

Method for Solving Fuzzy Inventory Model with Space Constraint under Robust Ranking Technique

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Abstract: In this paper, an EOQ (economic order quantity) model with space constraint has been analyzed in a fuzzy environment. The costs namely holding cost, shortage cost, setup cost, the ware house capacity and the objective are usually deterministic in nature. In this paper these have been considered to be triangular fuzzy numbers which are more realistic in nature. Robust's ranking method [8] has been used for ranking the fuzzy numbers. The fuzzy inventory problem has been transformed in to crisp inventory problem. For this EOQ and minimum total cost have been calculated. This method has been illustrated by means of numerical example

Keywords: Fuzzy inventory problem, EOQ, Triangular fuzzy number, ware house capacity, fuzzy ranking

1. Introduction

In the real world, keeping an inventory for future sale or use is very common in business. Often uncertainties may occur in business. These uncertainties may be associated with demand or various relevant costs. After the publication of classical lot-size formula by Harris in 1915, many researches utilized EOQ model and currently these results are available in reference books and survey papers such as Hadley and within [2], Ha x and canda[3] and Waters[7].

In a realistic situation, total expenditure for an inventory model and the space available to store the inventory may be limited. The inventory costs, Holding, Shortage, Setup costs and the ware house space capacity may be flexible with some vagueness for their values. All these parameters in any inventory model are normally variable uncertain, imprecise and adoptable to the optimum decision making process and the determination of optimum order quality becomes a vague decision making process.

The vagueness pertained in the above parameters induces to analyzed the inventory problem in a fuzzy environment.

The method is to rank the fuzzy costs by some ranking method for fuzzy numbers. On the basis of this idea the Robust's ranking method[8] has been adopted to transform the fuzzy inventory problem to a crisp one so that we can find the optimum solutions.

2. Preliminaries

Zadeh[9] in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition:

A fuzzy set is characterized by a membership function mapping elements of a domain, space or universe of discourse X to the unit interval [0,1] (i.e) $A = \{X, \mu_A(x) : x \in X\}$, here $\mu_A : X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These

membership grades are often represented by real numbers ranking from [0,1].

Definition:

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

Definition:

The fuzzy set A is convex if and only if, for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality $\mu_A(x) \{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, $0 \leq \lambda \leq 1$.

Definition (Triangular fuzzy number):

For a triangular fuzzy number A(x), it can be represented by A(a,b,c:1) with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)} & ; a \leq x \leq b \\ 1 & ; x = b \\ \frac{(c-x)}{(c-b)} & ; c \leq x \leq d \\ 0 & ; \text{otherwise} \end{cases}$$

Definition(α -cut of a fuzzy number):

The α -cut of a fuzzy number A(x) is defined as $A(\alpha) = \{x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$

Robust's Ranking Technique

In a crisp EOQ model, the problem is to choose the order level Q(>0) which minimizes the average cost C(Q) per unit time. That is

$$\min C(Q) = \frac{1}{2} C_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} C_2 \left(\frac{Q_2^2}{Q} \right) + C_3 \left(\frac{D}{Q} \right)$$

Subject to : $AQ_1 \leq B$ (1)

Where

C₁- Holding cost per unit time per unit quantity.

C₂- Shortage cost per unit time per unit quantity.

C₃- Setup cost per period

D- demand per unit time

A- The space required by each unit (in sq.mt)

B- Maximum available ware house space (in sq.mt)

Q₁- The amount which goes into inventory

Q₂-The unfilled demand

$$Q = Q_1 + Q_2$$

The above can be solved by using non-linear programming technique and the optimum values for Q, Q₁ and C(Q) can be found.

$$Q_2^* = Q^* - Q_1^*$$

Case: I Demand is fixed.

When the above costs are fuzzy numbers, then the total cost becomes a fuzzy number. The said problem is transformed to,

$$\min \tilde{C}(Q) = \frac{1}{2} \tilde{C}_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} \tilde{C}_2 \left(\frac{Q_2^2}{Q} \right) + \tilde{C}_3 \left(\frac{D}{Q} \right)$$

Subject to:

$$AQ_1 \leq \tilde{B} \quad (2)$$

Hence we cannot solve this directly. We defuzzify the fuzzy costs and the storage space into crisp one by means of a fuzzy number ranking method.

Case: II demand is a fuzzy number.

$$\min \tilde{C}(Q) = \frac{1}{2} \tilde{C}_1 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} \tilde{C}_2 \left(\frac{Q_2^2}{Q} \right) + \tilde{C}_3 \left(\frac{\tilde{D}}{Q} \right)$$

Subject to: $AQ_1 \leq \tilde{B} \quad (3)$

Hence we cannot solve this directly. We defuzzify the fuzzy costs, the demand and the storage space into crisp one by means of a fuzzy number ranking method

Robust's ranking technique [8] which satisfies compensation, linearity and additively properties and provides results which are consistent with human intuition. Give a convex fuzzy number \tilde{a} , the Robust's ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U) d\alpha,$$

where (a_α^L, a_α^U) is the α -level cut of the fuzzy number \tilde{a} .

In this paper we use this method for ranking the fuzzy numbers. The Robust's ranking index $R(\tilde{a})$ gives the representative value of the fuzzy number

\tilde{a} . It satisfies the linearity and additive property. We apply Robust's ranking method[8] to defuzzify the fuzzy costs, the storage space and then we can get the minimum total cost $\tilde{C}^*(Q)$.

For Case I

$$R(\tilde{C}^*(Q)) = \min C(Q) = \frac{1}{2} R(\tilde{C}_1) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} R(\tilde{C}_2) \left(\frac{Q_2^2}{Q} \right) + R(\tilde{C}_3) \left(\frac{D}{Q} \right)$$

Subject to: $AQ_1 \leq R(\tilde{B})$

For case II

$$R(\tilde{C}^*(Q)) = \min C(Q) = \frac{1}{2} R(\tilde{C}_1) \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} R(\tilde{C}_2) \left(\frac{Q_2^2}{Q} \right) + C_3 \left(\frac{R(\tilde{D})}{Q} \right)$$

Subject to: $AQ_1 \leq R(\tilde{B})$

Since $R(\tilde{C}_1), R(\tilde{C}_2), R(\tilde{C}_3), R(\tilde{D})$ and $R(\tilde{B})$ are crisp values, this problem is obviously the crisp inventory problem, for which we can find the optimum order quantity Q^* and the optimum total cost $C^*(Q)$.

3. Numerical Example

Case: I Let us consider $C_1=Rs.5, C_2=Rs.25, C_3=Rs.100, D=5000, A=0.5$ sq.mt, $B=150$ sq.mt

Suppose the costs are consider as triangular fuzzy numbers, $\tilde{C}_1=(3.5,4,6), \tilde{C}_2=(20,30,35), \tilde{C}_3=(91,100,117)$ and $\tilde{B}=(146,151,161)$.

$$R(\tilde{C}_1) = R(3.5, 4, 6) = \int_0^1 0.5(C_{1\alpha}^L, C_{1\alpha}^U) d\alpha = 4.375$$

$$R(\tilde{C}_2) = R(20, 30, 35) = 26.25.$$

$$R(\tilde{C}_3) = R(91, 100, 117) = 102.$$

$$R(\tilde{B}) = R(146, 151, 161) = 152.25$$

So,

$$R(\tilde{C}^*(Q)) = \min C(Q) = \frac{1}{2} \times 4.375 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} \times 26.25 \left(\frac{Q_2^2}{Q} \right) + 102 \times \left(\frac{5000}{Q} \right)$$

Subject to : $0.5 Q_1 \leq 152.25 \quad (4)$

Case II let us consider

$\tilde{D}=(4200,5000,6000)$ and $C_3= 100$.

$$R(\tilde{D}) = R(4200, 5000, 6000)$$

$$R(\tilde{C}^*(Q)) = \min C(Q)$$

$$= \frac{1}{2} \times 4.625 \left(\frac{Q_1^2}{Q} \right) + \frac{1}{2} \times 26.5 \left(\frac{Q_2^2}{Q} \right) + 100 \times \left(\frac{5050}{Q} \right)$$

Subject to : $0.5 Q_1 \leq 152.25 \quad (5)$

4. Results and Discussion

The solutions obtained from (1),(4)and (5) are given in table 1,2 and 3

Table 1:

Q*	Q ₁ *	Min C(Q)
384.71	300	2117.69

Table 2: Optimum Results for case I

Q*	Q ₁ *	Min C(Q)
383.45	304.5	2072.33

Table 3: Optimum results for case II

Q*	Q ₁ *	Min C(Q)
382.95	304.5	2059.28

5. Conclusion

In this paper, the costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy inventory model has been transformed in to crisp inventory problem using Robust's ranking indices[8]. Numerical example shows that by this method we can have the optimal total cost. By using Robust's [8] ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking method effectively.

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