.3: Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\emptyset, X,\ \{x_1\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$ and ideal $I = \{\emptyset,\ \{x_1\}\}$ on X. Suppose $A = \{x_1, x_4\}$. Then $c_g\left(i_g\ (A)\right) = c_g(i_g\{x_1, x_4\}) = c_g(\{x_1\}) = \{x_1, x_4\}$. Thus $A = c_g\left(i_g\ (A)\right)$. Hence A is g-semi -open set in X. Now $c_g^*\ \left(i_g(A)\right) = c_g^*\ (i_g\{x_1, x_4\})$= $c_g^*\{x_1\}$=$\{x_1\} \cup \{x_1\}^*$=$\{x_1\} \cup \emptyset = \{x_1\}$. Thus $A \nsubseteq c_g^*\ \left(i_g(A)\right)$. Therefore we see that A is g-semi-open but not g- semi- I-open set in X.

Proposition 4.4: In a generalized ideal topological space arbitrary union of g-semi-I-open sets is g-semi-I-open.

Proof: Let X be a generalized ideal topological space and let $\{A_\propto\}_{\propto \in \Lambda}$ be a family of g-semi-I-open sets in X. Then $A_\propto \subseteq c_g^*(i_g(A_\propto))$, for all $\propto \in \Lambda$. Suppose $A = \cup_{\propto \in \Lambda} A_\propto$. Now we have $c_g^*\left(i_g(A)\right) = c_g^*\left(i_g(\cup_{\propto \in \Lambda} A_\propto)\right) \supseteq c_g^*\left(\cup_{\propto \in \Lambda}(i_g(A_\propto))\right) \supseteq \cup_{\propto \in \Lambda}(c_g^*((i_g(A_\propto)) \supseteq \cup_{\propto \in \Lambda} A_\propto = A$. Thus $A = \cup_{\propto \in \Lambda} A_\propto$ is a g-semi-I-open sets in X.

However, intersection of two g-semi-I-open sets is not necessarily g-semi-I-open set. We have following example:

Example 4.4: Let us consider set $X = \{x_1, x_2, x_3\ x_4\}$ with generalized topology $\tau_g = \{\emptyset, X,\ \{x_1, x_2\},\ \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$ and ideal $I = \{\emptyset,\ \{x_3\}\}$ on X. Suppose $A = \{x_1, x_2, x_4\}$ and $B = \{x_2, x_3, x_4\}$. Then we see that the sets A and B are g-semi-I-open sets in X, but $A \cap B = \{x_2, x_4\}$ is not g-semi-I-open set in X.

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