

# Elastic Electron Scattering from $^{35}\text{Cl}$ , $^{37}\text{Cl}$ and $^{39}\text{K}$ Nuclei

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**Abstract:** *The ground state proton momentum distributions and elastic charge form factors for some odd  $2s - 1d$  shell nuclei, such as  $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$ , and  $^{39}\text{K}$  have been studied utilizing the Coherent Density Fluctuation Model and formulated by means of the fluctuation function (weight function)  $|f(x)|^2$ . The fluctuation function has been connected to the charge density distribution of the nuclei and obtained from the theory and experiment. The feature of the long-tail behavior at high momentum region of the PMD has been determined by both the theoretical and experimental fluctuation functions. It is found that the inclusion of the quadrupole form factors  $F_{C2}(q)$  in all nuclei under study, which are described by the undeformed  $2s - 1d$  shell model, is essential for obtaining a notable accord between the theoretical and experimental form factors.*

**Keyword:** density distributions; elastic electron scattering, quadrupole form factors; momentum distributions;  $2s - 1d$  shell nuclei; root mean square radii.

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## 1. Introduction

It is well known that electron scattering is one of the most powerful tools to study the properties of nuclei. Up to now, the scattering of high-energy electrons from stable nuclei on and near the stability line has given us the most precise information about nuclear size and charge density distribution. The interest in charge densities results from the very important fact that they reflect the behavior of wave functions of protons in nuclei, where the charge density distribution is the sum of the proton wave functions squared. Charge density distributions for stable nuclei have been well studied by [1- 3]. For unstable nuclei, although studies of electron scattering have not been realized so far, nuclear physicists have already planned to explore the structure of unstable nuclei with electron-nucleus scattering. Based on the new techniques for producing high-quality Radioactive Ion Beam (RIB), new electron-nucleus colliders are now under construction at RIKEN in Japan [4] and at GSI in Germany [5]. One of the main subjects of the new colliders is the measurement of charge form factors for unstable nuclei.

A number of theoretical methods have been utilized for investigating elastic electron-nucleus scattering, such as the plan-wave Born approximation (PWBA), the eikonal approximation and the phase-shift analysis method [6-10]. The PWBA method is able to offer qualitative outcome and has been employed extensively for its ease. To take account of the Coulomb distortion effect, which is ignored in PWBA, the other two methods may possibly be utilized. In the past few years, some theoretical investigations on elastic electron scattering of exotic nuclei have been carried out [6-10]. Wang et al. [6, 7] investigated such scattering along some isotopic and isotonic chains by joining the eikonal approximation with the relativistic mean field theory. Roca-Mazza et al, [8] systematically investigated elastic electron scattering for both stable and exotic nuclei with the phase-shift analysis method. Karataglidis and Amos [9] analyzed

the elastic electron scattering form factors, longitudinal and transverse, from exotic ( $He$  and  $Li$ ) isotopes and from  $^8B$  nucleus using large space shell models. Chu et al. [10] studied the elastic electron scattering along  $O$  and  $S$  isotopic chains and demonstrated that the phase-shift analysis method can reproduce the experimental data very well for both light and heavy nuclei. Al-Rahmani and Hussein [11] have studied the CDD and elastic electron scattering form factors of some  $2s - 1d$  shell nuclei utilizing the PWBA and illustrated that the inclusion of the higher  $1f - 2p$  shell in the calculations leads to produce a good results in comparison with those of the experimental data.

In the coherent density fluctuation model (CDFM), which is characterized by the work of Antonov et al. [12, 13], the local nucleon density distribution (NDD) and the nucleon momentum distribution (NMD) are simply linked and specified by an experimentally obtainable fluctuation function (weight function)  $|f(x)|^2$ . They [12, 13] studied the NMD of ( $^4He$  and  $^{16}O$ ),  $^{12}C$  and ( $^{39}K$ ,  $^{40}Ca$  and  $^{48}Ca$ ) nuclei employing weight functions  $|f(x)|^2$  specified by the two parameter Fermi (2PF) NDD [14], the data of Reuter et al. [15] and the model independent NDD [14], respectively. It is significant to remark that all above studies, employed the framework of the CDFM, proved a high momentum tail in the NMD. Elastic electron scattering from  $^{40}Ca$  nucleus was also studied in Ref. [12], where the calculated elastic differential cross sections ( $d\sigma/d\Omega$ ) were found to be in good agreement with those of 2PF [14].

Nearly all the CDFM investigations are based on the use of weight functions originated in terms of the experimental NDD. In the present study, we utilize the CDFM with

weight functions originated in terms of theoretical CDD. We first try to derive a theoretical form for the CDD, applicable through out the upper region of the 2s-1d shell nuclei with  $Z \geq 16$ , based on the use of the single particle harmonic oscillator wave function and the occupation numbers of the states. The derived form of the CDD is employed in determining the theoretical weight function  $|f(x)|^2$  which is then used in the CDFM to study the proton momentum distribution (PMD) and elastic electron scattering charge form factors for  $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$ , and  $^{39}\text{K}$  nuclei. The effect of considering the quadrupole form factor in these nuclei is also studied. It is found that the theoretical weight function  $|f(x)|^2$  based on the derived CDD is capable to give information about the PMD and elastic charge form factors as do those of the experimental data.

## 2. Theory

The CDD of the one-body operator can be written respectively, as [16]

$$\rho_c(r) = \frac{1}{4\pi} \left\{ 2|R_{10}(r)|^2 + 6|R_{11}(r)|^2 + (2-\alpha)|R_{20}|^2 + (Z-10+\alpha)|R_{12}|^2 \right\}, \quad (2)$$

where  $Z$  is the atomic number of nuclei,  $\alpha$  is the deviation of the proton occupation numbers from the prediction of the simple shell model. After introducing the form of  $R_{nl}(r)$  with a harmonic oscillator size parameter  $b$  in Eq.(2), an analytical form for the ground state CDD of the 2s-1d shell nuclei is expressed as

$$\rho_c(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3} \left\{ 5 - \frac{3}{2}\alpha + 2\alpha \left(\frac{r}{b}\right)^2 + \left(\frac{4Z-20}{15} - \frac{2}{5}\alpha\right) \left(\frac{r}{b}\right)^4 \right\} \quad (3)$$

The normalization condition of the  $\rho_c(r)$  is given by

$$Z = 4\pi \int_0^\infty \rho_c(r) r^2 dr, \quad (4)$$

and the mean square radius (MSR) of the considered 2s-1d shell nuclei is given by

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_c(r) r^4 dr. \quad (5)$$

The central  $\rho_c(r=0)$  is obtained from eq. (3) as

$$\rho_c(0) = \frac{1}{\pi^{3/2}b^3} \left\{ 5 - \frac{3}{2}\alpha \right\}, \quad (6)$$

then  $\alpha$  is obtained from eq. (6) as

$$\alpha = \frac{2}{3} \left\{ 5 - \pi^{3/2}b^3 \rho_c(0) \right\}. \quad (7)$$

In Eq. (7), the values of the central density  $\rho_c(0)$  are taken from the experiments whereas the oscillator size parameter  $b$  is chosen in such away so as to reproduce the experimental root mean square radii of nuclei. The PMD,  $n(k)$ , for the 2s-1d shell nuclei is studied using two distinct methods. In the first method, it is determined by the shell model using

$$\rho_c(r) = \frac{1}{4\pi} \sum_{nl} \eta_{nl} 2(2l+1) |R_{nl}(r)|^2 \quad (1)$$

where  $\rho_c(r)$  is the CDD of nuclei,  $\eta_{nl}$  is the proton occupation probability of the state  $nl$  ( $\eta_{nl} = 0$  or 1 for closed shell nuclei and  $0 < \eta_{nl} < 1$  for open shell nuclei) and  $R_{nl}(r)$  is the radial part of the single-particle harmonic oscillator wave function. To derive an explicit form for the CDD of 2s-1d shell nuclei, we assume that there is a core of filled 1s and 1p shells and the proton occupation numbers in 2s and 1d orbitals are equal to  $(2-\alpha)$  and  $(Z-10+\alpha)$ , respectively, but not to 2 and  $(Z-10)$  as in the simple shell model. Using this assumption in eq. (1), we get

the single-particle harmonic oscillator wave functions in momentum representation and expressed as

$$n(k) = \frac{b^3 e^{-b^2 k^2}}{\pi^{3/2}} \left[ 5 - \frac{3}{2}\alpha + 2\alpha (bk)^2 + \left( \frac{4Z-20}{15} - \frac{2}{5}\alpha \right) (bk)^4 \right] \quad (8)$$

whereas in the second method, the  $n(k)$  is determined by the CDFM, where the mixed density is given by [12, 13]

$$\rho(\vec{r}, \vec{r}') = \int_0^\infty |f(x)|^2 \rho_x(\vec{r}, \vec{r}') dx, \quad (9)$$

since

$$\rho_x(\vec{r}, \vec{r}') = 3\rho_0(x) \frac{j_1(k_F(x)|\vec{r}-\vec{r}'|)}{k_F(x)|\vec{r}-\vec{r}'|} \times \theta\left(\bar{x} - \frac{|\vec{r}+\vec{r}'|}{2}\right), \quad (10)$$

is the density matrix for  $Z$  protons uniformly distributed in the sphere with radius  $x$  and density  $\rho_0(x) = 3Z/4\pi x^3$ . The Fermi momentum is defined as [12, 13]

$$k_F(x) = \left( \frac{3\pi^2}{2} \rho_0(x) \right)^{1/3} \equiv \frac{V}{x}; \quad V = \left( \frac{9\pi Z}{8} \right)^{1/3} \quad (11)$$

and the step function  $\theta$ , in Eq. (10), is defined by

$$\theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases} \quad (12)$$

According to the density matrix definition of Eq.(9), one-particle density  $\rho(r)$  is given by its diagonal element as [12, 13]

$$\rho_c(r) = \rho_c(r, r')|_{r=r'} = \int_0^\infty |f(x)|^2 \rho_x(r) dx. \quad (13)$$

In Eq. (13),  $\rho_x(r)$  and  $|f(x)|^2$  have the following forms [12, 13]

$$\rho_x(r) = \rho_0(x)\theta(x - |\vec{r}|) \quad (14)$$

$$|f(x)|^2 = \frac{-1}{\rho_0(x)} \frac{d\rho_c(r)}{dr} \Big|_{r=x}. \quad (15)$$

The weight function  $|f(x)|^2$  of Eq. (15), determined in terms of the ground state  $\rho_c(r)$ , satisfies the following normalization condition [12, 13]

$$\int_0^\infty |f(x)|^2 dx = 1, \quad (16)$$

and holds only for monotonically decreasing  $\rho_c(r)$ , i.e.

$$\frac{d\rho_c(r)}{dr} < 0.$$

On the basis of Eq. (13), the PMD,  $n(k)$ , is given by [12, 13]

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k) dx, \quad (17)$$

where

$$n_x(k) = \frac{4}{3} \pi x^3 \theta(k_F(x) - |\vec{k}|), \quad (18)$$

is the Fermi-momentum distribution of the system with density  $\rho_0(x)$ . By means of Eqs. (15), (17) and (18), an explicit form for the PMD is expressed in terms of  $\rho_c(r)$  as

$$n^{CDFM}(k; [\rho_c]) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{Z} \times \left[ 6 \int_0^{V/k} \rho_c(x) x^5 dx - \left(\frac{V}{k}\right)^6 \rho_c\left(\frac{V}{k}\right) \right], \quad (19)$$

with normalization condition

$$Z = \int n^{CDFM}(k) \frac{d^3k}{(2\pi)^3}. \quad (20)$$

The elastic monopole charge form factors  $F_{C0}(q)$  of the target nucleus are also expressed in the CDFM as [12, 13]

$$F_{C0}(q) = \frac{1}{Z} \int_0^\infty |f(x)|^2 F(q, x) dx, \quad (21)$$

where the form factor of uniform charge density distribution is given by

$$F(q, x) = \frac{3Z}{(qx)^2} \left[ \frac{\sin(qx)}{(qx)} - \cos(qx) \right]. \quad (22)$$

Inclusion of the correction due to the finite nucleon size  $f_{fs}(q)$  and the center of mass correction  $f_{cm}(q)$  in the calculations requires multiplying the form factor of Eq. (21) by these corrections. Here,  $f_{fs}(q)$  is considered as free nucleon form factor which is assumed to be the same for protons and neutrons [17]

$$f_{fs}(q) = \exp\left[-\frac{0.43q^2}{4}\right]. \quad (23)$$

The correction  $f_{cm}(q)$  removes the spurious state arising from the motion of the center of mass when shell model wave function is used and is given by [17]

$$f_{cm}(q) = \exp\left[\frac{q^2 b^2}{4A}\right]. \quad (24)$$

Multiplying the right hand side of Eq. (21) by these corrections yields:

$$F_{C0}(q) = \frac{1}{Z} \int_0^\infty |f(x)|^2 F(q, x) dx f_{fs}(q) f_{cm}(q). \quad (25)$$

It is important to point out that all physical quantities studied above in the framework of the CDFM such as  $n(k)$  and  $F_{C0}(q)$ , are expressed in terms of the weight

function  $|f(x)|^2$ . In the previous work [12, 13], the weight function was obtained from the NDD of the 2PF, extracted by analyzing elastic electron-nuclei scattering experiments. In the present work, the theoretical weight function  $|f(x)|^2$  is expressed, by introducing the derived CDD of Eq. (3) into Eq. (15), as

$$|f(x)|^2 = \frac{8\pi x^4}{3Zb^2} \rho_c(x) - \frac{16x^4 e^{-x^2/b^2}}{3Zb^3 \pi^{1/2}} \left\{ \alpha + \left[ \frac{4Z}{15} - \frac{4}{3} - \frac{2}{5} \alpha \right] \left( \frac{x}{b} \right)^2 \right\}. \quad (26)$$

Here, the quadrupole charge from factors are described by the undeformed sd-shell model, where the ground state charge density distributions of these deformed nuclei are described by [18]

$$\rho_{ch}(\vec{r}) = \rho_{0ch}(r) + \rho_{2ch}(r) Y_{20}(\cos \theta) + \dots \quad (27)$$

The normalization of the spherically symmetric part  $\rho_{0ch}(r)$  gives  $4\pi \int \rho_{0ch}(r) r^2 dr = Ze$ . here, the

$\rho_{0ch}(r)$  is calculated by Eq. (3), i.e.,  $\rho_{0ch}(r) \equiv \rho_c(r)$ . The quadrupole part of the charge density  $\rho_{2ch}(r)$  is related to the electric quadrupole moment  $Q$  by [18]

$$Q = 2 \left( \frac{4\pi}{5} \right)^{1/2} \int \rho_{2ch}(r) r^4 dr \quad (28)$$

The quadrupole charge form factor, which contains the non-spherical part of the charge density distribution, is then given by [19]

$$F_{C2}(q) = \frac{\langle r^2 \rangle}{Q} \left( \frac{4}{5P_J} \right)^{1/2} \int j_2(qr) \rho_{2ch}(r) r^2 dr, \quad (29)$$

where  $j_2(qr)$  is the spherical Bessel function of order two,  $P_J$  is a quadrupole projection factor given by  $P_J = J(2J-1)/(J+1)(2J+3)$ , and  $J$  is the ground state angular momentum ( $J = 3/2$  for all nuclei under study). According to the undeformed *sd*-shell model [20], where the quadrupole moment arises from protons moving in the *sd*-shell of undeformed potential, the radial dependence of the quadrupole charge density

distributions  $\rho_{2ch}(r)$  is assumed to be the same as that of the  $sd$  – shell part  $\rho_{0ch}(r)$ . In this study, the quadrupole moment  $Q$  is considered as a free parameter so as to fit the theoretical form factors with those of experimental data.

### 3. Results and Discussion

In this study, the CDFM is used to study the proton momentum distributions  $n(k)$  and elastic form factors for some odd  $2s-1d$  shell nuclei. The distribution  $n^{CDFM}(k)$  of Eq. (19) is calculated by means of the CDD obtained firstly from theoretical consideration as in Eq. (3) and secondly from experiments, such as 3PF [14]. The size parameters  $b$  are chosen in such a way so as to imitate the experimental root mean square (rms) charge radii of nuclei. The values of  $\alpha$  are determined by Eq. (7). The values of the parameters  $b$ ,  $\alpha$  and  $Q$  together with the experimental values of the parameters  $c$ ,  $z$  and  $w$  of 3PF distribution as well as the corresponding value of the central densities  $\rho_{exp}^{3PF}(0)$  and the root mean square charge radii  $\langle r^2 \rangle_{exp}^{1/2}$  for  $^{35}Cl$ ,  $^{37}Cl$  and  $^{39}K$  nuclei are presented in table 1. The occupation numbers of protons in the orbitals  $2s$  and  $1d$ , which are equal to  $(2-\alpha)$  and  $(Z-10+\alpha)$  respectively, of the considered odd  $2s-1d$  shell nuclei are compared in table 2 with those obtained by the shell model calculations, utilizing the shell model OXBASH code [21], using the USDB [22] realistic effective interactions.

In Fig. 1, we explore the dependence of the CDD (in  $fm^{-3}$ ) on  $r$  (in  $fm$ ) for  $^{35}Cl$  [Fig. 1(a)],  $^{37}Cl$  [Fig. 1(b)] and  $^{39}K$  [Fig. 1(c)] nuclei. The dashed and solid curves correspond to the calculated CDD, using Eq. (3) with  $\alpha = 0$  and  $\alpha \neq 0$ , respectively whereas the dotted symbols correspond to the experimental data [14]. It is noticeable that the dashed curves are in poor agreement with the experimental data, particularly for small  $r$ . Inclusion of the parameter  $\alpha$  into our calculations leads to a good agreement with the experimental data as demonstrated by the solid curves.

In Fig. 2, we display the dependence of the  $n(k)$  (in  $fm^3$ ) on  $k$  (in  $fm^{-1}$ ) for  $^{35}Cl$  [Fig. 2(a)],  $^{37}Cl$  [Fig. 2(b)] and  $^{39}K$  [Fig. 2(c)] nuclei. The long-dashed curves correspond to the PMD's of Eq. (8) evaluated by the shell model using the single particle harmonic oscillator wave functions in the momentum space. The dotted symbols and solid curves correspond to the PMD's obtained by the CDFM of Eq. (19) employing the experimental and theoretical CDD, respectively. It is evident that the behavior of the long-dashed distributions estimated by the shell model is in contrast with the distributions imitated by the CDFM. The significant property of the long-dashed distributions is the steep slope mode, when  $k$  increases. This behavior is in disagreement with the studies [12, 13, 23- 25] and it is

attributed to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into account the important effects of the short range dynamical correlation functions. Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the PMD [23, 24]. It is noted that the general structure of the dotted and solid distributions at the region of high momentum components is almost the same for  $^{35}Cl$ ,  $^{37}Cl$  and  $^{39}K$  nuclei, where these distributions have the property of long-tail manner at momentum region  $k \geq 2 fm^{-1}$ . The property of long-tail manner obtained by the CDFM, which is in agreement with the studies [12, 13, 23- 25], is connected to the presence of high densities  $\rho_x(r)$  in the decomposition of Eq. (13), though their fluctuation functions  $|f(x)|^2$  are small.

The elastic electron scattering charge form factors from the considered nuclei are calculated in the framework of the CDFM through introducing the theoretical weight functions  $|f_c(x)|^2$  of Eq. (26) into Eq. (25). In Fig. 3, we present the dependence of the form factors  $F(q)$  on the momentum transfer  $q$  (in  $fm^{-1}$ ) for  $^{35}Cl$  [Fig. 3(a)],  $^{37}Cl$  [Fig. 3(b)] and  $^{39}K$  [Fig. 3(c)] nuclei. Here, the effect of the quadrupole form factor  $F_{C2}(q)$  is considered by the undeformed  $2s-1d$  shell model as given in Eq. (29). The dashed and long-dashed curves correspond to the contributions of the monopole form factors  $|F_{C0}(q)|^2$  and quadrupole form factors  $|F_{C2}(q)|^2$ , respectively, whereas the solid curves correspond to the total contribution, which is obtained as the sum of the monopole and quadrupole form factors. In Figs. 3(a) and 3(b), the experimental data [26] (open circles symbols) are very well explained by the dashed curves up to the momentum transfer  $q \approx 1.2 fm^{-1}$  while for  $q > 1.2 fm^{-1}$  these curves under predict these data. In Fig. 3(c), the dashed curve agrees well the experimental data [27] at the regions of  $q \approx 0-1.2, 1.5-2$  and  $2.7-3.1 fm^{-1}$  and disagrees them at the other regions of considered momentum transfer. Inclusion the effect of quadrupole form factors in the calculations leads to improve the calculated form factors, especially at the regions where the experimental data are not explained by the dashed curve. The locations of the diffraction minima in Figs. 3(a)-3(c) are approximately located in the correct places when the contribution of the quadrupole form factors is considered in the calculations. This figure gives the conclusion that the contribution of the quadrupole form factors gives a strong modification to the monopole form factors and brings the calculated values very close to the experimental data.

#### 4. Conclusions

The PMD and elastic charge form factors  $F(q)$ , which are evaluated by the *CDFM*, are formulated via the weight function  $|f(x)|^2$ . The weight function, which is related with the local density  $\rho_c(r)$ , is obtained from experiment and from theory. The property of the long-tail mode of the PMD, which is in agreement with the other studies [12, 13, 24, 25, 28], is achieved by both theoretical and experimental weight functions and is connected to the presence of high

densities  $\rho_x(r)$  in the decomposition of Eq. (13), though their weight functions are small. It is noticed that the inclusion of the quadrupole form factors in  $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$  and  $^{39}\text{K}$  nuclei, which are characterized by the undeformed  $2s-1d$  shell model, is necessary for getting a notable accord between the theoretical and experimental form factors. It is observed that the theoretical CDD of Eq. (3) utilized in obtaining the theoretical weight function of Eq. (26) is able to provide information about the PMD and elastic charge form factors as do those of the experimental data.

**Table 1:** The Values of various parameters employed in the present calculations together with the value of  $\rho_{\text{exp}}(0)$  and  $\langle r^2 \rangle_{\text{exp}}^{1/2}$ .

| Nucleus          | Model      | $c$<br>[14] | $z$<br>[14] | $w$<br>[14] | $\rho_{\text{exp}}(0)$ [14]<br>( $\text{fm}^{-3}$ ) | $\langle r^2 \rangle_{\text{exp}}^{1/2}$ [14] ( $\text{fm}$ ) | $b$<br>( $\text{fm}$ ) | $\alpha$  | $Q$ |
|------------------|------------|-------------|-------------|-------------|---|---|------------------------|-----------|-----|
| $^{35}\text{Cl}$ | <i>3PF</i> | 3.476       | 0.599       | -0.10       | 0.08288272  | 3.388   | 1.986                  | 0.9217771 | 190 |
| $^{37}\text{Cl}$ | <i>3PF</i> | 3.554       | 0.588       | -0.13       | 0.08134359  | 3.384   | 1.983                  | 0.9772688 | 175 |
| $^{39}\text{K}$  | <i>3PF</i> | 3.743       | 0.585       | -0.20       | 0.08568579  | 3.408   | 1.976                  | 0.8776901 | 320 |

**Table 2:** Occupation numbers of nucleons in 1d and 2s orbitals of some odd 2s-1d shell nuclei.

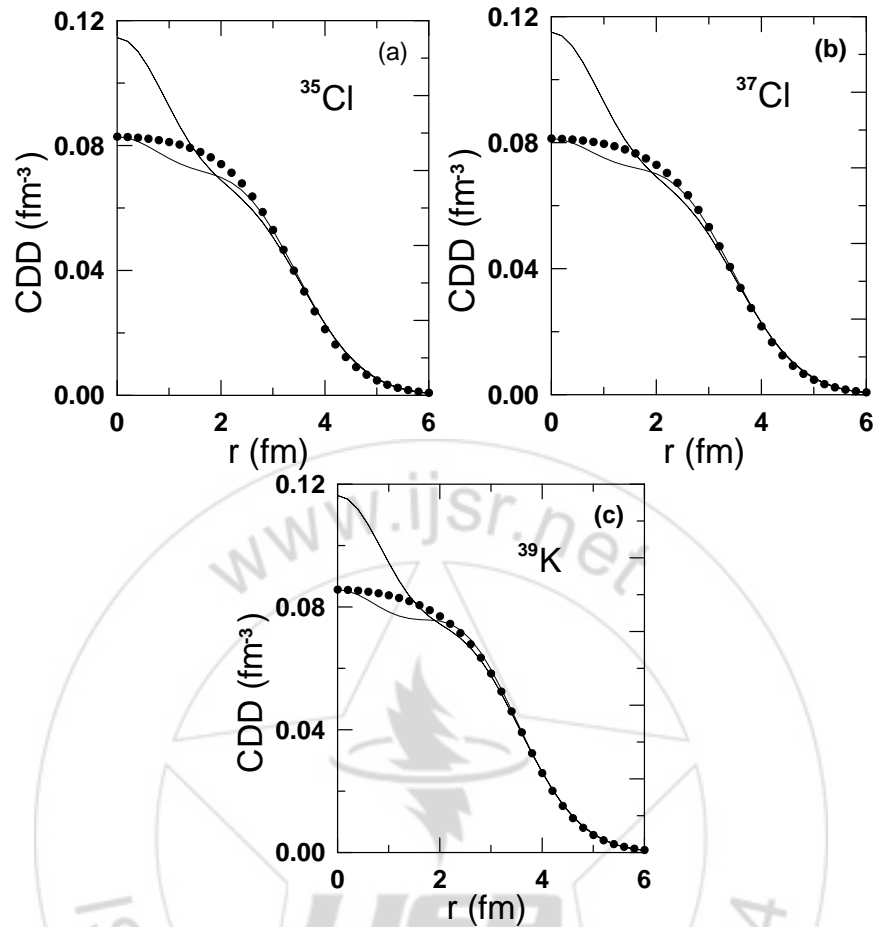
| Nuclei           | Occupation numbers of protons in 1d             |                        | Occupation numbers of protons in 2s        |                       |
|------------------|---|------------------------|--|-----------------------|
|                  | Present results obtained by $(Z - 10 + \alpha)$ | Realistic results [22] | Present results obtained by $(2 - \alpha)$ | Realistic result [22] |
| $^{35}\text{Cl}$ | 7.921777  | 7.980                  | 1.078223                                   | 1.020                 |
| $^{37}\text{Cl}$ | 7.977268  | 7.099                  | 1.022732                                   | 1.902                 |
| $^{39}\text{K}$  | 9.877690  | 9.000                  | 1.122310                                   | 2.000                 |

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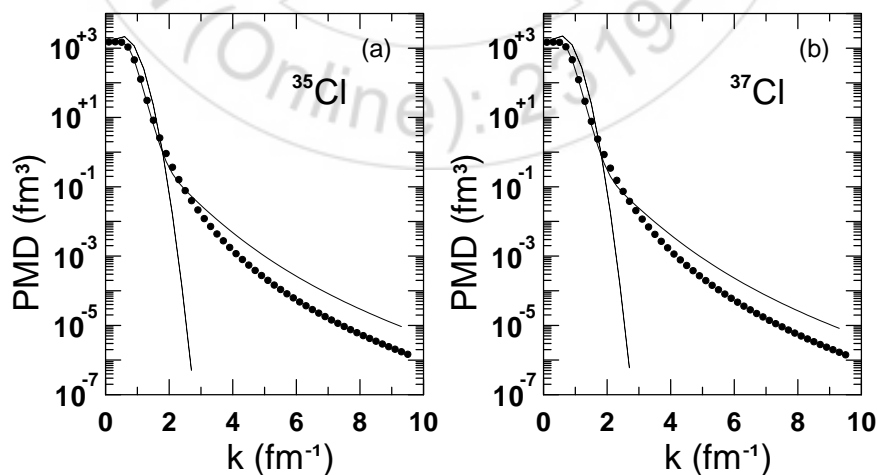
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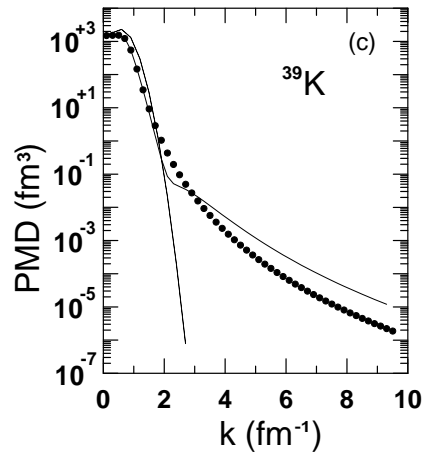
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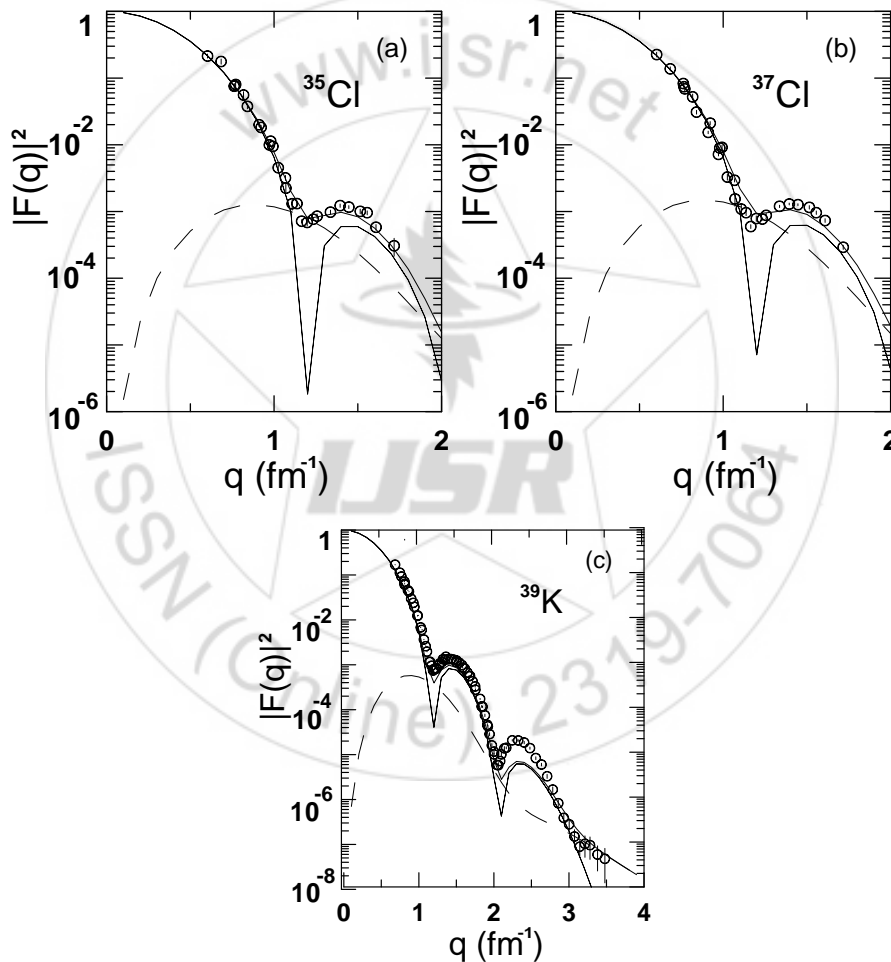


**Figure 1:** Dependence of the CDD on  $r$  for (a)  $^{35}\text{Cl}$ , (b)  $^{37}\text{Cl}$  and (c)  $^{39}\text{K}$  nuclei. The dashed and solid curves are the calculated CDD of Eq. (3) when  $\alpha = 0$  and  $\alpha \neq 0$ , respectively. The dotted symbols are the experimental data taken from ref. [14].





**Figure 2:** Dependence of PMD on  $k$  for (a)  $^{35}\text{Cl}$ , (b)  $^{37}\text{Cl}$  and (c)  $^{39}\text{K}$  nuclei. The solid curves and dotted symbols are the calculated PMD obtained in terms of the CDFM of Eq. (19) using the theoretical CDD of Eq. (3) and the experimental data of ref. [14], respectively. The long-dashed curves are the calculated PMD of Eq. (8) obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum representation.



**Figure 3:** Dependence of the charge form factors on  $q$  for (a)  $^{35}\text{Cl}$ , (b)  $^{37}\text{Cl}$  and (c)  $^{39}\text{K}$  nuclei. The dashed and long-dashed curves represent the contributions of the monopole form factors  $|F_{C_0}(q)|^2$  and the quadrupole form factors  $|F_{C_2}(q)|^2$ , respectively. The solid curves represent the total form factors for both contributions. The experimental data (open circles) for  $^{35}\text{Cl}$ ,  $^{37}\text{Cl}$  are taken from ref. [26] and for  $^{39}\text{K}$  are taken from ref. [27].