A Survey on Graph Partitioning Techniques

Santosh Nage[1], Girish Potdar[2]

[1,2] Department of Computer Engineering, Pune Institute of Computer Technology, Savitribai Phule Pune University, Pune 411043, India

Abstract: ***In real life, there are many problems like shortest path, graph coloring, travelling Salesmen problem (TSP) etc, thus providing solution to each problem is nearly or highly impossible with the help of traditional methods in reasonable amount of time. But it may be possible with the help of heuristic approach. It provides solution but don't guarantee optimal solution. Graph partitioning problems are NP-Complete problems, partitioning graph into p-partitions using multilevel method, spectral method etc. for various purposes .Here we are studying several techniques to partition graph.***

Keywords: graph partition, spectral, NP-complete, multi-level

1. Introduction

Graph partitioning is an important problem in the field of computer science as it has many applications in real life. To find shortest path in a given graph, partitioning will lead to find optimal shortest path using suitable heuristic [9]. Partitioning graph having exact solution is possible only with very small graph having limited vertices and edges. If there are „2n" vertices then total number of possibilities is $(2n)! / (n!)^2$ i.e. for 100 vertices 5 x 10^{28} possibilities[4]. To overcome this problem heuristic graph partitioning performs well and deploying it on multi-core systems will give good graph partitions.

2. Graph partitioning

Graph partitioning is dividing a graph into two or more parts based on certain condition. It can be defined as dividing vertices V into union of partitions P such that , disjoint set of $V_1,V_2,\ldots V_n$ such that $V_1 \Pi V_2=\Phi$ for $i \neq j$. Graphs are partitioned based on number of vertices and edges. Balanced graphs that are having equal number of vertices on each side, and partitioning it is NP-Complete[5][7], so we need good heuristic and approximation algorithms for partitioning graph to get optimal partition which will lead for further application base.

3. Heuristic

It is an approach towards finding solutions in the reasonable amount of time. Traditional methods aren"t performing well over large space to give results in a reasonable amount of time. According to G. Potdar et.al [13] heuristic approach doesn"t give guarantee to find optimal solution always. Many real world problems like NP-hard can be solved by using heuristic approach. Heuristic value is denoted as h(n) and formulated to find the total estimated cost.

Heuristic function

$f(n)=g(n)+h(n)$
Where,
f(n)- Total estimated cost
g(n)- Cost up to node n from starting point
h(n)- Estimated cost from n-node to goal.

4. Graph partitioning techniques

Graphs are based on two basic geometric figures

4.1 With nodal co-ordinate

4.1.1 Partitioning Planar graph[15][24]

A planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints Lipton-Tarjan proposed a partitioning algorithm in 1977. It is based on √n vertex separator theorem. Ungar(1951) having complexity O (√n log n), Hopcroft-Tarjan(1973) based on depth first search tried for partitioning planar graph.

4.1.2 Inertial Partitioning[5][15]

This graph bisection algorithm is very simple: For a graph with 2D coordinates, it chooses a line such that half the nodes are on one side of the line, and half are on the other. For a graph with 3D coordinates, it chooses a plane with the same property.

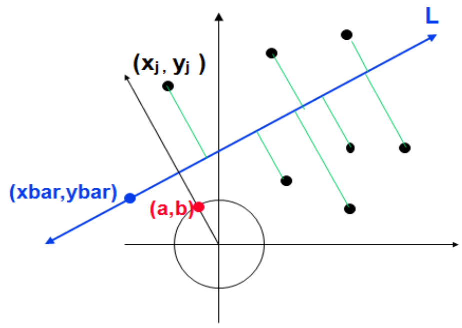

Figure.2: Inertial Graph Partitioning

4.1.3 Partitioning with random lines and circles[15][22]

The Lipton-Tarjan planar graph separator theorem provides intuition for the success of inertial graph partitioning. It is generalized idea of nearest neighbor of planar graph to higher dimension.

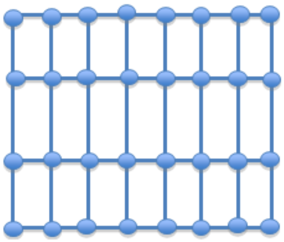

Figure 3: Partitioning with lines and circles.

4.2 Without nodal co-ordinate

4.2.1 Exact Solution[4]

A large amount of literature is available on methods that solve graph partitioning problem (GPP) optimally. Most of the methods rely on the branch-and-bound framework. Linear programming is used by Brunetta et al. Hager et al. formulate GPP in form of a continuous quadratic program on which the branch and bound technique is applied.

4.2.2 Breadth First Search[6][19][23]

It is simple approach based on Breadth First Search of a graph. This technique is effective on planar graphs and performs well on overlap graphs. It produce sub graph GP_i of G, where GP_i is a tree with root node.

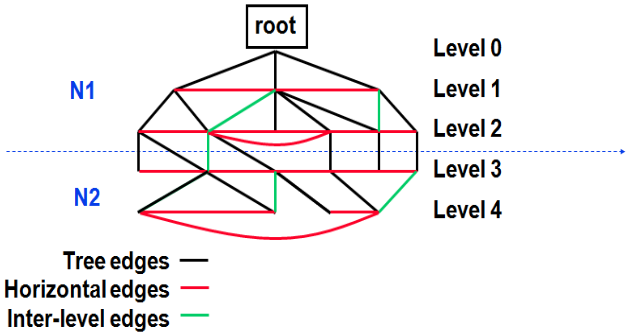

Figure 4: BFS partitioning

4.2.3 Kernighan-Lin algorithm(KL)[4]

KL algorithm is proposed one of the earliest methods for graph partitioning. More recent methods are based on local improvement methods, often variation of this method. Given an initial bisection, the KL method tries to find a sequence of node pair exchanges that leads to an improvement of the cut size.

Let $\{N_1,N_2\}$ –be the bisection of graph G(V,E) for all V Є N we define,

In KL algorithm V(G) calculate gain value of vertices for which need to calculate internal distance and external of the vertex with other vertex in the same partition or in another partition. These equations are given below,

$$\text{Int}(v) = \sum_{(v,u)\in E \& GP(v)=GP(u)} w(v,u) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{Ext}(v) = \sum_{(v,u)\in E \& GP(u)} w(v,u) \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$V(G) = \text{Ext}(v) - \text{Int}(v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

In above equations, equation (1), calculate internal distance and equation (2), calculate external distance of vertex in a given sub graphs.

4.2.4 Fiduccia and Mattheyes(FM)[14]

It is KL further improved algorithm introduced in 1982, which iteration can be done on O|E| time. Like KL method, FM method performs iterations during which each node moves at most once and best bisection observed during iteration is used as an input to next iteration. However instated of selecting a pair of nodes, FM method select single node i.e. at each step of an iteration unmark node with maximum gain value is alternatively selected from N_1 and N_2.

4.2.5 Spectral Partitioning[8][21]

It is the most powerful and expensive method. This technique is coined by Fiedler in 1970. It is motivated by analogy to vibrating string having modes of vibration or harmonics.

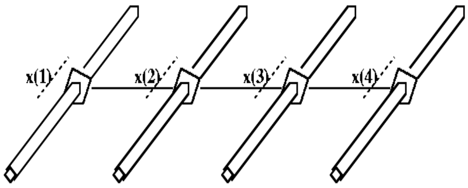

Figure 5: Vibrating string for spectral bisection.

4.2.6 Multilevel approach[10][11][16]

The graph G can be bisected using a multilevel algorithm. The basic structure of a multilevel algorithm is very simple. The graph G is first coarsened down to a few hundred vertices, a bisection of this much smaller graph is computed, and then this partition is projected back towards the original graph, by periodically refining the partition. Since the finer graph has more degrees of freedom, such refinements usually decrease the edge-cut. Formally, a multilevel graph bisection algorithm works as follows:

Consider a weighted graph $G_0 = (V_0, E_0)$, with weights both on vertices and edges. A multilevel graph bisection algorithm consists of the following three phases

a) Coarsening Phase
   1) A graph G_0 is transformed into sequence of smaller graphs $G_1, G_2, \ldots G_n$. such that
   2) $|V_0|>|V_1|>|V_2|\ldots|V_m|$.
b) Partitioning Phase
   1) Two partitions GP_1 and GP_2 are formed of a graph G(V,E) where each partition containing half of the vertices of G.
c) Un coarsening Phase

Partitioned graphs are projected to form original graph G_0.

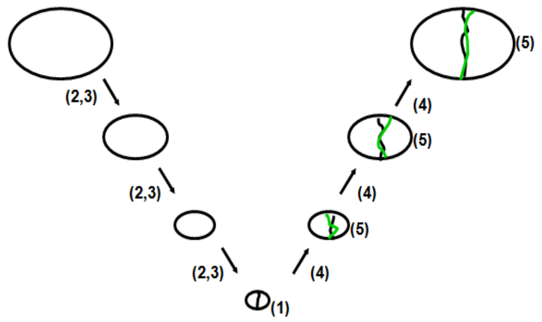

Figure 6: Multi-level graph partitioning

4.2.7 Flow based approach[9]

The well-known max-flow min-cut theorem can be used to separate two node sets in a graph by computing a maximum flow and hence a minimum cut between them. This approach completely ignores balance, and it is not obvious how to apply it to the balanced graph partitioning problem. The minimum cut of a weighted graph G is the cut of the graph with minimum weight. The minimum cut between two vertices v and w in G is the minimum weight cut of G that puts v and w in different partitions.

5. Conclusion

Here we have seen various methods of graph partitioning i.e. with co-ordinate and without co-ordinate. Based on graph methods performance may vary. Graph partitioning based on heuristic methods and approximations in large complex network lead to have optimal solution.

References

[1] Peter Sanders,Christian Schulz, "High Quality Graph Partitioning " Journal of Citeseer 2013, pp. 1-10.

[2] Yingxia Shao, Bin Cui Lin Ma "A Partition Aware Engine for Parallel Graph Computation" Knowledge and Data Engineering, IEEE Transactions on (Volume:27 , Issue: 2)2015 pp. 518-530.

[3] Sungpack Hong,Oguntebi, T., Olukotun, K. "Efficient Parallel Graph Exploration on Multi-Core CPU and GPU" Parallel Architectures and Compilation Techniques (PACT), 2011 International Conference on pp.78-88.

[4] Kernighan, B.W.Lin S. "An Efficient Heuristic Procedure for partitioning", Bell System Technical Journal, IEEE (Volume: 49, Issue: 2) 1970 pp. 291-307.

[5] Hendrickson, B. Leland, R. "A Multi-Level Algorithm For Partitioning Graphs", Supercomputing, 1995. Proceedings of the IEEE/ACM SC95 Conference pp. 1-28.

[6] Chao-Wei Ou; Ranka, S. "Parallel incremental graph partitioning" Parallel and Distributed Systems, IEEE Transactions on Year: 1997, Volume: 8, Issue: 8pp. 884-896.

[7] Gupta A. "Fast and effective algorithms for graph partitioning and sparse-matrix ordering" IBM Journal of Research and Development 1997 pp.171-183.

[8] Jigang Wu; Guiyuan Jiang; Lili Zheng; Suiping Zhou "Algorithms for Balanced Graph Bi-partitioning" High Performance Computing and Communications, 2014 IEEE 6th Intl Symp on Cyberspace Safety and Security, 2014 pp.185-188.

[9] Bruce Hendrickson Tamara G. Kolda "Graph partitioning models for parallel computing" Journal of parallel computing Volume 26 Issue 12, Nov. 2000 pp.1519-1534.

[10] Peter Sanders, Christian Schulz , "Engineering multilevel Graph Partitioning Algorithms" 19th Annual European Symposium, Saarbrücken, Germany, September 5-9, 2011, Proceedings pp. 469-480.

[11] George Karypis, Vipin kumar , "Parallel Multilevel Graph Partitioning" ,Parallel Processing Symposium, 1996., Proceedings of IPPS '96, The 10th International pp. 314-319.

[12] Jiangyu Liu, Jian Sun, "Parallel graph-cuts by Adaptive Bottom-up Merging" Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on 2010 pp. 2181-2188.

[13] Girish P. Potdar and Dr. R. C. Thool, (July 2014) "Comparison of Various Heuristic Search Techniques for Finding Shortest Path", *International Journal of Artificial Intelligence & Applications (IJAIA)*, Volume 5, No. 4, pp. 63-74.

[14] Fiduccia, C.M. Mattheyses R.M., "A Linear-time heuristic for improving Network Partitions" Design Automation, 1982. 19th Conference on pp.175-181.

[15] Alberto Moraglio, Yong-Hyuk Kim, Yourim Yoon, Byung-Ro Moon, "Geometric crossovers for multiway graph partitioning", Journal Evolutionary Computation archive Volume 15 Issue 4, Winter 2007 pp. 445-474.

[16] Padmavathi S, Gorge A, "Multilevel hybrid graph partitioning algorithm", *IEEE International Advance Computing Conference (IACC)*, 2014, pp.85-89.

[17] Mehrjerdi, H., Asber D, Saad M, "Graph partitioning of power network for emergency voltage control" *9th IEEE Asian Control Conference (ASCC)*, 2013, pp.1-6.

[18] Khan A A, Iqbal M, Khan M U, "Multilevel Graph Partitioning Scheme to Solve Traveling Salesman Problem", *Ninth International Conference on Information Technology: New Generations (ITNG)*, 2012, pp.458-463.

[19] Delling D, Goldberg A.V, Razenshteyn I, Werneck R.F. "Graph Partitioning with Natural Cuts", *IEEE International Parallel & Distributed Processing Symposium (IPDPS)*, 2011, pp.1135-1146.

[20] Barbu A., Songchun Zhu, "Graph partition by Swendsen-Wang cuts", *Ninth IEEE International Conference on Computer Vision*, Vol 1, 2003 pp.320-327.

[21] Charles J Alpert, Andrew B Kahngb, So-Zen Yaoc "Spectral partitioning with multiple eigenvectors", *The Journal of Combinatorial Algorithms, Informatics and Computational Sciences* Volume 90, Issues 1–3, 15 January 1999, Pages 3–26.

[22] Anne Condon , Richard M. Karp, "Algorithms for Graph Partitioning on the Planted Partition Model", *Randomization, Approximation, and Combinatorial Optimization. Algorithms and Techniques Lecture Notes in Computer Science* Volume 1671, 1999, pp 221-232.

[23] D. Delling, P. Sanders, D. Schultes, and D. Wagner. "Engineering Route Planning Algorithms" *In Algorithmics of Large and Complex Networks , volume 5515 of LNCS* , Springer, 2009, pp. 117-139.

[24] Thang Nguyen Bui, Andrew Peck ,"Partitioning Planar Graphs", *SIAM Journal on Computing*, 1992, Vol. 21, No. 2 : pp. 203-215.

[25] Kamesh Madduri, David A. Bader, Jonathan W. Berry, Joseph R. Crobak, "An Experimental Study of parallel Shortest Path Algorithm for Solving Large-Scale Graph Instances" *Workshop on Algorithm Engineering and Experiments (ALENEX),*2007 pp.1-10.

Author Profile

Santosh G. Nage is pursuing M.E in the Computer Engineering department at Pune Institute Computer Technology, Pune. He has done B.E from Bharati Vidhapeeth`s College of Engineering, under Shivaji University, Kolhapur, India.

Prof. Girish Potdar is HOD of Computer Engineering Department at Pune Institute of Computer Technology in Pune, India. He has research in Algorithms, Data Structure, Theory of Computation, Compiler Constructions, and Heuristics also wrote books in this area.