# On Generalized Semi-I-Open Sets

## H. K. Tripathi[1], J. K. Maira[2]

[1]Govt.Women's Polytechnic College, Jabalpur, India

[2]Department of Mathematics and Computer Science, Rani Durgawati University, Jabalpur, India

**Abstract:** *In this paper we have introduced the notion of g-semi-I-open sets in generalized ideal topological spaces and obtained its significant properties. We have also investigated the concept of the generalized closure operator $c_g^*$ and observed useful results*

**Keywords**: Ideal topological space, Generalized ideal topological space, Local function, g-semi-open set.

## 1. Introduction

The notion of generalized topology was introduced by Csaszar [1] in 2002. Jancovic and Hamlett [4] have studied the concept of local function in Ideal topological spaces. Using the concepts of local function, Hatir and Noiri [3] have introduce the notion of semi-I-open sets. Maitra and Tripathi [6] have studied the concept of local function in generalized ideal topological spaces.

In this paper we have introduced the notion of g-semi-I-open sets and obtained its several properties. Further we have introduced $c_g^*$ operator on a subset of generalized ideal topological spaces and obtained significant results.

## 2. Preliminaries

First we recall definition of generalized topological space.

**Definition 2.1** [1]: Let X be a non-empty set and let $\tau_g$ be a family of subsets of X. Than $\tau_g$ is said to be **generalized topology** on X if following two conditions are satisfied viz,;
(i) $\emptyset, X \in \tau_g$,
(ii) If $G_\lambda \in \tau_g$ for $\lambda \in \wedge$ then $\cup_{\lambda \in \wedge} G_\lambda \in \tau_g$.

The pair $(X, \tau_g)$ is called **generalized topological space**. The elements of family $\tau_g$ are called **g-open sets** and their complements are called **g-closed sets**.

**Example 2.1:** Let us consider set $X = \{x_1, x_2, x_3\}$. Then we see that $\tau_g = \{\emptyset, X, \{x_1, x_2\}\{x_2, x_3\}\}$ is a generalized topology on $X$.

**Proposition 2.1**: Let $(X, \tau_g)$ be a generalized topological space. Then the following conditions are satisfied:
(i) $\phi$ *and* $X$ are g-closed sets in X.
(ii) Arbitrary intersection of g-closed sets is a g-closed set in X.

**Remark**: We note that union of two g-closed sets in X may not be a g-closed set in X.

**Definition 2.2** [1]: Let $X$ be a generalized topological space and $A \subseteq X$. Then the **g-closure** of A is defined as the intersection of all g-closed sets in $X$ containing $A$. The g-closure of $A$ is denoted by $c_g(A)$.

**Remark:** In a generalized topological space $X$ we note that a set $A$ is g-closed iff $c_g(A) = A$. Further we note that $c_g(A)$ is the smallest g-closed set in $X$ containing $A$.

**Proposition 2.2:** Let $X$ be a generalized topological space and let $\{A_\propto\}_{\propto \in \wedge}$ be a family of subsets of $X$. Then
   (i) $\cup_{\propto \in \wedge} c_g (A_\propto) \subseteq c_g (\cup_{\propto \in \wedge} A_\propto)$, and
   (ii) $c_g (\cap_{\propto \in \wedge} A_\propto) \subseteq \cap_{\propto \in \wedge} c_g (A_\propto)$.

**Definition 2.3** [1]: Let $X$ be a generalized topological space and $A \subseteq X$. Then the **g-interior** of $A$ is defined as the union of all g-open sets in X contained in A. The g-interior of $A$ is denoted by $i_g(A)$.

**Remark:** In a generalized topological space $X$ we note that a set $A$ is g-open iff $i_g(A) = A$. Further we note that $i_g(A)$ is the largest g-open set in $X$ contained in $A$.

**Proposition 2.3:** Let $X$ be a generalized topological space and let $\{A_\propto\}_{\propto \in \wedge}$ be a family of subsets of X. Then
(i) $\cup_{\propto \in \wedge} i_g (A_\propto) \subseteq i_g (\cup_{\propto \in \wedge} A_\propto)$, and
(ii) $i_g (\cap_{\propto \in \wedge} A_\propto) \subseteq \cap_{\propto \in \wedge} i_g (A_\propto)$.

**Proposition 2.4:** Let $X$ be a generalized topological space and $A \subseteq X$. Then
(i) $i_g (X - A) = X - c_g(A)$, and
(ii) $c_g (X - A) = X - i_g (A)$.

## 3. Generalized Closure Operator on Subsets of X

In this section we have introduced the notion of $c_g^*$, the generalized closure operator on the family of all subsets of $X$, where X is a generalized topological space. We have obtained significant results of the $c_g^*$ operator. We begin with the definition of Ideal on a generalized topological space.

**Definition 3.1** [6]: Let $(X, \tau_g)$ be a generalized topological space and let $I$ be a family of subsets of $X$. Then $I$ is said to be an **Ideal** on X if following two conditions are satisfied viz,;
(i) Hereditary property:- If $A \in I$ and $B \subseteq A$ then $B \in I$.
(ii) Finite additivity:- If $A, B \in I$ then $A \cup B \in I$.

The triplet $(X, \tau_g, I)$ is called **generalized ideal topological space**.

**Example 3.1**: Let us consider set $X = \{x_1, x_2, x_3\}$ and let $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_2, x_3\}\}$ be generalized topology on $X$. Let $I = \{\emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$. Then $I$ is an ideal on the generalized topological space $X$, and $(X, \tau_g, I)$ is a generalized ideal topological space.

The collection of Ideals on a generalized topological space X is closed under arbitrary intersection operation, viz.:

**Proposition 3.1**: Let $X$ be a generalized topological space and let $\{I_\lambda\}_{\lambda \in \Lambda}$, where $\Lambda$ is an index set, be any arbitrary collection of ideals on X. Then $I = \underset{\lambda \in \Lambda}{\cap} I_\lambda$ is an ideal on X.

**Proof**: Suppose $A \in I$ and $B \subseteq A$. Then $A \in I_\lambda$ for each $\lambda \in \Lambda$. Since $I_\lambda$ is an ideal on X and $B \subseteq A$, we have $B \in I_\lambda$. Therefore $B \in I_\lambda$ for each $\lambda \in \Lambda$. Hence $B \in \underset{\lambda \in \Lambda}{\cap} I_\lambda = I$. Further suppose that $A_1, A_2 \in I$. This implies $A_1, A_2 \in I_\lambda$ for each $\lambda \in \Lambda$. As $I_\lambda$ is an ideal on X, we have $A_1 \cup A_2 \in I_\lambda$ for each $\lambda \in \Lambda$. Hence $A_1 \cup A_2 \in \underset{\lambda \in \Lambda}{\cap} I_\lambda = I$. Thus $I = \underset{\lambda \in \Lambda}{\cap} I_\lambda$ is an ideal on X.

However, collection of ideals on a generalized topological space $X$ is not closed under the operation of union. We have following Example.

**Example 3.2**: Let us consider generalized topological space $X = \{x_1, x_2, x_3\}$ with generalized topology $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_1, x_3\}\}$. Further let us consider ideals $I_1 = \{\emptyset, \{x_1\} \{x_2\}, \{x_1, x_2\}\}$ and $I_2 = \{\emptyset, \{x_1\} \{x_3\}, \{x_1, x_3\}\}$ on X. Then we can see that $I_1 \cup I_2 = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}$ is not an ideal on $X$.

**Definition 3.2**: Let $(X, \tau_g, I)$ be a generalized ideal topological space, and let $A \subseteq X$. Then the **local function** of A is denoted by $A^*$ and is defined as $A^* = \{x \in X : A \cap U \notin I$ for each $g$−open set $U$ containing $x$.

**Definition 3.3**: Let $(X, \tau_g, I)$ be a generalized ideal topological space. Then the operator $c_g^* : P(X) \longrightarrow P(X)$, where P(X) is a power set of X, defined as $c_g^*(A) = A \cup A^*$ for all $A \in P(X)$ is called **generalized closure operator**.

**Example 3.3**: Let $X = \{x_1, x_2, x_3\}$ be a generalized topological space with generalized topology $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_2, x_3\}\}$ and ideal $I = \{\emptyset, \{x_1\}\}$ on X. Then generalized closure operator an each subsets of X is defined as follows:
- $c_g^*(\emptyset) = \emptyset$.
- $c_g^*\{x_1\} = \{x_1\}$.
- $c_g^*\{x_2\} = X$.
- $c_g^*\{x_3\} = \{x_3\}$.
- $c_g^*\{x_1, x_2\} = X$.
- $c_g^*\{x_1, x_3\} = \{x_1, x_3\}$.
- $c_g^*\{x_2, x_3\} = X$.
- $c_g^*(X) = X$.

**Proposition 3.2**: Let $X$ be a generalized ideal topological space and $A \subseteq X$. Then $c_g^*(A) \subseteq c_g(A)$.

**Proof:** Let $X$ be a generalized ideal topological space and $A \subseteq X$. Let $x \in A^*$ and F be a g-closed set such that $A \subseteq F$. Then $X - F$ is a g-open set and it is disjoint from $A$. This implies $x \notin X - F$. i.e., $x \in F$. Hence $A^* \subseteq F$ and we have, $A^* \subseteq c_g(A)$. Since $c_g^*(A) = A \cup A^* \subseteq A \cup c_g(A)$, it follows that $c_g^*(A) \subseteq c_g(A)$.

**Proposition 3.3**: Let $X$ be a generalized ideal topological space and let A, B be subsets of $X$. Then
(i)   $c_g^*(\phi) = \phi, c_g^*(X) = X$.
(ii)  If $A \subseteq B$ then $c_g^*(A) \subseteq c_g^*(B)$.
(iii) $c_g^*(A) \cup c_g^*(B) \subseteq c_g^*(A \cup B)$.
(iv)  $c_g^*(A \cap B) \subseteq c_g^*(A) \cap c_g^*(B)$.

**Proof**: (i) Since $c_g^*(\phi) = \phi \cup \phi^* = \phi \cup \phi = \phi$. Thus $c_g^*(\phi) = \phi$.
    Now $c_g^*(X) = X \cup X^* = X$.
(ii) Let $X$ be a generalized ideal topological space and $A \subseteq B \subseteq X$. Then we have, $A^* \subseteq B^*$. Now $c_g^*(A) = A \cup A^* \subseteq B \cup B^* = c_g^*(B)$. Thus $c_g^*(A) \subseteq c_g^*(B)$.

(iii) Since $A \subseteq A \cup B, B \subseteq A \cup B$ from (ii) we have $c_g^*(A) \subseteq c_g^*(A \cup B)$ and $c_g^*(B) \subseteq c_g^*(A \cup B)$. This implies $c_g^*(A) \cup c_g^*(B) \subseteq c_g^*(A \cup B)$.

(iv) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ from (ii) we have $c_g^*(A \cap B) \subseteq c_g^*(A)$ and $c_g^*(A \cap B) \subseteq c_g^*(B)$. This implies $c_g^*(A \cap B) \subseteq c_g^*(A) \cap c_g^*(B)$.

**Proposition 3.4**: Let $X$ be a generalized ideal topological space and let $\{A_\propto\}_{\propto \in \Lambda}$ be a family of subsets of X. Then
(i)  $\cup_{\propto \in \Lambda} c_g^* (A_\propto) \subseteq c_g^* (\cup_{\propto \in \Lambda} A_\propto)$, and
(ii) $c_g^* (\cap_{\propto \in \Lambda} A_\propto) \subseteq \cap_{\propto \in \Lambda} c_g^*(A_\propto)$.

**Proof:** (i) Let $\{A_\propto\}_{\propto \in \Lambda}$ be a collection of subsets of generalized topological space $X$. Then we have $A_\propto \subseteq \cup_{\propto \in \Lambda} A_\propto$, for each $\propto \in \Lambda$. This implies $c_g^* (A_\propto) \subseteq c_g^*(\cup_{\propto \in \Lambda} A_\propto)$, for each $\propto \in \Lambda$. Therefore $\cup_{\propto \in \Lambda} c_g^* (A_\propto) \subseteq c_g^*(\cup_{\propto \in \Lambda} A_\propto)$.
(ii) We have $\cap_{\propto \in \Lambda} A_\propto \subseteq A_\propto$, for each $\propto \in \Lambda$. This implies $c_g^*(\cap_{\propto \in \Lambda} A_\propto) \subseteq c_g^* (A_\propto)$, for each $\propto \in \Lambda$. Hence we conclude that $c_g^* (\cap_{\propto \in \Lambda} A_\propto) \subseteq \cap_{\propto \in \Lambda} c_g^*(A_\propto)$.

## 4. g-semi-I- open sets

In this section, we have studied the notion of g-semi-I- open sets in a generalized ideal topological space and observed its properties. First we recall the definition of g-semi- open set.

**Definition 4.1**[2]: Let $(X, \tau_g)$ be a generalized topological space and $A \subseteq X$. Then, A is said to be **g-semi-open set** if $A \subseteq c_g (i_g(A))$.

**Proposition 4.1**: In a generalized topological space each g-open set is g-semi-open.

**Proof:** X be a generalized topological space and let A be a g-open set in X. Then $A = i_g(A)$. Since $A \subseteq c_g(A)$, we have $A \subseteq c_g(i_g(A))$. Hence A is a g-semi-open set in X.

The converse of above result is not necessarily true. We have following example:

**Example 4.1:** Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\phi, X, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}$ on X. Then the family of g-closed sets is given by $\tau_g^c = \{\phi, X, \{x_4\}, \{x_3, x_4\}, x1, x4$. Suppose $A = x1, x2, x4$. Then $igA = \{x1, x2\}$. Now $cg(igA = cgx1, x2 = X$. Therefore $A \subseteq cg(ig(A))$. Hence A is a g-semi-open set in X, but $A$ is not g-open set in $X$.

**Definition 4.2**: Let $(X, \tau_g, I)$ be a generalized ideal topological space and $A \subseteq X$. Then A is said to be **g –semi-I-open set** if $A \subseteq c_g^*(i_g(A))$.

**Proposition 4.2:** In a generalized ideal topological space $(X, \tau_g, I)$ each g-open set is g-semi-I-open.

**Proof:** Let $(X, \tau_g, I)$ be a generalized ideal topological space and let A be a g-open set in X. Then $A = i_g(A)$. Now $c_g^*(i_g(A)) = c_g^*(A) = A \cup A^* \supseteq A$. Thus we have $A \subseteq c_g^*(i_g(A))$. Hence A is g-semi-I-open set in X.

The converse of above result is not necessarily true. We have following example:

**Example 4.2:** Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\phi, X, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}$ and $I = \{\phi, \{x_3\}\}$ on X. Let $A = \{x_2, x_3, x_4\}$. Then $i_g(A) = \{x_2, x_3\}$. Now $c_g^*(i_g(A) = c_g^*(\{x_2, x_3\}) = \{x_2, x_3\} \cup \{x_2, x_3\}^* = \{x_2, x_3\} \cup X = X$. Therefore $A \subseteq c_g^*(i_g(A))$. Hence A is a g-semi-I-open set in X but $A$ is not g-open set in $X$.

**Proposition 4.3:** In a generalized ideal topological space each g-semi$-I-$open set is g-semi-open.

**Proof:** Let $(X, \tau_g, I)$ be a generalized ideal topological space let A be a g-semi-I-open set in X. Then we have $A \subseteq c_g^*(i_g(A))$. Since $c_g^*(i_g(A)) \subseteq c_g(i_g(A))$, it follows that $A \subseteq c_g(i_g(A))$. Hence A is g-semi-open set in X.

The converse of above result is not necessarily true. We have following example:

**Example 4.3**: Let us consider set $X = \{x_1, x_2, x_3, x_4\}$ with generalized topology $\tau_g = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$ and ideal $I = \{\emptyset, \{x_1\}\}$ on X. Suppose $A = \{x_1, x_4\}$. Then $c_g(i_g(A)) = c_g(i_g\{x_1, x_4\}) = c_g(\{x_1\}) = \{x_1, x_4\}$. Thus $A = c_g(i_g(A))$. Hence A is g-semi -open set in X. Now $c_g^*(i_g(A)) = c_g^*(i_g\{x_1, x_4\}) = c_g^*\{x_1\} = \{x_1\} \cup \{x_1\}^* = \{x_1\} \cup$

$\emptyset = \{x_1\}$. Thus $A \not\subseteq c_g^*(i_g(A))$. Therefore we see that A is g-semi-open but not g- semi- I-open set in X.

**Proposition 4.4:** In a generalized ideal topological space arbitrary union of g-semi-I-open sets is g-semi-I-open.

**Proof**: Let $X$ be a generalized ideal topological space and let $\{A_\alpha\}_{\alpha \in \Lambda}$ be a family of g-semi-I-open sets in X. Then $A_\alpha \subseteq c_g^*(i_g(A_\alpha))$, for all $\alpha \in \Lambda$. Suppose $A = \bigcup_{\alpha \in \Lambda} A_\alpha$. Now we have $c_g^*(i_g(A)) = c_g^*(i_g(\bigcup_{\alpha \in \Lambda} A_\alpha)) \supseteq c_g^*(\bigcup_{\alpha \in \Lambda}(i_g(A_\alpha))) \supseteq \bigcup_{\alpha \in \Lambda}(c_g^*((i_g(A_\alpha)) \supseteq \bigcup_{\alpha \in \Lambda} A_\alpha = A$. Thus $A = \bigcup_{\alpha \in \Lambda} A_\alpha$ is a g-semi-I-open sets in X.

However, intersection of two g-semi-I-open sets is not necessarily g-semi-I-open set. We have following example:

**Example 4.4**: Let us consider set $X = \{x_1, x_2, x_3 \ x_4\}$ with generalized topology $\tau_g = \{\emptyset, X, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$ and ideal $I = \{\emptyset, \{x_3\}\}$ on $X$. Suppose $A = \{x_1, x_2, x_4\}$ and $B = \{x_2, x_3, x_4\}$. Then we see that the sets A and B are g-semi-I-open sets in X, but $A \cap B = \{x_2, x_4\}$ is not g-semi-I-open set in X.

## References

[1] Csaszar; A; Generalized topology, generalized continuity, Acta Math. Hungar., 96 (2002), 351-357.
[2] Csaszar, A., Generalized open sets in generalized topologies. Acta Mathematica Hangarica, 2005, 106, 53-66.
[3] Hatir E. (Konya) and Noiri T. (Yatsushiro), On Semi-I-Open sets and Semi-I-Continuous functions, Acta. Math. Hungar. 107(4) (2005), 345-353.
[4] Jankovic D and Hamlett T.R., New Topologies from old via Ideals, The American Mathematical Monthly. Vol.97. No. 4 (Apr. 1990), pp. 295-310.
[5] Kuralowski, K. Topologic, Vol. 1 Wars zawa (1930).
[6] Maitra J.K. and Tripathi H.K., (2014), Local Function in Generalized ideal topological spaces, VISLESANA, 11(1), 191-195.
[7] Maitra J.K., Tiwari Rajesh and Tiwari V., (2015), A Note on Local Function in Generalized Ideal topological space. International Journal of Mathematics and Computer Applications and Research (IJMCAR) vol. 5, issue 5, oct,2015, 7-12.
[8] Samuels P., A Topology formed from a given topology and ideal, J. London Math. Soc., 10 (1975) 409-416.