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New Approach to Find the Solution for the Generalized Fuzzy Assignment Problem with Ranking of Generalized Fuzzy Numbers

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Abstract: ***In this paper, Generalized Fuzzy Assignment Problem with Generalized Trapezoidal fuzzy numbers is introduced. A new approach is proposed to find the solution of Generalized Fuzzy Assignment Problem. Ranking method based on the rank, perimeter, mode, divergence, left spread and right spread is utilized to order the generalized fuzzy numbers. Numerical example is provided to illustrate the new approach.***

Keywords: Trapezoidal fuzzy numbers, Generalized trapezoidal fuzzy numbers, Ranking of generalized fuzzy numbers, Generalized Fuzzy Assignment Problem.

1. Introduction

Assignment problem is a particular case of the transportation problem in which the number of jobs (or origins or sources) is equal to the number of facilities (destinations or machines or persons and so on). The objective is be to minimize the total time to complete a set of jobs or to maximize skill rating, maximize the total satisfaction of the group or minimize the cost of the assignment. Various algorithms such as linear programming by Balinski (1986), Barr etal (1977), Hung& Rom(1980), Hungarian algorithm by Kuhn(1955) have been developed to find the solution of assignment problems. In recent years, fuzzy assignment problems have received much attention. Chen (1985) projected a fuzzy assignment model that considers all persons to have same skills. Lin and Wen (2004) proposed an efficient algorithm based on the labelling method for solving the linear fractional programming case. ChenLiang-Hsuan and Lu Hai-Wen (2007) developed a procedure for resolving assignments problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model to determine the assignments with the maximum efficiency. Sakthi Mukherjee and Kajla Basu (2010) introduced fuzzy ranking method for solving assignment problems with fuzzy costs. Sagaya and Henry (2012) proposed Generalized fuzzy Hungarian method for Generalized fuzzy assignment problem

In this paper, Generalized Fuzzy Assignment Problem (GFAP) with generalized trapezoidal fuzzy numbers is introduced. Generalized Fuzzy Approximation Approach is proposed to solve GFAP. The ranking method of generalized trapezoidal fuzzy numbers proposed by Sagaya & Henry (2015) is utilized to order the generalized trapezoidal fuzzy numbers and the approach is illustrated by a numerical example.

2. Preliminaries

In this section basic definitions are reviewed according to Kaufmann A. and Gupta (1988) **.**

Definition1: The characteristic function μ_A of a crisp set A $\subseteq$ X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\cdot}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\cdot} : X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) ; x \in X \}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition2: A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:
(i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty , a] \cup [d , \infty)$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a , b] and strictly decreasing on [c , d].
(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b , c]$, where $a \leq b \leq c \leq d$.

Definition3: A fuzzy number $\tilde{A} = (a , b , c ,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Definition4: A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be Generalized Fuzzy Number if its membership function has the following characteristics
(i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
(iv) $\mu_{\tilde{A}}(x) = w$, for all $x \in [b,c]$, where $0 \leq w \leq 1$.

Definition5:
A Generalized Fuzzy Number $\tilde{A} = (a,b,c,d;w)$ is said to be a Generalized Trapezoidal Fuzzy Number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w(x-a)}{(b-a)}, a \le x \le b \\ w, b \le x \le c \\ \dfrac{w(x-d)}{(c-d)}, c \le x \le d \end{cases}$$

3. Arithmetic Operations of Generalized Fuzzy Numbers

The Arithmetic Operations between two Generalized Trapezoidal Fuzzy Numbers, defined on universal set of real numbers R, are reviewed as in Chen& Chen (2009),
If $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ are two Generalized Trapezoidal Fuzzy Numbers then

(i) $\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d2;$ minimumw1,w2

(ii) $\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a2;$ minimumw1,w2

(iii) $\tilde{A} \otimes \tilde{B} = (a', b', c', d';\ \text{minimum}(w_1, w_2))$ where $a' = \text{minimum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$ $b' = \text{minimum}(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$
$c' = \text{maximum}\ (b_1b_2, b_1c_2, c_1b_2, c_1c_2)$ $d' = \text{maximum}\ (a_1a_2, a_1d_2, a_2d_1, d_1d_2)$

(iv) $\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1), \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1), \lambda < 0 \end{cases}$

4. Ranking of Generalized fuzzy numbers

An efficient method for ordering the fuzzy numbers is by the use of a ranking function, $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Jain (1976), proposed the concept of ranking function for comparing normal fuzzy numbers. Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the ranking method proposed by Sagaya & Henry (2015) and it as follows

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ be a generalized trapezoidal fuzzy number then

1) Rank $\Re(\tilde{A}) = \frac{w_1}{2}(c_1 - b_1 + d_1 - a_1)$
2) Perimeter $P(\tilde{A}) = \sqrt{(a_1 - b_1)^2 + w_1{}^2} + \sqrt{(d_1 - c_1)^2 + w_1{}^2} + (c_1 - b_1) + (d_1 - a_1)$
3) mode $(\tilde{A}) = \frac{w_1(b_1 + c_1)}{2}$,
4) divergence $(\tilde{A}) = w_1(d_1 - a_1)$
5) Left spread $(\tilde{A}) = w_1(b_1 - a_1)$, (vi) Right spread $(\tilde{A}) = \text{w}(d_1 - c_1)$

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the following approach

Step 1: Find $\Re(\tilde{A})$ and $\Re(\tilde{B})$
Case (i) If $\Re(\tilde{A}) > \Re(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If $\Re(\tilde{A}) < \Re(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If $R(\tilde{A}) = R(\tilde{B})$ then go to step 2

Step 2: Find $P(\tilde{A})$ and $P(\tilde{B})$
Case (i) If $P(\tilde{A}) > P(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If $P(\tilde{A}) < P(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If $P(\tilde{A}) = P(\tilde{B})$ then go to step 3

Step 3: Find mode $(\tilde{A})$ and mode $(\tilde{B})$
Case (i) If mode $(\tilde{A})$ > mode $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If mode $(\tilde{A})$ < mode $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If mode $(\tilde{A})$ = mode $(\tilde{B})$ then go to step 4

Step 4: Find divergence $(\tilde{A})$ and divergence $(\tilde{B})$
Case (i) If divergence $(\tilde{A})$ > divergence $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If divergence $(\tilde{A})$ < divergence $(\tilde{B})$then $\tilde{A} \prec \tilde{B}$
Case (iii) If divergence $(\tilde{A})$ =divergence $(\tilde{B})$ then go to step 5

Step 5: Find Left spread $(\tilde{A})$ and Left spread $(\tilde{B})$
Case (i) If Left spread $(\tilde{A})$ > Left spread $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If Left spread $(\tilde{A})$ < Left spread $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If Left spread $(\tilde{A})$ = Left spread $(\tilde{B})$ then go to step 6

Step 6: Find right spread $(\tilde{A})$ and right spread $(\tilde{B})$
Case (i) If right spread $(\tilde{A})$ > right spread $(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If right spread $(\tilde{A})$ < right spread $(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If right spread $(\tilde{A})$ = right spread $(\tilde{B})$ then go to step 7

Step 7: Find w_1 and w_2
Case (i) If $w_1 > w_2$ then $\tilde{A} \succ \tilde{B}$
Case (ii) If $w_1 < w_2$ then $\tilde{A} \prec \tilde{B}$
Case (iii) If $w_1 = w_2$ then $\tilde{A} \sim \tilde{B}$

5. The Generalized fuzzy Assignment Problem

The Generalized Fuzzy Assignment Problem (GFAP) can be stated in the form of nxn fuzzy cost table $[\tilde{c}_{ij}]$ as given in the following table:

		Jobs					
		1	2	...	j	...	n
Persons	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	...	$\tilde{c}_{1j}$	...	$\tilde{c}_{1n}$
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	...	$\tilde{c}_{2j}$	...	$\tilde{c}_{2n}$
	.	.	.	.	.	.	.
	i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	...	$\tilde{c}_{ij}$	...	$\tilde{c}_{in}$
	.	.	.	.	.	.	.
	n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	...	$\tilde{c}_{nj}$	...	$\tilde{c}_{nn}$

The costs or time $\tilde{c}_{ij}$ are generalized trapezoidal fuzzy numbers $\tilde{c}_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{ij}]$ is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons. (one job to one person).

6. Generalized Fuzzy Approximation Approach

An efficient approach, Generalized Fuzzy Approximation approach is proposed to find the optimal assignment for GFAP. The steps of the solution procedure are summarized as follows

Step 1: Test whether the given GFAP is balanced or not.
(i) If it is a balanced one (i.e., the number of persons are equal to the number of jobs) then go to step 3.
(ii) If it is an unbalanced one (i.e., the number of persons are not equal to the number of jobs) then go to step 2.

Step 2: Add dummy rows or dummy columns, so that the generalized fuzzy cost matrix becomes a generalized fuzzy square matrix. The generalized fuzzy cost entries of dummy rows/columns are always generalized fuzzy zero.

Step 3: Find the rank of each cell $\tilde{c}_{ij}$ and determine the minimum and the next minimum generalized fuzzy costs of each row and column of GFAP by using the ranking method.

Step 4: Determine the generalized fuzzy difference between the minimum and next minimum generalized fuzzy costs in each row and column and display them alongside the Generalized fuzzy assignment matrix by enclosing them in the parenthesis against the respective rows. Similarly compute the differences for each column

Step 5: Using the ranking mehod **i**dentify the row or column with the largest generalized fuzzy difference among all the rows and columns. If a tie occurs use arbitrary tie-breaking choice. Let the greatest difference correspond to ith row and let $\tilde{c}_{ij}$ be the smallest generalized fuzzy cost in the ith row. Assign the corresponding generalized fuzzy entry and cross off the ith row and jth column.

Step 6: Recompute the column and row generalized fuzzy differences for the reduced GFAP and go to step5. Repeat the procedure until each row and column has exactly one generalized fuzzy assignment.

7. Numerical Example

Consider a generalized fuzzy assignment problem with rows representing three persons P_1, P_2, P_3 and columns representing the three jobs J_1, J_2, J_3. The following is the cost matrix $[\tilde{C}_{ij}]$ whose elements are generalized trapezoidal fuzzy numbers.

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} J_1 & J_2 & J_3 \end{array} \\ \begin{pmatrix} [2,3,4,5,0.2] & [2,4,5,7,0.5] & [1,2,3,4,0.1] \\ [2,3,6,13,0.3] & [1,2,3,6,0.1] & [2,3,6,14,0.1] \\ [2,6,10,17,0.2] & [2,6,8,12,0.2] & [1,2,4,6,0.1] \end{pmatrix} \end{array}$$

The given problem is a balanced one.

Using Step3 of the generalized fuzzy approximation Method, the following table gives the rank of each cell of the GFAP.

	J₁	**J₂**	**J₃**
P₁	**0.4**	**1.5**	**0.2**
P₂	**2.1**	**0.3**	**0.75**
P₃	**1.9**	**1.2**	**0.35**

By Step 4 of the generalized fuzzy approximation Method, the following generalized fuzzy assignment matrix is obtained.

	J_1	J_2	J_3	
P_1	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[-2,0,2,4,0.1] (0.4)
P_2	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[-4,0,4,13,0.1] (1.05)
P_3	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[1,2,4,6,0.1]	[-4,2,6,11,0.1] (0.95)
	[-3,2,7,15,0.2] * (2.3)	[-4,3,6,11,0.1] (0.9)	[-3,-1,2,5,0.1] (0.55)	

Now using the Step 5 and 6 of the generalized fuzzy approximation Method, the following generalized fuzzy optimal assignment matrix is obtained.

	J_1	J_2	J_3			
P_1	[2,3,4,5,0.2]	[2,4,5,7,0.5]	[1,2,3,4,0.1]	[-2,0,2,4,0.1] (0.4)		
P_2	[2,3,6,13,0.3]	[1,2,3,6,0.1]	[2,3,6,14,0.1]	[-4,0,4,13,0.1] (1.05)	[-4,0,4,13,0.1] (1.05)	[1,2,3,6,0.1] *(0.3)
P_3	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[1,2,4,6,0.1]	[-4,2,6,11,0.1] (0.95)	[-4,2,6,11,0.1] (0.95)	
	[-3,2,7,15,0.2] * (2.3)	[-4,3,6,11,0.1] (0.9) [-4,3,6,11,0.1] (0.9) [1,2,3,6,0.1] *(0.3)	[-3,-1,2,5,0.1] (0.55) [-4,-1,4,13,0.1] *(1.1)			

Therefore, the generalized fuzzy optimal assignment for the GFAP is

$P_1 \longrightarrow J_1,\ P_2 \longrightarrow J_2,\ P_3 \longrightarrow J_3$

And the generalized fuzzy optimal total cost is

[2,3,4,5,0.2] + [1,2,3,6,0.1] + [1,2,4,6,0.1] = [4,7,11,17,0.1]

8. Conclusion

In this paper, a new approach, Generalized Fuzzy Approximation Approach is proposed to find the optimal assignment for GFAP with generalized trapezoidal fuzzy numbers. To order the generalized trapezoidal fuzzy numbers, ranking method based on the rank, perimeter, mode, divergence, left spread and right spread is utilized. Finally a numerical example is provided to discuss the new approach and one can apply this approach to solve any IFAP.

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