# Temperature of a Rotating Black Hole

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**Abstract:** ***This science article deals with the temperature of a rotating black hole. In this article I have proved that temperature of a rotating black hole depends not only on mass but also the linear velocity of the black hole and derivative of rotational velocity with respect to the mass of rotating black hole unlike the temperature of non-rotating black hole. In this article I have used the concepts of quantum mechanics, hawking radiation, benkenstien entropy formula and by going through all of them I have derived the above stated fascinating result, temperature of a rotating black hole is a function of mass, rotational velocity, and derivative of rotational velocity with respect to the mass, and enhanced the our knowledge about the rotating black hole. I have derived a new result that temperature of rotating black hole is apart from non rotating black hole.***

$$T = \frac{-hvc^3}{8\pi^2 km^2 G * \frac{d(v)}{dm}}$$

**Keywords:** hawking radiation, benkenstien‟s entropy, angular momentum, Schwarzschild radius, hawking temperature.

## 1. Introduction

Hawking radiation explains that when laws of quantum mechanics are applied just near the event horizon of the black hole we get some weird results. When pair production takes place just near the event horizon negative energy particle falls into the black hole and reduces its mass and surface area whereas positive energy particle has just enough energy to escape and emits as radiation. Let a rotating black hole has radius „r‟ and time period "t". Surface area „A‟ of rotating black hole will be

$$A = 4\pi r^2$$

Dividing and multiplying the above equation by the mass of the rotating black hole „m‟.

$$A = 4\pi (m*r^2)/m$$

Inertia $I = m*r^2$ putting this value of $m*r^2$in. We get

$$A = 4\pi\ I/m \quad \text{………………………(1)}$$

For rotating black hole let angular velocity is „ω‟. Then

$$\omega = 2 * \pi/t$$

Or

$$2* \pi = \omega *t$$

Putting this value of 2* π in (1). We get

$$A = 2(I\omega)t/m \quad \text{……………..............(2)}$$

We know that angular momentum

$$J = I*\omega$$

Putting the value of I*ω in (2). We get

$$A = 2J*t/m \quad \text{…………………………(3)}$$

Time period „t‟ = 2 π r/v

Where „v‟ represents linear velocity (from ahead I‟ll just say velocity instead of linear velocity). For rotating black hole Schwarzschild radius

$$r = 2Gm/c^2$$

putting its value in (4) and get

$$t = 4\pi\ Gm/v\ c^2$$

Putting this value of „t‟ in (3).

$$A = 8\pi JGm/v\ c^2\ m$$

Or

$$A = 8\pi JG/v\ c^2 \quad \text{……………….(4)}$$

This is the formula for surface area of a rotating black hole in terms of velocity. When the hawking radiation takes place black hole lose mass in from of radiation and gets reduced in size (radius). So change in surface area with respect to the mass , given by

$$\frac{dA}{dm} = \frac{8\pi JG}{c^2} * \frac{d(1/v)}{dm} \quad \text{(where J = constant)}$$

Angular momentum of rotating black hole must remain conserved because there is no external force is being applied on the black hole it is just evaporating by itself.

$$\frac{dA}{dm} = \frac{-8\pi JG}{c^2 v^2} * \frac{d(v)}{dm} \quad \text{……….(5)}$$

Entropy „S‟ of a rotating black hole is directly proportional to the surface area of black hole total entropy of black hole is given by **(Benkenstien- hawking entropy formula)**

$$S = \frac{\pi kc^3 * A}{2hG}$$

Where k represents Boltzmann constant.
Differentiating above equation with respect to mass.

$$\frac{dS}{dm} = \frac{\pi kc^3}{2hG} * \frac{dA}{dm}$$

putting the value of $\frac{dA}{dm}$ from (5) in to the above equation and get

$$\frac{dS}{dm} = \frac{\pi kc^3}{2hG} * \left(\frac{-8\pi JG}{c^2 v^2} * \frac{d(v)}{dm}\right)$$

$$\frac{dS}{dm} = -\frac{4\pi^2 kcJ}{hv^2} * \frac{d(v)}{dm} \quad \text{……..(6)}$$

Energy of a rotating black hole is given by Einstein‟s equation

$$E = mc^2$$

Change in energy is

$$\frac{dE}{dm} = c^2 \quad \text{…………….(7)}$$

We know that change temperature „T‟ is given by (basic entropy temperature relationship)

$$T = dE/dS$$

Or it can be written as

$$T = \frac{\frac{dE}{dm}}{\frac{dS}{dm}}$$

putting the values of dE/dm and dS/dm from (7) and (6) respectively.

$$T = \frac{-c^2}{\frac{4\pi^2 kcJ}{hv^2} * \frac{d(v)}{dm}}$$

$$T = \frac{-cv^2h}{4\pi^2 kJ * \frac{d(v)}{dm}}$$

Putting

J = m v r (constant)

Angular momentum of rotating black hole must remain conserved because there is no external force is being applied on the black hole it is just evaporating by itself.

$$T = -\frac{cv^2h}{4\pi^2 k * mvr * \frac{d(v)}{dm}}$$

$$T = -\frac{hvc}{4\pi^2 kmr * \frac{d(v)}{dm}} \qquad \text{............(8)}$$

further putting $r = 2Gm/c^2$ in (8).

$$T = -\frac{hvc}{4\pi^2 km * \frac{2Gm}{c^2} * \frac{d(v)}{dm}}$$

## 2. Result

$$\boxed{T = -\frac{hvc^3}{8\pi^2 km^2 G * \frac{d(v)}{dm}}} \qquad \text{....... (9)}$$

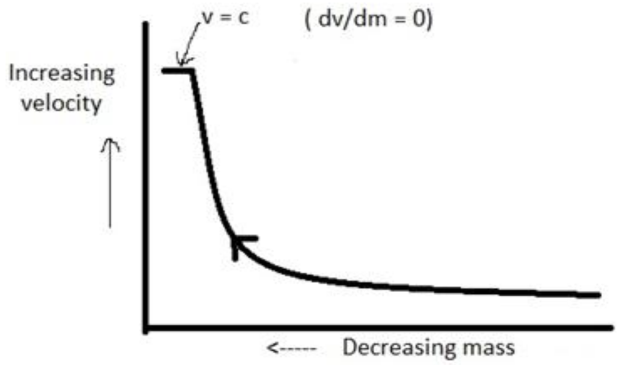
This is the formula I promised for. Temperature of a rotating black hole is different from non -rotating black. It depends on linear velocity as well . Hence conclusions are

## 3. Conclusions

- Temperature of a rotating black hole is directly proportional to its velocity.
- Temperature of a rotating black hole is inversely proportional to square of its mass (unlike non-rotating black hole whose temperature is inversely proportional to single power of mass).
- Temperature of a rotating black hole is inversely proportional derivative of linear velocity with respect to its mass.
- Negative sign does not indicate that temperature is negative. Its negative sign will be canceled out by negative sign we‟ll get d(v)/dm.
- d(v)/dm is always negative because mass of rotating black hole always decreases as it evaporates. Angular momentum „J‟ of a rotating black hole always remain constant because no external force in being applied on it. So

$$J = mvr = W \text{ (some constant)}$$
$$r = 2Gm/c^2$$
$$mv*2Gm/c^2 = W$$
$$v = w/m^2 (w = \text{constant})$$

Graph of „v‟ against „m‟ will be like-




velocity-mass graph of rotating black hole

At the moment when velocity becomes equal to the velocity of light. At that point dv/dm = 0. Rotating black hole‟s temperature becomes infinity that means at infinite temperature black hole will emit all its radiation at once. There will be bang, an explosion that will be end of rotating black hole. In nut shell when velocity of rotating black hole equals to the velocity of light that will be moment of its death. It will break apart in to particles and emit all its mass at once.

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## Author Profile

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