Bayesian and Non Bayesian Estimations for Birnbaum-Saunders Distribution under Partially Accelerated Life Testing Based on Censoring Sampling

Mohamed S. Hamouda

Department of Statistics, Faculty of Commerce, South Valley University, Qena, Egypt

Abstract: ***In this paper, the maximum likelihood and Bayes estimators for the Constant-stress partially accelerated life testing are considered when the lifetimes of test units are assumed to follow the two-parameter Birnbaum-Saunders distribution. Based on type-II censored samples, the maximum likelihood estimates (MLEs) for the parameters of the model are derived. Simulation study is carried out to investigate the precision of the MLEs for the parameters involved. Bayesian estimation has been considered using reference prior with partial information for the parameters of the model under squared error loss function. Due to the complexity of the model, Markovchain Monte Carlo using Gibbs sampling are used to develop a Bayesian estimation for Constant-stress partially accelerated life testing model.***

Keywords: Constant-stress partially Accelerated life testing; Birnbaum-Saunders distribution; Type- II censoring; Simulation study; Gibbs sampling

1. Introduction

Under continuing quest for improvement in the manufacturing design, it is more difficult to obtain failure information quickly for products tested at the normal use condition since test units have a long life and lengthy applied tests tend to be far too expensive. For this reason, all or some of test units may be subjected to more severe conditions than normal ones. These conditions are called stresses with may be in the form temperature, voltage, pressure, load, humidity, vibration …etc, or some combination of them. This kind of testing is called accelerated life testing (ALT) or partially accelerated life testing (PALT). In ALT, the test units are tested only at accelerated conditions; data collected at accelerated conditions are then extrapolated through a physically appropriate statistical model to estimate the life distribution at design condition. The major assumption in ALT is that the relationship between life and stress must be known or can be assumed. There are situations where a life stress relationship is not known and cannot be assumed, i.e., the data obtained from ALT cannot be extrapolated to use conditions. In such situations, partially accelerated life testing (PALT) is used. In PALT, test units are run at both use and accelerated conditions. The object of a PALT is to collect more failure data in a limited time without necessarily using high stress to all test units. Constant-stress PALT and step-stress PALT are two commonly used methods. In constant-stress PALT products are tested at either normal use or accelerated condition only until the test is terminated. In step-stress PALT, a sample of test units is first run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until a prespecified numbers of failures are obtained or a prespecified time has reached.

A few research articles are available for constant-stress PALT, which is the main topic of this paper, (for example, see, Bai & Chung[1992], Abdel-Ghaly etal.[2008], Abdel-Hamid[2009], Ismail[2004, 2009], Ismail et al.[2011], Cheng and Wang [2012], Zarrin et al.[2012], Srivastava and Mittal [2013], Srivastava and Sharma [2014],).

Birnbaum and Saunders [1969a] proposed a two-parameter failure time distribution for fatigue failure caused under cyclic loading. Fatigue failure based on a model which assumes that failure is due to the development and growth of a dominant crack. This distribution is known as the two-parameter Birnbaum-Saunders (BS) distribution or as the fatigue life distribution.

Statistical analysis for the BS distribution has for the most part, been limited to complete data by several authors, (for example, see Dupuis and Mills [1998], Kevin [1999], Xu and Tang [2010, 2011]). little work has been based on censored data, (for example, see Rieck [1995], Jeng [2003], Ng et al. [2006], Wang et al[2006] and Artur et al. [2011]). Constant-stress ALT for two-parameter BS distribution is introduced by Owen [1997] for complete data. Recently, Constant-stress ALT and step-stress PALT for two-parameter BS distribution based on censoring are introduced by Attia et al. [2013a, b] and Abd el Sattar [2014]).

In this paper, we discuss the MLE and the Bayesian estimators of constant-stress PALT for two-parameter BS distribution under type-II censored data. This model is described in detail and the MLEs of the parameters of the model are derived and its simulation study in section 2. In section 3, explains the Bayesian estimation for this model under a squared error loss function and the steps of the Markov chain Monte Carlo simulation are presented. Finally, conclusions are included in section 4.

2. Model Description

The lifetime T is assumed to have a two parameters BS distribution with the shape and scale parameters α and β respectively. So, the probability density function of T is

$$f(t;\alpha,\beta)=\frac{1}{2\alpha\sqrt{2\pi\beta}}\frac{(t+\beta)}{\left(t^{\frac{3}{2}}\right)}exp\left[\frac{-1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right], t\geq o,\alpha,\beta > o \quad (1)$$

The scale parameter β is the median of the BS distribution which has a wide use in reliability studies. The cumulative distribution function is as follows:

$$F(t;\alpha,\beta)=\Phi\left(\frac{1}{\alpha}\left(\sqrt{\left(\frac{t}{\beta}\right)}-\sqrt{\left(\frac{\beta}{t}\right)}\right)\right), t\geq o,\alpha,\beta > o \quad (2)$$

Where Φ (.) denotes the standard normal distribution.
And the reliability function of BS distribution in (1) take the form:

$$R(t)=1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\left(\frac{t}{\beta}\right)}-\sqrt{\left(\frac{\beta}{t}\right)}\right)\right), t\geq o,\alpha,\beta > o \quad (3)$$

Specifically, in a constant-stress PALT the test units run at either normal use or accelerated condition only. That is, the n sample units are divided into two groups according to a certain proportion p such that np units are randomly selected and allocated to run under normal use conditions while the remaining n(1-p) units are allocated to run under accelerated conditions. Suppose that the lifetime of a unit at normal use conditions is denoted by T, then the lifetime of that unit at accelerated conditions is $=\lambda^{-1}T$. Each test unit runs until failures without altering the test conditions.

For an unit tested at normal use condition, the probability density function is given by:

$$f_T(t)=\frac{1}{2\alpha\sqrt{2\pi\beta}}\frac{(t+\beta)}{\left(t^{\frac{3}{2}}\right)}exp\left[\frac{-1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right], t\geq o,\alpha,\beta > o \quad (4)$$

While for a unit tested at accelerated condition, the probability density function is given by:

$$f_Z(z)=\frac{\lambda}{2\alpha\sqrt{2\pi\beta}}\frac{(\lambda z+\beta)}{(\lambda z)^{\frac{3}{2}}}exp\left[\frac{-1}{2\alpha^2}\left(\frac{\lambda z}{\beta}+\frac{\beta}{\lambda z}-2\right)\right] \quad (5)$$

Where, $z\geq 0,\alpha,\beta > 0$, and $\lambda > 1$

The constant-stress PALT under type-II censoring takes place by running np test units under normal use conditions while n(1-p) units will be running under accelerated conditions and the test continues until a pre-specified number of failures, r, occurs. Then the observed lifetimes $t_{(1)}\leq t_{(2)}\leq\cdots t_{(n_u)}\leq R$ and $z_{(1)}\leq z_{(2)}\leq\cdots z_{(n_a)}\leq R$ are the ordered failures times at normal use and accelerated conditions, respectively, where $R=X_{(r)}$ which is the time of the r^{th} failures at which the test is terminated, n_u and n_a are the numbers of items failed at use and accelerated conditions, respectively. The following two indicators are then defined, $\delta_{u_i}\equiv I(t_i\leq R), i=1,2,\dots,np$

and $\delta_{a_j}\equiv I(z_j\leq R), j=1,2,\dots,n(1-p)$.

where $\sum_{i=1}^{np}\delta_{ui}=n_u$, $\sum_{j=1}^{n(1-p)}\delta_{aj}=n_a$, $\sum_{i=1}^{np}\overline{\delta_{ui}}=np-n_u$ and $\sum_{j=1}^{n(1-p)}\overline{\delta_{aj}}=nq-n_a$.

2.1 Maximum Likelihood Estimation

The maximum likelihood estimators of α, β and λ are those values of the parameters that maximize likelihood function or, equivalently, its nature logarithm. The natural logarithm of the likelihood function is given by:

$$L(y;\beta,\alpha,\lambda)=\prod_{i=1}^{np}L_{u_i}(t_i;\alpha,\beta)\prod_{j=1}^{nq}L_{a_j}(z_i;\alpha,\beta,\lambda)$$

$$=\prod_{i=1}^{np}\left[\frac{1}{2\alpha\sqrt{2\pi\beta}}\left(\frac{t_i+\beta}{t_i^{\frac{3}{2}}}\right)exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_i}{\beta}+\frac{\beta}{t_i}-2\right)\right)\right]^{\delta_{u_i}}\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)\right]^{\overline{\delta_{u_i}}}$$

$$*\prod_{j=1}^{nq}\left[\frac{\lambda}{2\alpha\sqrt{2\pi\beta}}\left(\frac{\lambda z_j+\beta}{(\lambda z_j)^{\frac{3}{2}}}\right)exp\left(-\frac{1}{2\alpha^2}\left(\frac{\lambda z_j}{\beta}+\frac{\beta}{\lambda z_j}-2\right)\right)\right]^{\delta_{a_j}}\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{\lambda R}{\beta}}-\sqrt{\frac{\beta}{\lambda R}}\right)\right)\right]^{\overline{\delta_{a_j}}} \quad (6)$$

$$\mathrm{LnL}=-(n_u+n_a)\ln\alpha-\frac{(n_u+n_a)}{2}\ln\beta+n_a\ln\lambda+\sum_{i=1}^{np}\ln(t_i+\beta)-\frac{3}{2}\sum_{i=1}^{np}\ln t_i+(np-n_u)\ln\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)\right]$$

$$+\sum_{j=1}^{nq}\ln\left(\lambda z_j+\beta\right)-\frac{3}{2}\sum_{j=1}^{nq}\ln\,\lambda z_j-\frac{1}{2\alpha^2}\left[\frac{1}{\beta}\left(\sum_{i=1}^{np}t_i+\lambda\sum_{j=1}^{nq}z_j\right)+\beta\left(\sum_{i=1}^{np}\frac{1}{t_i}+\frac{1}{\lambda}\sum_{j=1}^{nq}\frac{1}{z_j}\right)-2(n-(n_u+n_a))\right]+(nq-n_a)\ln\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{\lambda R}{\beta}}-\sqrt{\frac{\beta}{\lambda R}}\right)\right)\right](7)$$

$$LnL=-n_0\ln\alpha-\frac{n_0}{2}\ln\beta+n_a\ln\lambda+\sum_{i=1}^{np}\ln\left(t_i+\beta\right)-\frac{3}{2}\sum_{i=1}^{np}\ln\,t_i+(np-n_u)\,W(R)$$

$$+\sum_{j=1}^{nq}\ln\left(\lambda z_j+\beta\right)-\frac{3}{2}\sum_{j=1}^{nq}\ln\,\lambda z_j-\frac{Q}{2\alpha^2}+(nq-n_a)\,W(\lambda R)\quad(7)$$

Where : $n_0=n_u+n_a$

$$W(R)=\ln\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)\right],\omega(R)=\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)$$

$$Q=\frac{1}{\beta}\left(\sum_{i=1}^{np}t_i+\lambda\sum_{j=1}^{nq}z_j\right)+\beta\left(\sum_{i=1}^{np}\frac{1}{t_i}+\frac{1}{\lambda}\sum_{j=1}^{nq}\frac{1}{z_j}\right)-2(n-n_0)$$

$$W(\lambda R)=\ln\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{\lambda R}{\beta}}-\sqrt{\frac{\beta}{\lambda R}}\right)\right)\right],\omega(\lambda R)=\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)$$

The first derivatives of the (7) with respect to α,β *and* λ are given by:

$$\frac{\partial \ln L}{\partial\alpha}=\frac{-n_0}{\alpha}+(np-n_u)\frac{\partial W(R)}{\partial\alpha}+\frac{Q}{\alpha^3}+(nq-n_a)\frac{\partial W(\lambda R)}{\partial\alpha}\quad(8)$$

$$\frac{\partial \ln L}{\partial\beta}=\frac{-n_0}{2\beta}+\sum_{i=1}^{np}\frac{1}{(t_i+\beta)}+(np-n_u)\frac{\partial W(R)}{\partial\beta}+\sum_{j=1}^{nq}\frac{1}{(\lambda z_j+\beta)}-\frac{1}{2\alpha^2}\frac{\partial Q}{\partial\beta}+(nq-n_a)\frac{\partial W(\lambda R)}{\partial\beta}\quad(9)$$

$$\frac{\partial \ln L}{\partial\lambda}=\frac{n_a}{\lambda}+\sum_{j=1}^{nq}\frac{z_j}{(\lambda z_j+\beta)}-\frac{3nq}{2\lambda}-\frac{1}{2\alpha^2}\frac{\partial Q}{\partial\lambda}+(nq-n_a)\frac{\partial W(\lambda R)}{\partial\lambda}\quad(10)$$

Where $\frac{\partial W(R)}{\partial\alpha}=H(\omega(R))\frac{1}{\alpha}\omega(R)$

$$\frac{\partial W(\lambda R)}{\partial\alpha}=H(\omega(\lambda R))\frac{1}{\alpha}\omega(\lambda R)$$

$$\frac{\partial W(R)}{\partial\beta}=H(\omega(R))\frac{1}{2\alpha}\left(\sqrt{\frac{R}{\beta^3}}+\sqrt{\frac{1}{R\beta}}\right)$$

$$\frac{\partial Q}{\partial\beta}=\frac{-1}{\beta^2}\left(\sum_{i=1}^{np}t_i+\lambda\sum_{j=1}^{nq}z_j\right)+\left(\sum_{i=1}^{np}\frac{1}{t_i}+\frac{1}{\lambda}\sum_{j=1}^{nq}\frac{1}{z_j}\right)$$

$$\frac{\partial W(\lambda R)}{\partial\beta}=H(\omega(\lambda R))\frac{1}{2\alpha}\left(\sqrt{\frac{\lambda R}{\beta^3}}+\sqrt{\frac{1}{\lambda R\beta}}\right)$$

$$\frac{\partial W(\lambda R)}{\partial\lambda}=H(\omega(\lambda R))\frac{-1}{2\alpha}\left(\sqrt{\frac{R}{\lambda\beta}}+\sqrt{\frac{\beta}{\lambda^3 R}}\right)$$

$$\frac{\partial Q}{\partial\lambda}=\frac{1}{\beta}\sum_{j=1}^{nq}z_j-\frac{\beta}{\lambda^2}\sum_{j=1}^{nq}\frac{1}{z_j}$$

Such that : $H(y)=\dfrac{\phi(y)}{1-\Phi(y)}$

To obtain $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$ at which the log likelihood function is maximized, equating the equations (8-10) to zero, Since the closed form solution to thesis equations do not exist. Iterative method will be used to solve these equations numerically. To estimate the variance-covariance matrix of the estimated parameters, we use the second derivatives of the logarithm of the likelihood function defined in equation (7). The second derivatives are used to get the information matrix and by substituting $\hat{\alpha}$ for, $\alpha,\hat{\beta}$ for β and $\hat{\lambda}$ for λ, hence the asymptotic variance-covariance matrix is its inverse. Then, the second partial derivates are given as follows:

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = \frac{n_0}{\alpha^2} + (np - n_u)\frac{\partial^2 W(R)}{\partial \alpha^2} - \frac{3Q}{\alpha^4} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \alpha^2} \quad (11)$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \frac{n_0}{2\beta^2} - \sum_{i=1}^{np}\frac{1}{(t_i+\beta)^2} + (np - n_u)\frac{\partial^2 W(R)}{\partial \beta^2} - \sum_{j=1}^{nq}\frac{1}{(\lambda z_j+\beta)^2} - \frac{1}{2\alpha^2}\frac{\partial^2 Q}{\partial \beta^2} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \beta^2} \quad (12)$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{-n_a}{\lambda^2} - \sum_{j=1}^{nq}\frac{z_j}{(\lambda z_j+\beta)^2} + \frac{3}{2}\frac{nq}{\lambda^2} - \frac{1}{2\alpha^2}\frac{\partial^2 Q}{\partial \lambda^2} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \lambda^2} \quad (13)$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} = (np - n_u)\frac{\partial^2 W(R)}{\partial \alpha \partial \beta} + \frac{1}{\alpha^3}\frac{\partial Q}{\partial \beta} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \alpha \partial \beta} \quad (14)$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = \frac{1}{\alpha^3}\frac{\partial Q}{\partial \lambda} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \alpha \partial \lambda} \quad (15)$$

$$\frac{\partial^2 lnL}{\partial \beta \partial \lambda} = -\sum_{j=1}^{nq}\frac{1}{(\lambda z_j+\beta)^2} - \frac{1}{2\alpha^2}\frac{\partial^2 Q}{\partial \beta \partial \lambda} + (nq - n_a)\frac{\partial^2 W(\lambda R)}{\partial \beta \partial \lambda} \quad (16)$$

Where

$$\frac{\partial^2 W(R)}{\partial \alpha^2} = \grave{H}(\omega(R))\frac{1}{\alpha}\omega(R) - H(\omega(R))\frac{1}{\alpha^2}\omega(R)$$

$$\frac{\partial^2 W(\lambda R)}{\partial \alpha^2} = \grave{H}(\omega(\lambda R))\frac{1}{\alpha}\omega(\lambda R) - H(\omega(\lambda R))\frac{1}{\alpha^2}\omega(\lambda R)$$

$$\frac{\partial^2 W(R)}{\partial \beta^2} = \grave{H}(\omega(R))\frac{1}{2\alpha}\left(\sqrt{\frac{R}{\beta^3}} + \sqrt{\frac{1}{R\beta}}\right) - H(\omega(R))\frac{1}{4\alpha}\left(3\sqrt{\frac{R}{\beta^5}} + \sqrt{\frac{1}{R\beta^3}}\right)$$

$$\frac{\partial^2 Q}{\partial \beta^2} = \frac{2}{\beta^3}\left(\sum_{i=1}^{np} t_i + \lambda \sum_{j=1}^{nq} z_j\right)$$

$$\frac{\partial^2 W(\lambda R)}{\partial \beta^2} = \grave{H}(\omega(\lambda R))\frac{1}{2\alpha}\left(\sqrt{\frac{\lambda R}{\beta^3}} + \sqrt{\frac{1}{\lambda R\beta}}\right) - H(\omega(\lambda R))\frac{1}{4\alpha}\left(3\sqrt{\frac{\lambda R}{\beta^5}} + \sqrt{\frac{1}{\lambda R\beta^3}}\right)$$

$$\frac{\partial^2 Q}{\partial \lambda^2} = \frac{2\beta}{\lambda^3}\sum_{j=1}^{nq}\frac{1}{z_j}$$

$$\frac{\partial^2 W(\lambda R)}{\partial \lambda^2} = \grave{H}(\omega(\lambda R))\frac{-1}{2\alpha}\left(\sqrt{\frac{R}{\lambda\beta}} + \sqrt{\frac{\beta}{\lambda^3 R}}\right) + H(\omega(\lambda R))\frac{1}{4\alpha}\left(\sqrt{\frac{R}{\lambda^3\beta}} + \sqrt{\frac{\beta}{\lambda^5 R}}\right)$$

$$\frac{\partial^2 W(R)}{\partial \alpha \partial \beta} = \grave{H}(\omega(R))\frac{1}{\alpha}\omega(R) - H(\omega(R))\frac{1}{2\alpha^2}\left(\sqrt{\frac{R}{\beta^3}} + \sqrt{\frac{1}{R\beta}}\right)$$

$$\frac{\partial^2 W(\lambda R)}{\partial \alpha \partial \beta} = \grave{H}(\omega(\lambda R))\frac{1}{\alpha}\omega(\lambda R) - H(\omega(\lambda R))\frac{1}{2\alpha^2}\left(\sqrt{\frac{\lambda R}{\beta^3}} + \sqrt{\frac{1}{\lambda R\beta}}\right)$$

$$\frac{\partial^2 W(\lambda R)}{\partial \alpha \partial \lambda} = \grave{H}(\omega(\lambda R))\frac{1}{\alpha}\omega(\lambda R) + H(\omega(\lambda R))\frac{1}{2\alpha^2}\left(\sqrt{\frac{R}{\lambda\beta}} + \sqrt{\frac{\beta}{\lambda^3 R}}\right)$$

$$\frac{\partial^2 Q}{\partial \beta \partial \lambda} = \frac{-1}{\beta^2}\sum_{j=1}^{nq} z_j - \frac{1}{\lambda^2}\sum_{j=1}^{nq}\frac{1}{z_j}$$

$$\frac{\partial^2 W(\lambda R)}{\partial \beta \partial \lambda} = \grave{H}(\omega(\lambda R))\frac{1}{2\alpha}\left(\sqrt{\frac{\lambda R}{\beta^3}} + \sqrt{\frac{1}{\lambda R\beta}}\right) + H(\omega(\lambda R))\frac{1}{4\alpha}\left(\sqrt{\frac{R}{\lambda\beta^3}} - \sqrt{\frac{1}{\lambda^3 R\beta}}\right)$$

Such that : $H^{`}(y) = -yH(y) + H^2(y)$

The approximate confidence intervals of the parameters are derived based on the asymptotic distribution of the MLEs of the elements of the vector of unknown parameters $\Theta = (\alpha, \beta, \lambda)$. It is known that the asymptotic distribution of the MLEs of $\frac{\widehat{\Theta} - \Theta}{\sqrt{var(\widehat{\Theta})}}$ can be approximated by a standard normal distribution, where $var(\widehat{\Theta})$ is estimated as the asymptotic variance, then, the approximate $100(1-\gamma\%$ two sided confidence interval for α, β, λ are, respectively, given by:

$$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{var(\hat{\alpha})}, \hat{\beta} \pm Z_{\gamma/2}\sqrt{var(\hat{\beta})} \text{ and } \hat{\lambda} \pm Z_{\gamma/2}\sqrt{var(\hat{\lambda})} \quad (17)$$

Where $Z_{\gamma/2}$ is the $100(\gamma/2)$standard normal percentile.

2.2 Simulation Study

In this section simulation study has been performed using Mathcad (14) and is conducted to investigate the performances of the MLEs through their estimators, their absolute relative bias (RABias) and mean square error (MSE).The Simulation procedures will described as follows:
Step1. 1000 random samples of sizes 100(100)500 were generated from the BS distribution. Different initial values for all sets of parameters (α_0, β_0 and λ_0) are selected.

Step2. For all sample sizes and for all sets of parameters, the parameters of the model were estimated under type-II censored samples, Where $p = 0.25$ *and* $r = 0.7\ n$.

Step3. All equations under type-II censored samples were solved by using the numerical iteration method.

Step4. The estimators, RABias and MSE were tabulated for all sets of parameters in tables (1-3) for type-II censoring.

Step5. The confidence limit with confidence level $(1-\gamma) = 0.95$ were tabulated for all sets of parameters in table (4-6) for type-II censoring. So, simulation results are summarized in Tables (1-6).

3. Bayesian Estimation of the Parameters

For Bayesian estimation, Desmond (1986) pointed out, statistical analysis of the two parameters BS distribution is more difficult. In this section we will introduce Bayesian estimation of the constant-stress PALT model for the BS distribution when the data are type-II censored. The BS distribution has for the most part, been limited to complete data. Achcar [1993], used the Jeffrey's priors and reference priors to obtain the marginal posterior densities of parameters of interest using Laplace s method for approximation, Xu and Tang [2010], used the reference priors to obtain the marginal posterior densities of parameters of interest using Lindley's method and Gibbs sampling for approximations, and showed through simulations that these two methods outperform the Laplace's method. Xu and Tang [2011], proved that the reference prior is not suitable to proceed Bayesian estimation and introduced reference prior with partial information for type-I censoring data. Attia et al.(2013), used the reference prior with partial information for type-I censoring data to obtain the marginal posterior densities of parameters of interest using Gibbs sampling.

In Bayesian estimation we assume that α is independent of λ and β, so we use gamma prior for α and the reference prior with partial information for λ and β, see, (Xu and Tang (2011) and Attia et al.(2013)). Suppose that there is a subjective prior density (Gamma prior) for λ and the conditional noninformative prior for β, then the joint prior of the parameters is

$$\pi(\alpha,\beta,\lambda) \propto \pi(\alpha) * \pi(\lambda) * \pi(\beta|\lambda)$$

$$\pi(\alpha,\beta,\lambda) \propto \alpha^{a_1-1} * \lambda^{a_2-1} * \frac{1}{\beta} exp(-\alpha b1 - \lambda b_2) \quad (18)$$

Combined with (6), the joint posterior density of α, β *and* λ can be written as:

$$\pi(\alpha,\beta,\lambda\,|t) = \frac{L(t|\alpha,\beta,\lambda)\pi(\alpha,\beta,\lambda)}{\int_\sigma^\infty\int_\sigma^\infty\int_\sigma^\infty L(t|\alpha,\beta,\lambda)\pi(\alpha,\beta,\lambda)d\alpha d\beta d\lambda}$$

Then $\pi(\alpha,\beta,\lambda\,|t)$ is proportional to

$$\pi(\alpha,\beta,\lambda\,|t) \propto \alpha^{a_1-n_0-1} * \lambda^{a_2+n_a-1} * \frac{1}{\beta^{n_0+1}} exp(-\alpha b1 - \lambda b_2)$$
$$\prod_{i=1}^{np}\left[\left(\frac{t_i+\beta}{t_i^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_i}{\beta}+\frac{\beta}{t_i}-2\right)\right)\right]^{\delta_{u_i}}\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{R}{\beta}}-\sqrt{\frac{\beta}{R}}\right)\right)\right]^{\overline{\delta_{u_i}}}$$
$$\prod_{j=1}^{nq}\left[\left(\frac{\lambda z_j+\beta}{(\lambda z_j)^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{\lambda z_j}{\beta}+\frac{\beta}{\lambda z_j}-2\right)\right)\right]^{\delta_{a_j}}\left[1-\Phi\left(\frac{1}{\alpha}\left(\sqrt{\frac{\lambda R}{\beta}}-\sqrt{\frac{\beta}{\lambda R}}\right)\right)\right]^{\overline{\delta_{a_j}}} \quad (19)$$

Therefore, the posterior mean of any function of α, β and λ say g(α, β, λ), under the mean squared error loss function is:

$$E(g(\alpha,\beta,\lambda)) \propto \int_0^\infty\int_0^\infty\int_0^\infty g\,(\alpha,\beta,\lambda)\pi(\alpha,\beta,\lambda|t)d\alpha d\beta d\lambda \quad (20)$$

Generally, it is impossible for [20] to have a closed form. Therefore, Monte Carlo integration is used for the approximate evaluation of the integral [20]. So, we develop a Bayesian estimation of the BS model using Gibbs sampling, this technique aims to find a Markov chain which has as a limiting distribution the target posterior, and then the simulated sample or chain can be used to compute any desired characteristic. This means that rather than direct computation of the posterior, the simulated Markov chain after some burn- in will have realizations that are viewed as simulated from the posterior distribution. Then the generated samples are used to estimate the parameters of BS model. In order to apply the Gibbs sampling, we have to derive the full conditional distributions for unknown parameters form the posterior by removing all factors that are unrelated to the parameter of interest.

The Gibbs sampling can be carried out using the Win BUGS program (Bayesian inference using Gibbs sampling). With Win BUGS we need only to make some general specification about the model of interest and the software will computes all the required univariate marginal's. In order to apply the Gibbs sampling, the full conditional posterior distribution of α, β and λ are obtained from the posterior given in [19]. Then the full conditional distribution of α is proportional to

$$\pi(\alpha|t,\beta,\lambda) \propto \alpha^{a_1-n_0-1} exp(-\alpha b1)$$
$$\prod_{i=1}^{np}\left[\left(\frac{t_i+\beta}{t_i^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_i}{\beta}+\frac{\beta}{t_i}-2\right)\right)\right]^{\delta_{u_i}}[1-F(R)]^{\overline{\delta_{u_i}}}$$
$$\prod_{j=1}^{nq}\left[\left(\frac{\lambda z_j+\beta}{(\lambda z_j)^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{\lambda z_j}{\beta}+\frac{\beta}{\lambda z_j}-2\right)\right)\right]^{\delta_{a_j}}[1-F(\lambda R)]^{\overline{\delta_{a_j}}} \quad (21)$$

The full conditional distribution for β is proportional to

$$\pi(\beta|t,\alpha,\lambda) \propto \frac{1}{\beta^{n_0+1}} \prod_{i=1}^{np}\left[\left(\frac{t_i+\beta}{t_i^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_i}{\beta}+\frac{\beta}{t_i}-2\right)\right)\right]^{\delta_{u_i}}[1-F(R)]^{\overline{\delta_{u_i}}}$$

$$\prod_{j=1}^{nq}\left[\left(\frac{\lambda z_j+\beta}{(\lambda z_j)^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{\lambda z_j}{\beta}+\frac{\beta}{\lambda z_j}-2\right)\right)\right]^{\delta_{a_j}}[1-F(\lambda R)]^{\overline{\delta_{a_j}}} \quad (22)$$

The full conditional distribution for λ is proportional to

$$\pi(\lambda|t,\alpha,\beta) \propto \lambda^{a_2+n_a-1} * exp(-\lambda b_2)\prod_{i=1}^{np}\left[\left(\frac{t_i+\beta}{t_i^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{t_i}{\beta}+\frac{\beta}{t_i}-2\right)\right)\right]^{\delta_{u_i}}[1-F(R)]^{\overline{\delta_{u_i}}}$$

$$\prod_{j=1}^{nq}\left[\left(\frac{\lambda z_j+\beta}{(\lambda z_j)^{\frac{3}{2}}}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{\lambda z_j}{\beta}+\frac{\beta}{\lambda z_j}-2\right)\right)\right]^{\delta_{a_j}}[1-F(\lambda R)]^{\overline{\delta_{a_j}}} \quad (23)$$

Now, we write the Gibbs sampling steps as follows:

1) Derive the full conditional distributions for the α, β and λ from the posterior as was explained on equation [21-23].
2) Select the initial values such as MLEs, that are close to the center of the posterior distribution, say α_0, β_0 and λ_0, respectively, and initialize the counter j=0
3) Move the counter from j to j+1 and generate α_1, β_1 and λ_1 respectively as follows:

$\alpha_1 \sim \pi(\alpha|t,\beta_0, \lambda_0)$
$\beta_1 \sim \pi(\beta|t,\alpha_1, \lambda_0)$
$\lambda_1 \sim \pi(P|t,\alpha_1,\beta_1)$

4- Move the counter from j+1 to j+2 and generate α_2, β_2 and λ_2 as follows

$\alpha_2 \sim \pi(\alpha|t,\beta_1,\lambda_1)$
$\beta_2 \sim \pi(\beta|t,\alpha_2,\lambda_1)$
$\lambda_2 \sim \pi(\lambda|t,\alpha_2,\beta_2)$

5) Repeat steps 3 and step 4 until get $\alpha_{5000}, \beta_{5000}$ and λ_{5000}. This sequence of draws constitutes a Markov chain because the values at step N depend on the values at step N-1.
6) Discard the first K iterates, where (K=1000) is the number of burn-in sample
7) Apply Monte Carlo estimation to the generated sample or chain to obtain the Bayesian estimators of $(\Theta|t)$ by $\frac{1}{N-k}\sum_{J=k+1}^{N} g(\Theta_{N-K})$, also, standard deviation, MC errors and credible intervals of $(\alpha, \beta$ and $\lambda)$. Bayesian MCMC results are summarized in Table (7).Table (7) give the point estimates, standard deviation, MC errors and credible intervals of all parameters**.**

4. Conclusion

In this paper, we present Bayesian estimation for the unknown parameters of the constant-stress PALT model for the BS distribution when the data are type-II censored and perform two simulation studies to assess the performance of MLE and Bayesian estimators. It is observed that the maximum likelihood estimators cannot be obtained in closed form and we have proposed to use the numerical method to compute them. The performances of the MLEs are investigated by simulation study and from results of Tables (1-6). It is observed that:

1) The maximum likelihood estimators for the all sets of initial values of parameters have good statistical properties for all sample sizes. As the sample size increases the MSE of estimators decreases.
2) As the sample size increases the interval of the estimators decreases.
3) As the acceleration factor increases it is evident that the MSE of the estimated parameters tend to increase and the variance increases for all values of n.
4) As the value of α_0 and β_0 decrease, it is evident that the MSE of the estimates decrease.
5) The results of Bayesian analysis obtained from WinBUGs are showed in Table (7).It is observed that: The Bayesian estimators have good statistical properties.

Table 1: The Estimates, RABias and MSE of the parameters (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = .5, \beta_0 = 1, \lambda_0 = 1.1)$		
		Estimators	RABias	MSE	Estimators	RABias	MSE
100	α	0.102	0.592	0.022	0.199	0.601	0.091
	β	0.7	0.3	0.091	0.496	0.504	0.255
	λ	0.764	0.305	0.114	0.54	0.509	0.315
200	α	0.106	0.572	0.021	0.21	0.583	0.085
	β	0.698	0.302	0.091	0.494	0.506	0.257
	λ	0.83	0.244	0.072	0.637	0.421	0.215
300	α	0.111	0.556	0.019	0.218	0.564	0.08
	β	0.698	0.302	0.091	0.493	0.507	0.258
	λ	0.858	0.22	0.059	0.677	0.385	0.179
400	α	0.113	0.548	0.019	0.222	0.557	0.078
	β	0.697	0.303	0.092	0.492	0.508	0.259
	λ	0.876	0.203	0.05	0.706	0.359	0.156
500	α	0.117	0.534	0.018	0.23	0.544	0.074
	β	0.696	0.304	0.093	0.5	0.509	0.26
	λ	0.884	0.197	0.047	0.718	0.347	0.146

Table 2: The Estimates, RABias and MSE of the parameters (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = .25, \beta_0 = 2, \lambda_0 = 1.1)$		
		Estimators	RABias	MSE	Estimators	RABias	MSE
100	α	0.102	0.592	0.022	0.102	0.592	0.022
	β	0.7	0.3	0.091	1.4	0.3	0.362
	λ	0.764	0.305	0.114	0.76	0.305	0.114
200	α	0.106	0.572	0.021	0.106	0.572	0.021
	β	0.698	0.302	0.091	1.39	0.302	0.365
	λ	0.83	0.244	0.072	0.83	0.244	0.072
300	α	0.111	0.556	0.019	0.111	0.556	0.019
	β	0.698	0.302	0.091	1.39	0.302	0.366
	λ	0.858	0.22	0.059	0.858	0.22	0.059
400	α	0.113	0.548	0.019	0.113	0.548	0.019
	β	0.697	0.303	0.092	1.4	0.303	0.369
	λ	0.876	0.203	0.05	0.876	0.203	0.05
500	α	0.117	0.534	0.018	0.116	0.534	0.018
	β	0.696	0.304	0.093	1.4	0.303	0.368
	λ	0.884	0.197	0.047	0.884	0.196	0.047

Table 3: The Estimates, RABias and MSE of the parameters (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.5)$		
		Estimators	RABias	MSE	Estimators	RABias	MSE
100	α	0.102	0.592	0.022	0.102	0.592	0.022
	β	0.7	0.3	0.091	0.7	0.3	0.091
	λ	0.764	0.305	0.114	1.04	0.305	0.211
200	α	0.106	0.572	0.021	0.106	0.575	0.021
	β	0.698	0.302	0.091	0.698	0.302	0.091
	λ	0.83	0.244	0.072	1.14	0.244	0.134
300	α	0.111	0.556	0.019	0.111	0.556	0.019
	β	0.698	0.302	0.091	0.698	0.302	0.091
	λ	0.858	0.22	0.059	1.17	0.22	0.109
400	α	0.113	0.548	0.019	0.113	0.548	0.019
	β	0.697	0.303	0.092	0.7	0.303	0.092
	λ	0.876	0.203	0.05	1.2	0.203	0.93
500	α	0.117	0.534	0.018	0.116	0.534	0.018
	β	0.696	0.304	0.093	0.697	0.303	0.092
	λ	0.884	0.197	0.047	1.21	0.196	0.087

Table 4: Confidence bounds of the estimates at confidence levels.95 of (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = 0.5, \beta_0 = 1, \lambda_0 = 1.1)$		
		Estimators	Variance	(L, U)	Estimators	Variance	(L, U)
100	α	0.102	0.00007	(0.085, 0.118)	0.199	0.0003	(0.167, 0.232)
	β	0.7	0.0003	(0.668, 0.733)	0.496	0.0005	(0.452, 0.541)
	λ	0.764	0.0004	(0.723, 0.805)	0.54	0.0008	(0.484, 0.596)
200	α	0.106	0.00004	(0.094, 0.119)	0.21	0.0002	(0.184, 0.233)
	β	0.698	0.0002	(0.674, 0.723)	0.494	0.0003	(0.461, 0.527)
	λ	0.83	0.0003	(0.799, 0.865)	0.637	0.0006	(0.587, 0.687)
300	α	0.111	0.00003	(0.1, 0.122)	0.218	0.0001	(0.197, 0.239)
	β	0.698	0.0001	(0.678, 0.719)	0.493	0.0002	(0.464, 0.521)
	λ	0.858	0.0002	(0.829, 0.887)	0.677	0.0005	(0.632, 0.722)
400	α	0.113	0.00002	(0.104, 0.122)	0.222	0.00009	(0.203, 0.24)
	β	0.697	0.00009	(0.679, 0.715)	0.492	0.00016	(0.467, 0.516)
	λ	0.876	0.00018	(0.85, 0.903)	0.706	0.0004	(0.664, 0.747)
500	α	0.117	0.00001	(0.108, 0.125)	0.23	0.00008	(0.211, 0.245)
	β	0.696	0.00007	(0.679, 0.713)	0.5	0.00014	(0.468, 0.514)
	λ	0.884	0.00016	(0.859, 0.908)	0.718	0.0003	(0.679, 0.757)

Table 5: Confidence bounds of the estimates at confidence levels.95 of (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = .25, \beta_0 = 2, \lambda_0 = 1.1)$		
		Estimators	Variance	(L, U)	Estimators	Variance	(L, U)
100	α	0.102	0.00007	(0.085, 0.118)	0.102	0.00007	(0.085, 0.118)
	β	0.7	0.0003	(0.668, 0.733)	1.4	0.001	(1.336, 1.465)
	λ	0.764	0.0004	(0.723, 0.805)	0.76	0.0004	(0.723, 0.805)
200	α	0.106	0.00004	(0.094, 0.119)	0.106	0.00005	(0.094, 0.119)
	β	0.698	0.0002	(0.674, 0.723)	1.39	0.0006	(1.349, 1.445)
	λ	0.83	0.0003	(0.799, 0.865)	0.83	0.0003	(0.799, 0.865)
300	α	0.111	0.00003	(0.1, 0.122)	0.111	0.00004	(0.1, 0.122)
	β	0.698	0.0001	(0.678, 0.719)	1.39	0.0004	(1.36, 1.43)
	λ	0.858	0.0002	(0.829, 0.887)	0.858	0.0002	(0.829, 0.887)
400	α	0.113	0.00002	(0.104, 0.122)	0.113	0.00003	(0.104, 0.122)
	β	0.697	0.00009	(0.679, 0.715)	1.4	0.0003	(1.357, 1.43)
	λ	0.876	0.00018	(0.85, 0.903)	0.876	0.0002	(0.85, 0.903)
500	α	0.117	0.00001	(0.108, 0.125)	0.116	0.00002	(0.108, 0.125)
	β	0.696	0.00007	(0.679, 0.713)	1.4	0.0002	(1.36, 1.427)
	λ	0.884	0.00016	(0.859, 0.908)	0.884	0.0001	(0.86, 0.909)

Table 6: Confidence bounds of the estimates at confidence levels.95 of (α, β, λ) under type II censoring

n	Parameters	$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.1)$			$(\alpha_0 = .25, \beta_0 = 1, \lambda_0 = 1.5)$		
		Estimators	Variance	(Lower, Upper)	Estimators	Variance	(Lower, Upper)
100	α	0.102	0.00007	(0.085, 0.118)	0.102	0.00007	(0.085, 0.118)
	β	0.7	0.0003	(0.668, 0.733)	0.7	0.0003	(0.668, 0.733)
	λ	0.764	0.0004	(0.723, 0.805)	1.04	0.0008	(0.986, 1.098)
200	α	0.106	0.00004	(0.094, 0.119)	0.106	0.00004	(0.094, 0.119)
	β	0.698	0.0002	(0.674, 0.723)	0.698	0.0002	(0.674, 0.723)
	λ	0.83	0.0003	(0.799, 0.865)	1.14	0.0005	(1.09, 1.18)
300	α	0.111	0.00003	(0.1, 0.122)	0.111	0.00003	(0.1, 0.122)
	β	0.698	0.0001	(0.678, 0.719)	0.698	0.0001	(0.678, 0.719)
	λ	0.858	0.0002	(0.829, 0.887)	1.17	0.0004	(1.13, 1.21)
400	α	0.113	0.00002	(0.104, 0.122)	0.113	0.00002	(0.104, 0.122)
	β	0.697	0.00009	(0.679, 0.715)	0.7	0.00008	(0.679, 0.715)
	λ	0.876	0.00018	(0.85, 0.903)	1.2	0.0003	(1.159, 1.23)
500	α	0.117	0.00001	(0.108, 0.125)	0.116	0.00001	(0.108, 0.125)
	β	0.696	0.00007	(0.679, 0.713)	0.697	0.00007	(0.68, 0.713)
	λ	0.884	0.00016	(0.859, 0.908)	1.21	0.0002	(1.172, 1.24)

Table 7: Node statistics with initial values **(alpha=.25, beta=1.5, lamada=1)**

Node	*Mean*	*sd*	*MC_error*	*2.50%*	*Median*	*97.50%*	*Start*	*Sample*
Lamada	1.013	0.00946	0.00149	1	1.01	1.033	1001	4000
alpha	0.274	0.04671	0.01321	0.1633	0.2704	0.3402	1001	4000
beta	1.474	0.07315	0.02073	1.278	1.504	1.549	1001	4000

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