A Study on Multi Server Fuzzy Queuing Model in Triangular and Trapezoidal Fuzzy Numbers Using α Cuts

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Abstract: ***In this paper we study fuzzy multi server queuing model in triangular and trapezoidal fuzzy numbers using α cut method. The arrival and the service rate are fuzzy natures and also analyzed the performance measures of this model both in triangular and trapezoidal fuzzy numbers. In the numerical example we are deduce that the efficiency of this model.***

Keywords: Membership function, Triangular and Trapezoidal fuzzy numbers, α cuts, DSW algorithm, Interval analysis

1. Introduction

In general waiting line(queue) is one of the un avoid situation in our daily life (bank, reservation counter, railway station, hospitals etc.). queuing theory is studying about such waiting line through performance measures. Anger krarupErlang[8] introduced queuing theory in 1909. The queuing model comprises one or more queue and one or more service facilities under a set of rules. The parameters arrival rate λ and service rate μ follow poison and exponential distribution respectively.

Fuzzy queuing model was first introduced by R.J.Lie and E.S.Lee in 1989 further developed this model by many authors J.J.Buckely[4] in 1990, R.S.Negi and E.S.Lee[5] in 1992, S.P.Chen in 2005 and R.Srinivasan[11] in 2014. Here the parameters fuzzy arrival rate and fuzzy service rate are best described by linguistic terms very high, high, low, very low and moderate.

Our aim of this paper discussed about multi server queuing model and first come first served discipline using triangular and trapezoidal fuzzy numbers under α cut representation through DSW algorithm here fuzzy set decompose into distinct leaven points through α cut method. The approximate method DSW (Dong, Shah, Wong) algorithm is used to define a membership function of the performance measures in multi-server fuzzy queuing model.

2. Fuzzy Set Theory

2.1 Fuzzy set

A fuzzy set A in X is characterized by its membership function
$A: X \to [0,1]$. Here X is a non – empty set.

2.2 Triangular fuzzy number

A triangular fuzzy number $\tilde{A}$ is defined by (a_1, a_2, a_3) where $a_i \in R$ and $a_1 \le a_2 \le a_3$. Its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for \ \ x < a_1 \\ \dfrac{a_3-x}{a_3-a_2} & for \ a_1 \le x \le a_2 \\ 0 & otherwise \end{cases}$$

2.3 Trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}$ is defined by (a_1, a_2, a_3, a_4) where $a_i \in R$ and $a_1 \le a_2 \le a_3 \le a_4$. Its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for \ a_1 \le x \le a_2 \\ 1 & for \ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for \ a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

3. Solution Procedure

Let us consider a classical multi server queuing model infinite calling source and first come first served discipline. The inter arrival time A and service time S are described by the following fuzzy sets

$$A = \{(a, \tilde{\mu}_A(a)) / a \in X\}$$
$$S = \{(s, \tilde{\mu}_s(s)) / s \in X\}$$

Here X is the classical set of the inter arrival time and Y is the classical set of the service time.

$\tilde{\mu}_A(a)$ is the membership function of the inter arrival time.

$\tilde{\mu}_s(s)$ is the membership function of the service time.

The α cuts of inter arrival time and service time are represented as

$$A(\alpha) = \{a \in X / \tilde{\mu}_A(a) \ge \alpha\}$$
$$S(\alpha) = \{s \in Y / \tilde{\mu}_s(s) \ge \alpha\}$$

Using these α cuts we have to define the membership function $P(A,S)$ as follows

Triangular membership function

$$\mu_{P(A,S)}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for\ x < a_1 \\ \dfrac{a_3-x}{a_3-a_2} & for\ a_1 \le x \le a_2 \\ 0 & otherwise \end{cases}$$

Trapezoidal membership function

$$\mu_{P(A,S)}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for\ a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

Multi server fuzzy queue infinite calling source and first come first served discipline. In technically $(FM/FM/C):(FCFS/\infty/\infty)$

Multi server queue has two or more service facility in parallel providing identical service. All the customers in the waiting line can be served by more than one station. The arrival time and the service time follow poison and exponential distribution.

The performance measures of the Multi – server queuing system are,

1. Expected number of customers in the system

$$L_S = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$$

2. Expected number of customers waiting in the queue

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0$$

3. Average time a customer spends in the system

$$W_S = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0 + \frac{1}{\mu}$$

4. Average waiting time of a customer in the queue

$$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^C}{(C-1)!(C\mu-\lambda)^2}P_0$$

Here $P_0 = \dfrac{1}{\sum_{n=0}^{c-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!}\cdot\frac{c\mu}{c\mu-\lambda}}$

4. Interval Analysis Arithmetic

Let I_1 and I_2 be two interval numbers defined by ordered of real numbers with lower and upper bounds.

$$I_1 = [a,b], a \le b \qquad I_2 = [c,d],\ c \le d$$

Define a general arithmetic property with the symbol $*$ $=[+,-,\times,\div]$ symbolically the operation

$$I_1 * I_2 = [a,b]*[c,d]$$

Represents another interval. The interval calculation depends on the magnitudes and signs of the elements a,b,c,d

$$[a,b]+[c,d]=[a+c,b+d]$$

$$[a,b]-[c,d]=[a-d,b-c]$$

$$[a,b].[c,d]=[\min(ac,ad,bc,bd),\max(ac,ad,bc,bd)]$$

$$[a,b]\div[c,d]=[a,b].\left[\frac{1}{d},\frac{1}{c}\right] provided\ that\ \ 0 \notin [c,d]$$

$$\alpha[a,b] = \begin{cases}[\alpha a,\alpha b] & for\ \alpha > 0 \\ [\alpha b,\alpha a] & for\ \alpha < 0\end{cases}$$

5. DSW Algorithm

The DSW algorithm consists of the following steps:

1) Select a α cut value where $0 \le \alpha \le 1$.
2) Find the intervals in the input membership functions that correspond to this α.
3) Using standard binary interval operations compute the interval for the output membership function for the selected α cut level.
4) Repeat steps 1 – 3 for different values of α to complete α cut representation of the solution.

6. Numerical Example

6.1 Triangular fuzzy number

Consider a FM/FM/C queue, where the both the arrival rate and service rate are triangular fuzzy numbers represented by $\tilde{\lambda}$ = [16,17,18] and $\tilde{\mu}$ = [5,6,7]. The number of server C=4. The interval of confidence at possibility level α as [16+α , 18− α] and [5+ α , 7−α]

Where x = [16+α , 18− α] and$y$ = [5+ α , 7−α]

Table 1: The α cuts of L_q, L_s, W_q, W_s at α values

α	L_q	L_s	W_q	W_s
0	[0.3356, 7.0897]	[2.6213, 10.6897]	[0.0209, 0.3938]	[0.1638, 0.5938]
0.1	[0.3726, 5.3176]	[2.7059, 8.8274]	[0.0231, 0.2970]	[0.1680, 0.4931]
0.2	[0.4144, 4.1556]	[2.7968, 7.5787]	[0.0255, 0.2334]	[0.1726, 0.4257]
0.3	[0.4620, 3.3411]	[2.8948, 6.6807]	[0.0283, 0.1887]	[0.1775, 0.3774]
0.4	[0.5162, 2.7427]	[3.0010, 6.0020]	[0.0314, 0.1558]	[0.1829, 0.3410]
0.5	[0.5782, 2.2877]	[3.1167, 5.4695]	[0.0350, 0.1307]	[0.1888, 0.3125]
0.6	[0.6496, 1.9324]	[3.2433, 5.0395]	[0.0391, 0.1110]	[0.1953, 0.2896]
0.7	[0.7321, 1.6490]	[3.3829, 4.6841]	[0.0438, 0.0953]	[0.2025, 0.2707]
0.8	[0.8280, 1.4191]	[3.5377, 4.3846]	[0.0492, 0.0825]	[0.2105, 0.2549]
0.9	[0.9402, 1.2300]	[3.7107, 4.1283]	[0.0556, 0.0719]	[0.2195, 0.2414]
1	[1.0726, 1.0726]	[3.9059, 3.9059]	[0.0630, 0.0630]	[0.2297, 0.2297]

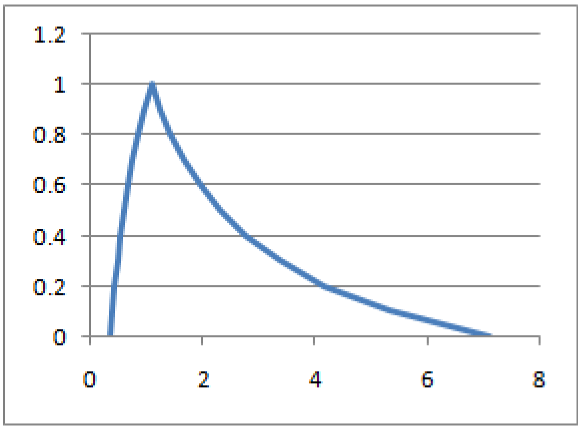


Figure 1: L_q

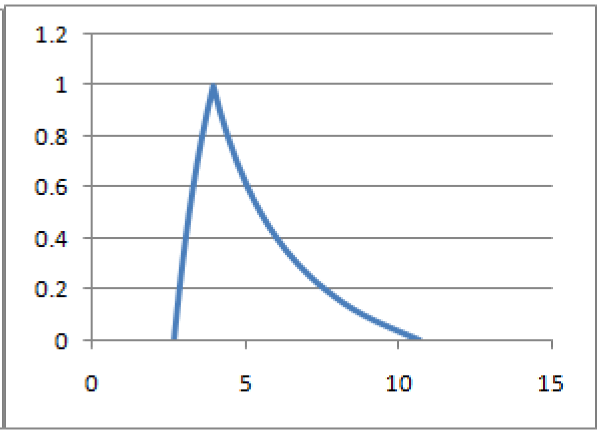


Figure 2: L_s

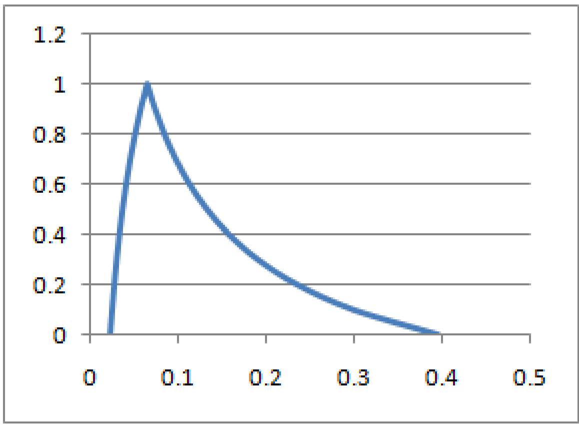


Figure 3: W_q

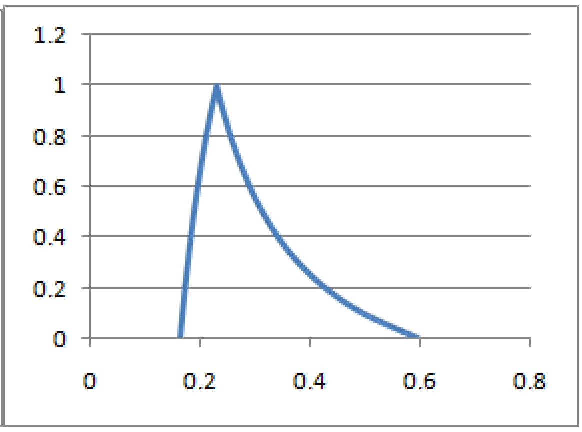


Figure 4: W_s

6.2 Trapezoidal Fuzzy Number

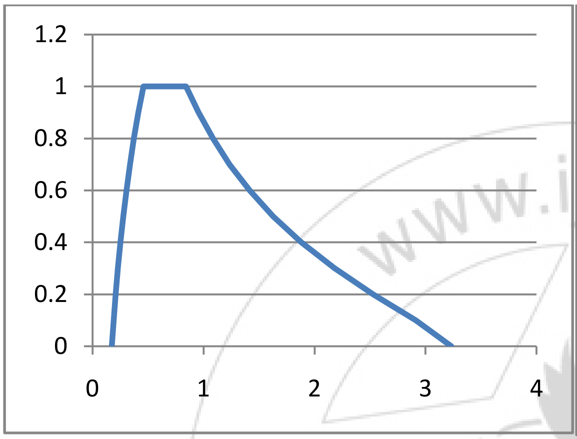
Take the both the arrival rate and service rate are trapezoidal fuzzy number represented by $\tilde{\lambda}$ = [16,17,18,19] and $\tilde{\mu}$ = [5,6,7,8,]. The number of server C=4 The interval of confidence at possibility level α as

$$[16+\alpha , 19-\alpha] \text{ and } [5+\alpha , 8-\alpha]$$

Where x = [16+α , 19 − α] and y = [5+ α , 8 −α]

Table 2 : The α cuts of L_q, L_s, W_q, W_s at α values

α	L_q	L_s	W_q	W_s
0	[0.1739, 3.2312]	[2.1739, 20.7369]	[0.0108, 0.8914]	[0.1358, 1.0914]
0.1	[0.1904, 2.9103]	[2.2284, 14.3151]	[0.0118, 0.5613]	[0.1384, 0.7574]
0.2	[0.2088, 2.5288]	[2.2857, 11.0935]	[0.0128, 0.3977]	[0.1410, 0.5900]
0.3	[0.2292, 2.1798]	[2.3461, 9.1521]	[0.0140, 0.3007]	[0.1439, 0.4894]
0.4	[0.2519, 1.8802]	[2.4098, 7.8509]	[0.0153, 0.2369]	[0.1469, 0.4220]
0.5	[0.2771, 1.6274]	[2.4771, 6.9156]	[0.0167, 0.1920]	[0.1501, 0.3738]
0.6	[0.3054, 1.4149]	[2.5486, 6.2091]	[0.0184, 0.1588]	[0.1535, 0.3374]
0.7	[0.3370, 1.2357]	[2.6247, 5.6553]	[0.0201, 0.1335]	[0.1571, 0.3090]
0.8	[0.3726, 1.0838]	[2.7059, 5.2083]	[0.0221, 0.1137]	[0.1610, 0.2861]
0.9	[0.4126, 0.9545]	[2.7929, 4.8391]	[0.0244, 0.0978]	[0.1652, 0.2673]
1	[0.4578, 0.8437]	[2.8863, 4.5283]	[0.0269, 0.0849]	[0.1697, 0.2515]



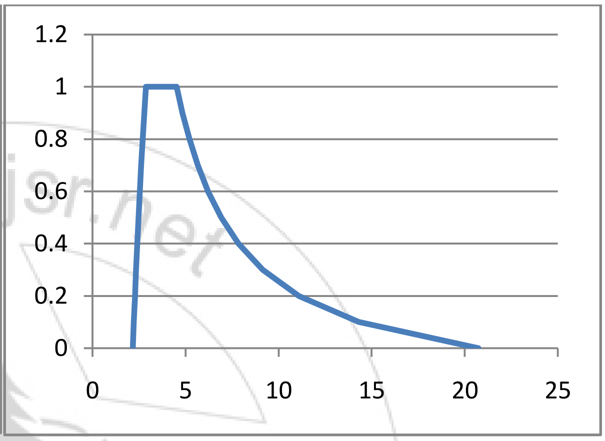
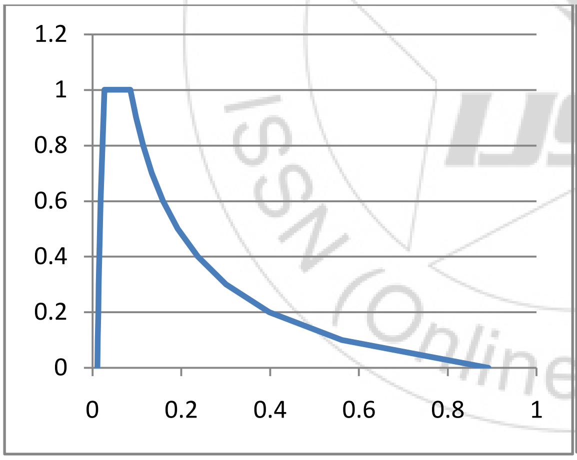


Figure 1: L_q **Figure 2:** L_s



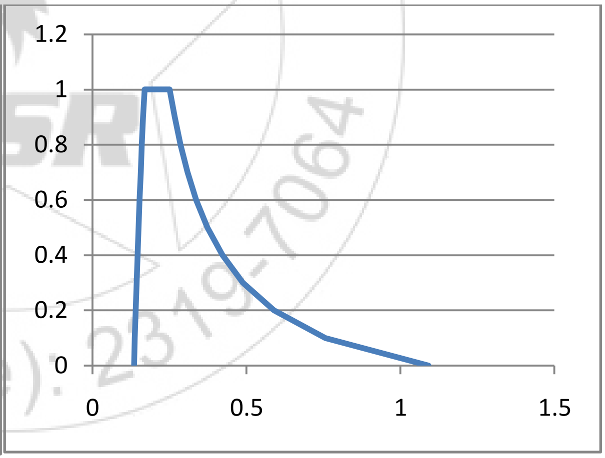


Figure 3: W_q **Figure 4:** W_s

we performed α cuts of arrival rate and service rate and fuzzy expected number of jobs in queue at eleven distinct α levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in table1and 2. The performance measures such as expected number of customers in the system (L_s), expected length of queue (L_q), the average waiting time in the system (W_s) and the average waiting time of a customer in the queue (W_q) also derived in table 1and 2.

From table 1:
1) Expected number of customers in the queue is 1.0726 and impossible falls outside [0.3356, 7.0897]
2) Expected number of customer in the system is 3.9059 and impossible falls outside [2.6213, 10.6897]
3) Average waiting time of a customer in the queue is 0.0630and impossible falls outside [0.0209, 0.3938]
4) Average waiting time of a customer in the system is 0.1638 and impossible falls outside [0.1638, 0.5938]

From table 2:
1) Expected number of customer in the queue falls between 0.4578 and 0.8437 and never falls outside [0.1739, 3.2312]
2) Expected number of customer in the system falls between 2.8863 and 4.5283 and never falls outside [2.1739, 20.7369]

3) Average number of customer in the queue falls between 0.0269 and 0.0849 and never falls outside [0.0108, 0.8914]
4) Average number of customer in the system falls between 0.1697 and 0.2515 and never falls outside [0.1358, 1.0914]

These above results are very useful for defining a queuing system.

7. Conclusion

In this paper we discussed about the performance measures of multi-server queuing model in triangular and trapezoidal fuzzy numbers. The arrival time and the service time are fuzzy nature. The performance of this model are also fuzzy nature. Numerical example shows that the efficiency of this model.

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