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Queuing Theory and its Application in Waste Management Authority (A Focus on Lawma Igando Dump Site, Lagos State)

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Abstract: ***This research work is a study of queuing theory and its application in waste management authority (A case study of LAWMA Igando dump site, Lagos state). The queuing model used was M/M/S. Both the service and the inter-arrival time made a good fit to Exponential distribution. Queuing performance measures was used to estimate the inter-arrival, service and waiting time of the queue. These results were inter-arrival time (λ) =30.8/hr, service time (μ)=3.49/truck, and waiting time in the queue = 4.0hrs.***

Keywords: Queuing, Arrival Time, Inter-arrival Time, Service Time, Waste management.

1. Introduction

Lagos Waste Management Authority (LAWMA) is mandated to handle waste collection, disposal and clearing of backlog of waste. Domestic, commercial and industrial waste collection and disposal are handled by the accredited Private Sector Participation (PSP) operators all over the state.

Their service provision and delivery is that all household are serviced at least once weekly. All highways are cleared of refuse by 8am daily, medical waste are parked in customized receptacle and taken to designated pre-treatment plants before disposal while market waste are cleared daily. Landfill sites and Transfer loading station must operate to support disposal services. All clients are to receive bills as at when due for services rendered.

LAWMA has about 365 private and domestic contractors and have about 6 fillable dumpsites in different locations. One of the dumpsites in Igando, Alimosho Local Government Area, Lagos state, is used as a case study to obtain data and also the research object, which led to the consideration of Queuing theory in waste management service.

Queuing theory also known as waiting line is a quantitative analysis technique. It is an everyday occurrence, affecting people such as making a bank deposit, trucks in line to be unloaded, airplanes lined up on a runaway waiting for permission to takeoff; passengers queue up in motor parks either for available buses. Waiting for service is a part of our daily life. Queuing theory is used to develop a more efficient queuing System that reduces customers' wait time and increases the number of customers that can be served. It also permits the derivation and calculation of several performance measures including the average waiting time in the queue or in the system, the expected number waiting or receiving service and the probability of encountering the system in certain states such as empty, full, having to wait a certain time to be served

2. Motivation

Considering the fact that waiting is difficult to estimate, it measures in terms of loss or gain in both services and human approach. In waste management services, when there is longer queue than services in dumpsite, it prevents work done in cleaning of the state and causes havoc in the society. Therefore, the study of queue will help in quantifying the phenomenon of waiting lines using representative measures of performances such as average queue length, average waiting time queue and average facility utilization to stabilize the system.

3. Literature Review

Queuing theory has become one of the most important, valuable and arguable of one of the most universal used tools by an operational researcher. It has applications in diverse fields.

Jeroen et al (2011) solved a problem on municipal solid waste collection, using a vehicle routing problem to serve a number of customers with a fleet of vehicles. They minimized cost, allocated trucks to various customers with the use of 3D-GIS model concluded that with the proper allocation of vehicles, there will be little or no waste in environment/society. Charisios et al (2013) used multi-criteria decision analysis to tackle waste management problems, in providing environmental managers and decision makers with thorough list of practical applications of the multi-criteria decision analysis techniques that are used to solve real-life waste management problems and were able to achieve the solution.

Omotoso et al (2013) applied queuing theory to the optimum control of the waiting times, idle times and queue in banks using (m/m/s/∞/∞/queuing system). The result of the research shows that the waiting time and queue length is kept at optimum with just two servers (cashier). Azmat (2007) also applied queuing theory to determine the sales checkout operation in ICA supermarket using a multiple queue multiple server model. This was used to obtain efficiency of the models in terms of utilization and waiting length, hence increasing the number of queues so customers will not have to wait longer when servers are too busy. The model contains five (5) servers which are checkout sales counters and it helps to reduce queue.

Morharchol et al (2010), solved a problem on multi-server queuing system with multiple priority classes using a technique which they refer to as Recursive Dimensionality Reduction (RDR) in which the M-dimensionally infinite markov chain representing the M classes state space is recursively reduced to a 1-dimensionally infinite markov chain, which is easily and quickly solved. They concluded that the RDR algorithm is efficient when the number of priority classes is small and becomes less practical when the number of priority classes grow (i.e for an m/m/2 with 10 priority classes, the running time can get as high as 10s of seconds). Hamid et al (2014) analyze a firm that operates as m/m/I queue and provides service to price and delay sensitive customers who are differentiated on both their value for the service and cost of waiting. Their main result is that offering two service grades is asymptotically optional with an optimality gap of \$0(n^ { $\frac{1}{4}$ })\$ relative to the overall optimal policy.

Gregor et al (2014) provides an analytical solution for the time dependent performance evaluation of truck handling operations at an air cargo terminal. Two heterogeneous handling facilities with multiple servers queuing system are available to handle trucks assuming exponentially distribution processing times. They develop a stationary backlog-carryover (SBC) approach which they use to solve the problem and they were able to route trucks to a handling facility. Mathias (2010) observed queuing for fast restaurants, specifically observed the actual waiting time for customers for a number of fast food restaurants and compared the metrics with waiting time that customer expects. They came up that during lunch time peak hours, customers spent on average 5.4 minutes waiting before they could get their order. The 5.4 minutes consisted of 2.42 minutes of queuing time and 2.98 minutes of queuing time and 2.98 minutes of service time. This total waiting time is only slightly below the actual expected waiting time of 5.42 minutes.

Mohammed (2013) used queuing theory model (m/m/s) multiple-channel queuing model with Poisson Arrival and Exponential Service Time. To solve for waiting line of a bank (a case on Islami Bank Bangladseh Limited Chittagong) and was able to present the total minimum expectation cost of waiting of the bank. Houda et al (2012) describe several common queuing situations and present mathematical models for analyzing waiting lines. The model used was multiple-channel queuing with Poisson Arrival and Exponential service time (m/m/s). they were able to resolve the waiting problem and a good linear programming was taken into consideration.

Robert et al (2000) applied queuing theory in the planning of the optimal number of servers (RAMPS) in closed parking system. They learn how to efficiently organize traffic areas, the size of parking capacities and to ensure a quality parking service to local population. They concluded that the optimal number of servers (RAMPS) in closed parking systems can be determined. Muhammed (2014) carry out queuing analysis to examine the multi-stage production line performance to facilitate more realistic resource planning leaving distribution data in the company showed an accuracy of 93.80%.

Adam et al (2012) worked on the study of queuing theory in low to high rework environment with process availability using G/G/I modelling techniques. The result they came out with is that the model performs well even under high rework condition. Tabar et al (2012) used queuing theory to reorganize the optimal number of required human resources in an educational institution carried out in Iran. Multi-queuing analysis was used to estimate the average waiting time, queue lengths, number of servers and service rates. The analysis was performed for different numbers of staff members. Finally, the result shows that the staff members in this department should be reduced.

Pouya (2009) worked on queuing model of Hospital congestion; he used queuing network modelling to determine the Intensive Care Unit (ICU) and Medical Unit (MU) of the Hospital. He determined the sufficient bed counts in each of these two units so as to guarantee certain access standards. Dhar and Tanzina (2013) worked on a case study for Bank ATM queuing model; they derive the arrival rate, service rate, utilization rate, waiting time in the queue. The model used was m/m/i. They concluded that the time arrival rate at a bank ATM on Sunday during banking time is 1 customer per minutes (cpm) while the service rate is 1.50cpm. The average number of customer in the ATM is 2 and the utilization period is 0.70

Parameters in queuing models (m/m/s)

λ :Mean arrival rate i.e (1/average no of trucks arriving in each queue in the system)

μ : Mean service rate i.e (1/average number of trucks been served at a server).

t : Mean inter-arrival time i.e $^1/_\lambda$

S : Number of parallel servers (2)

n : Number of total trucks in the system (in queue + in service)

ℓ : Mean number of trucks in the service i.e $\ell = {}^\lambda/_\mu$

ρ : Traffic intensity or utilization factor i.e $^\lambda/_{S\mu}$

P_0 : Steady state probability of all idle servers in the system

Ls : Average (expected) number of trucks in the system (waiting and service)

Lq : Average (expected) number of customers in the queue (queue length)

L : Average (expected) length of non-empty queue

Ws : Average(expected) waiting time in the system (waiting and in service)

Wq: Average (expected) waiting in the queue

Pw : Probability that an arrival customer has to wait (system being busy) I - $P_0(^\lambda/_\mu)$.

PERFORMANCE MEASURES

The expected number of customers waiting in the queue (length of line)

$$L_q = \sum_{n-s}^{\infty}(n-s)P_n = \sum_{n-s}^{\infty}(n-s)\frac{\rho^n}{S^{n-s,}S!}P_o$$

$= \frac{\rho^s P_o}{S!} \sum_{n-s}^{\infty} (n-s)\rho^{n-s} = \frac{\rho^s P_o}{S!} \sum_{m=o}^{\infty} m\rho^m, n-s=m, \rho = \lambda⍰$

$= \frac{\rho}{S!} \rho P_o \sum_{m=o}^{\infty} m\rho^{m-1} = \frac{\rho^s}{S!} . \rho P_o \frac{d}{dp}[\sum_{m=1}^{\infty} \rho^m]$

$= \frac{\rho^s}{S!} \rho P_o \frac{1}{(1-\rho)^2} = \left[\frac{1}{S!} \left(\frac{\lambda}{⍰}\right)^s \frac{\lambda.S⍰}{(S⍰-\lambda)^2}\right] P_o$

$L_q = \left[\frac{1}{(S-1)!} \left(\frac{\lambda}{⍰}\right)^s \frac{\lambda.S⍰}{(S⍰-\lambda)^2}\right] P_o$

The expected number of customers in the system

$L_s = L_q + \frac{\lambda}{⍰}$

The expected waiting time of a customer in the queue

$W_q = \left[\frac{1}{(S-1)!} \left(\frac{\lambda}{⍰}\right)^s \frac{⍰}{(S⍰-\lambda)^2}\right] P_o$

$W_q = \frac{L_q}{\lambda}$

The expected waiting time that a customer spends in the system

$W_s = W_q + \frac{1}{⍰}$

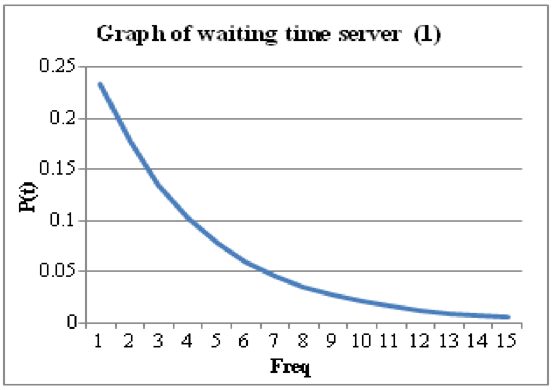
$= \frac{L_q}{\lambda} + \frac{1}{⍰}$

The probability that all customers are simultaneously busy

$P(n \geq s) = \sum_{n=s}^{\infty} P_n = \sum_{n=s}^{\infty} \frac{1}{S!^{n-s}} \left(\frac{\lambda}{⍰}\right)^n P_o$

$= \frac{1}{S!} \left(\frac{\lambda}{⍰}\right)^s P_o \sum_{m=o}^{\infty} \left(\frac{\lambda}{⍰}\right) = \frac{1}{S!} \left(\frac{\lambda}{⍰}\right)^s \frac{S⍰}{S⍰-\lambda} P_o$

$P(n \geq s) = \frac{1}{S!} \left(\frac{\lambda}{⍰}\right)^s \frac{S⍰}{S⍰-\lambda} P_o$



The inter-arrival time follows exponential distribution.

WAITING TIME ANALYSIS FOR SERVER TWO

Table 2

INTERVAL	F_i	X_i	F_iX_i	P(t)	$e_{i=NP(t)}$	$(O_i-e_i)^2/e_i$
0≤t≤6	3	3	9	0.189	5.67	1.257
7≤t≤13	6	10	60	0.149	4.47	0.524
14≤t≤20	8	17	136	0.117	3.51	5.744
21≤t≤27	3	24	72	0.091	2.73	0.026
28≤t≤34	0	31	0	0.071	2.13	2.130
35≤t≤41	5	38	190	0.056	1.68	6.561
42≤t≤48	1	45	45	0.043	1.29	0.065
49≤t≤55	0	52	0	0.033	0.99	0.990
56≤t≤62	1	59	59	0.027	0.81	0.045
63≤t≤69	0	66	0	0.021	0.63	0.63
70≤t≤76	0	73	0	0.017	0.51	0.51
77≤t≤83	1	80	80	0.012	0.36	1.137
84≤t≤90	1	87	87	0.01	0.30	1.633
91≤t≤97	0	94	0	0.007	0.21	0.21
98≤t≤104	0	101	0	0.006	0.18	0.18
105≤t≤111	0	108	0	0.004	0.12	0.12
112≤t≤118	0	115	0	0.003	0.09	0.09
119≤t≤125	1	122	122	0.002	0.06	14.726
	30		860			36.578

Mean X = $\frac{\sum_{i=1}^{n} f_x}{\sum_{i=1}^{n} F} = \frac{860}{30} = 28.6$

$\lambda = \frac{1}{x} = \frac{1}{28.6} = 0.035$

The probability density function (pdf) is given by

$f(t) = \int \lambda e^{-\lambda t} \quad t>0$

Otherwise

$P[L \leq t < U] = \int_L^U P(t)dt = \int_L^U \lambda e^{-\lambda t} dt$

$= e^{-}e^{\lambda e}/L^U = -e^{-0.035t}/0^6$

$P[0 \leq t \leq 6] = -[e^{-0.035x\,6} - e^{-0.035x\,0}] = 0.189$

$P[7 \leq t \leq 13] = -[e^{-0.035x\,13} - e^{-0.035x\,7}] = 0.149$

$P[14 \leq t \leq 20] = -[e^{-0.035x\,20} - e^{-0.035x\,14}] = 0.117$

$P[21 \leq t \leq 27] = -[e^{-0.035x\,27} - e^{-0.035x\,21}] = 0.091$

$P[28 \leq t \leq 34] = -[e^{-0.035x\,34} - e^{-0.035x\,28}] = 0.071$

$P[35 \leq t \leq 41] = -[e^{-0.035x41} - e^{-0.035x\,35}] = 0.056$

$P[42 \leq t \leq 48] = -[e^{-0.035x\,48} - e^{-0.035x\,42}] = 0.043$

$P[49 \leq t \leq 55] = -[e^{-0.035x\,55} - e^{-0.035x\,49}] = 0.033$

$P[56 \leq t \leq 62] = -[e^{-0.035x\,62} - e^{-0.035x\,56}] = 0.027$

$P[63 \leq t \leq 69] = -[e^{-0.035x\,69} - e^{-0.035x\,63}] = 0.021$

$P[70 \leq t \leq 76] = -[e^{-0.035x\,76} - e^{-0.035x\,70}] = 0.017$

$P[77 \leq t \leq 83] = -[e^{-0.035x\,83} - e^{-0.035x\,77}] = 0.012$

$P[84 \leq t \leq 90] = -[e^{-0.035x\,90} - e^{-0.035x\,84}] = 0.01$

$P[91 \leq t \leq 97] = -[e^{-0.035x\,97} - e^{-0.035x\,91}] = 0.007$

$P[98 \leq t \leq 104] = -[e^{-0.035x\,104} - e^{-0.035x\,98}] = 0.006$

$P[105 \leq t \leq 111] = -[e^{-0.035x\,111} - e^{-0.035x\,105}] = 0.004$

$P[112 \leq t \leq 118] = -[e^{-0.035x\,118} - e^{-0.035x\,112}] = 0.003$

$P[119 \leq t \leq 125] = -[e^{-0.035x\,125} - e^{-0.035x\,119}] = 0.002$

Chi-Square Calculated

$X^2 = \sum_{i=1}^{n} \frac{(oi-ei)^2}{ei}$

$X^2 = 36.578$

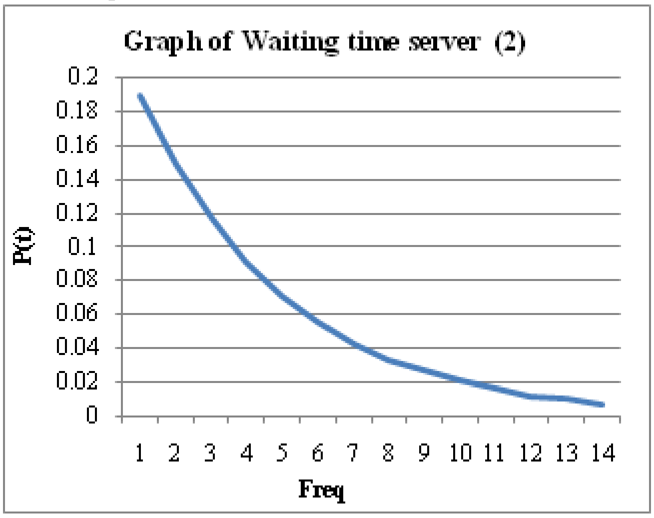
Therefore

$X^2_{cal} = 36.578$ and $X^2_{k-1-5} = X^2_{18-1-0.05} = X^2_{16(0.05)} = 26.30$

Conclusion

Since $X^2_{cal} = 36.578 > X^2_{16(0.05)} = 26.30$

We reject H_o and conclude that the inter-arrived time follows exponential distribution.



Fitting The Distribution Of Service Time Server 1

Hypothesis

H_o: Service time follows exponential distributions

H_1: Service time does not follow exponential distribution.

Level of Significant

'$\alpha = 0.05$

Test statistic $X^2 = \frac{(Oi-ei)}{e}$ $X^2_{k-s-1}(\alpha)$

Reject H_o if $X^2_{cal} > X^2_{k-s-1}(\alpha)$ accept if otherwise

4. Service Time Analysis for Server One

Table 3

INTERVAL	F_i	X_i	F_iX_i	P(t)	$e_{i=NP(t)}$	$(O_i-e_i)^2/e_i$
0≤t≤1	0	0.5	0	0.2883	8.649	8.649
2≤t≤3	23	2.5	57.5	0.146	4.38	79.1563
4≤t≤5	7	4.5	31.5	0.074	2.22	10.2921
	30					98.097S

Mean $X = \frac{\sum_{i=1}^{n} f_x}{\sum_{i=1}^{n} F} = \frac{89}{30} = 2.97$

$\lambda = \frac{1}{x} = \frac{1}{2.97} = 0.34$

The probability density function (pdf) is given by

$f(t) = \int \lambda e^{-\lambda t} \quad t>0$

Otherwise

$P[L \leq +U < U] = \int_L^U P(t)dt = \int_L^U \lambda e^{-\lambda t}\, dt$

$= e^{-}e^{\lambda e}/L^U = -e^{-0.34t}/0^1$

$P[0 \leq + \leq 1] = -[e^{-0.345 \times 1} - e^{-0.34 \times 0}] = 0.2883$

$P[2 \leq + \leq 3] = -[e^{-0.34 \times 3} - e^{-0.34 \times 2}] = 0.146$

$P[4 \leq + \leq 5] = -[e^{-0.34 \times 5} - e^{-0.34 \times 4}] = 0.07402$

Chi-Square Calculated

$X^2 = \sum_{i=1}^{n} \frac{(oi-ei)^2}{ei}$

$X^2 = \frac{(0-8.649)^2}{8.649} + \frac{(23-4.38)^2}{4.38} + \frac{(7-2.22)^2}{2.22}$

$= 8.649 + 79.15625 + 10.2921 = 98.097$

Therefore

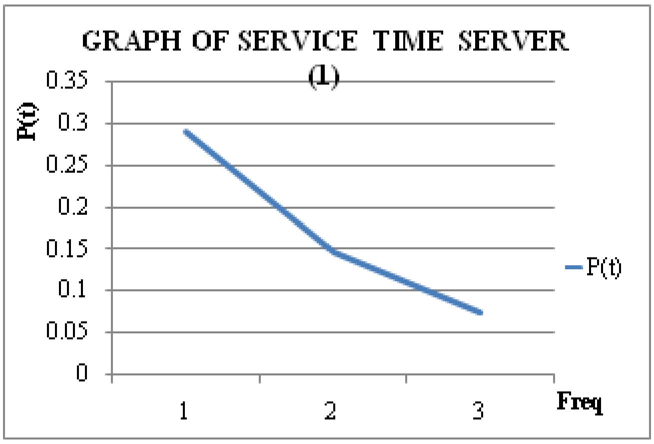
$X^2_{cal} = 98.097$

$X^2_{k-1-5} = X^2_{3-1-0.05} = X^2_{1(0.05)} = 3.84$

Conclusion

Since $X^2_{cal} = 98.097 > X^2_{1\,(0.05)} = 3.84$

We reject H_o and conclude that the inter-arrived time follows exponential distribution.



Fitting the Distribution of Service Time Server 2

Hypothesis

H_o: Service time follows exponential distributions

H_1: Service time does not follow exponential distribution.

Level of Significant

'$\alpha = 0.05$

Test statistic $X^2 = \frac{(Oi-ei)}{e}$ $X^2_{k-s-1}(\alpha)$

Reject H_o if $X^2_{cal} > X^2_{k-s-1}(\alpha)$accept if otherwise

5. Service Time Analysis for Server Two

Table 4

INTERVAL	F_i	X_i	F_iX_i	P(t)	$e_{i=NP(t)}$	$(O_i-e_i)^2/e_i$
0≤t≤1	0	0.5	0	0.2883	8.649	8.649
2≤t≤3	23	2.5	57.5	0.146	4.38	79.1563
4≤t≤5	7	4.5	31.5	0.074	2.22	10.2921

Mean $X = \frac{\sum_{i=1}^{n} f_x}{\sum_{i=1}^{n} F} = \frac{115}{30} = 3.83$

$\lambda = \frac{1}{x} = \frac{1}{3.83} = 0.26$

The probability density function (pdf) is given by

$f(t) = \int \lambda e^{-\lambda t} \quad t>0$

Otherwise

$P[L \leq +U < U] = \int_L^U P(t)dt = \int_L^U \lambda e^{-\lambda t}\, dt$

$= e^{-}e^{\lambda e}/L^U = -e^{-0.26t}/0^1$

$P[0 \leq t \leq 1] = -[e^{-0.26x\,1} - e^{-0.26x0}] = 0.229$

$P[2 \leq t \leq 3] = -[e^{-0.26x\,3} - e^{-0.26x\,2}] = 0.137$

$P[4 \leq t \leq 5] = -[e^{-0.26x\,5} - e^{-0.26x\,4}] = 0.080$

$P[6 \leq t \leq 7] = -[e^{-0.26x\,7} - e^{-0.26x\,6}] = 0.048$

$P[8 \leq t \leq 9] = -[e^{-0.26x\,9} - e^{-0.26x\,8}] = 0.029$

Chi-Square Calculated

$X^2 = \sum_{i=1}^{n} \frac{(oi-ei)^2}{ei}$

Therefore

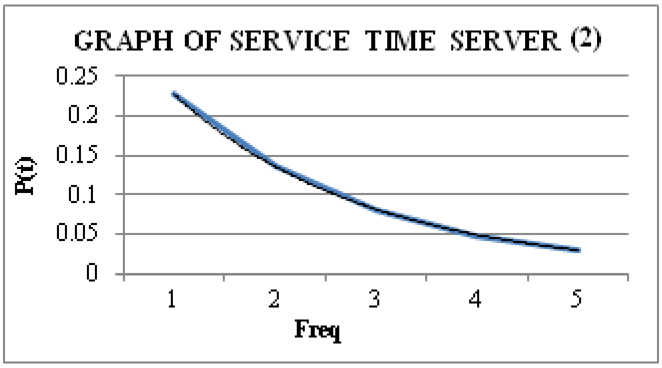
$X^2_{cal} = 92.4$

$X^2_{k-1-5} = X^2_{5-1-0.05} = X^2_{3(0.05)} = 7.81$

Conclusion

Since $X^2_{cal} = 92.4 > X^2_{1\,(0.05)} = 7.81$

This shows that the inter-arrived time follows exponential distribution.



Inter-Arrival Time Analysis For Server One

Table 5

INTERVAL	F_i	X_i	F_iX_i	P(t)	$e_{i=NP(t)}$	$(O_i-e_i)^2/e_i$
0≤t≤6	11	3	33	0.2724	8.4444	0.7734
7≤t≤13	5	10	50	0.1879	5.8249	0.1168
14≤t≤20	7	17	119	0.1297	4.0207	2.2076
21≤t≤27	2	24	48	0.0895	2.7745	0.2162
28≤t≤34	0	31	0	0.0618	1.9158	1.9158
35≤t≤41	1	38	38	0.0427	1.3237	0.0792
42≤t≤48	1	45	45	0.0293	0.9083	0.0093
49≤t≤55	1	52	52	0.0203	0.6293	0.2184
56≤t≤62	2	59	118	0.014	0.434	5.651
63≤t≤69	0	66	0	0.0097	0.3007	0.3007
70≤t≤76	0	73	0	0.0097	0.3007	0.3007
77≤t≤83	0	80	0	0.0045	0.1395	0.1395
84≤t≤90	1	87	87	0.0032	0.0992	8.1798
	31		590			20.1084

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Mean X = $\frac{\sum_{i=1}^{n} f_x}{\sum_{i=1}^{n} F}$ = $\frac{590}{31}$ = 19

$\lambda = \frac{1}{x} = \frac{1}{19} = 0.053$

The probability density function (pdf) is given by

$f(t) = \int \lambda e^{-\lambda t}$ t>0

Otherwise

$P[L \leq t < U] = \int_L^U P(t)dt = \int_L^U \lambda e^{-\lambda t}\, dt$

$= e^{-}e^{\lambda e}/L^U = -e^{-0053t}/0^6$

$P[0 \leq t \leq 6] = -[e^{-0.053 \times 6} - e^{-0.053 \times 0}] = 0.2724$

$P[7 \leq t \leq 13] = -[e^{-0.053 \times 13} - e^{-0.053 \times 7}] = 001879$

$P[14 \leq t \leq 20] = -[e^{-0.053 \times 20} - e^{-0.053 \times 14}] = 0.1297$

$P[21 \leq t \leq 27] = -[e^{-0.053 \times 27} - e^{-0.053 \times 21}] = 0.0895$

$P[28 \leq t \leq 34] = -[e^{-0.053 \times 34} - e^{-0.053 \times 28}] = 0.0618$

$P[35 \leq t \leq 41] = -[e^{-0.053 \times 41} - e^{-0.053 \times 35}] = 0.0427$

$P[42 \leq t \leq 48] = -[e^{-0.053 \times 42} - e^{-0.053 \times 48}] = 0.0293$

$P[49 \leq t \leq 55] = -[e^{-0.053 \times 55} - e^{-0.053 \times 49}] = 0.0203$

$P[56 \leq t \leq 62] = -[e^{-0.053 \times 62} - e^{-0.053 \times 56}] = 0.014$

$P[63 \leq t \leq 69] = -[e^{-0.053 \times 69} - e^{-0.053 \times 63}] = 0.0097$

$P[70 \leq t \leq 76] = -[e^{-0.053 \times 76} - e^{-0.053 \times 70}] = 0.0097$

$P[77 \leq t \leq 83] = -[e^{-0.053 \times 83} - e^{-0.053 \times 77}] = 0.0045$

$P[84 \leq t \leq 90] = -[e^{-0.053 \times 90} - e^{-0.053 \times 84}] = 0.0032$

Chi-Square Calculated

$X^2 = \sum_{i=1}^{n} \frac{(oi-ei)^2}{ei}$

$X^2 = \frac{(11-8.4444)^2}{8.44444} + \frac{(5-5.8249)^2}{5.8249} + \frac{(7-4.0207)^2}{4.0207} + \frac{(2-2.7745)^2}{2.7745} + \frac{(0-1.9158)^2}{1.9156} + \frac{(1-1.32372)^2}{1.3237} + \frac{(1-0.9083)^2}{0.9083} + \frac{(1-0.6293)^2}{0.6293} + \frac{(2-0.434)^2}{0.434} + \frac{(0-0.3007)^2}{0.3007} + \frac{(0.03007)^2}{0.3007} + \frac{(0.01395)^2}{0.1395} + \frac{(1-0.0992)^2}{0.0992}$

$X^2 = 0.7734 + 0.1168 + 2.2076 + 0.2162 + 1.9158 + 0.0792 + 0.0093 + 0.2184 + 5.651 + 0.3007 + 0.3007 + 0.1395 + 8.1798$

$X^2 = 20.1084$

Therefore

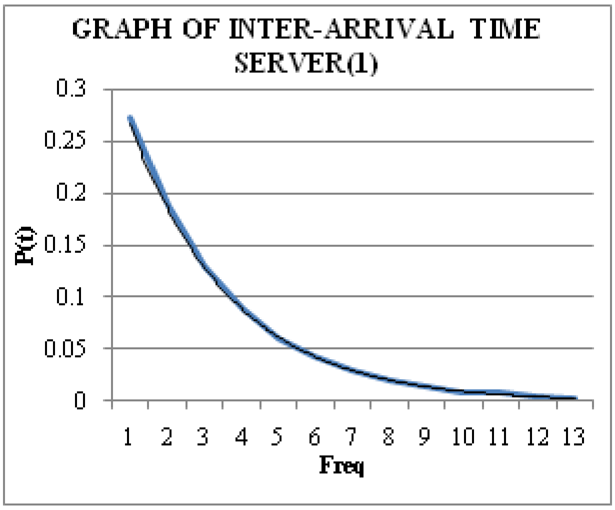
$X^2_{cal} = 20.1084$

$X^2_{k-1-5} = X^2_{13-1-0.05} = X^2_{11(0.05)} = 19.68$

Conclusion

Since $X^2_{cal} = 20.1084 > X^2_{11(0.05)} = 19.68$

We reject H_o and conclude that the inter-arrived time follows exponential distribution.



6. Inter-Arrival Time Analysis for Server Two

Table 6

INTERVAL	F_i	X_i	F_iX_i	P(t)	$e_{i=NP(t)}$	$(O_i-e_i)^2/e_i$
0≤t≤5	6	3	18	0.0405	1.215	18.8446
6≤t≤10	4	8	32	0.1188	3.564	0.0533
11≤t≤15	2	13	26	0.0966	2.898	0.2783
16≤t≤20	2	18	36	0.0786	2.358	0.0544
21≤t≤25	1	23	23	0.064	1.920	0.4408
26≤t≤30	5	28	140	0.0521	1.563	7.5578
31≤t≤35	2	33	66	0.0423	1.269	0.4211
36≤t≤40	3	38	114	0.0345	1.035	3.7306
41≤t≤45	1	43	43	0.028	0.840	0.0305
46≤t≤50	1	48	48	0.022	0.684	0.1459
51≤t≤55	0	53	0	0.0185	0.555	0.555
56≤t≤60	2	58	116	0.0184	0.552	3.7984
61≤t≤65	1	63	63	0.0122	0.366	1.0982
	30		725			37.0089

Mean X = $\frac{\sum_{i=1}^{n} f_x}{\sum_{i=1}^{n} F}$ = $\frac{725}{30}$ = 24.16

$\lambda = \frac{1}{x} = \frac{1}{24.16} = 0.0413$

The probability density function (pdf) is given by

$f(t) = \int \lambda e^{-\lambda t}$ t>0

Otherwise

$P[L \leq +U < U] = \int_L^U P(t)dt = \int_L^U \lambda e^{-\lambda t}\, dt$

$= e^{-}e^{\lambda e}/L^U = -e^{-0.0413t}/0^5$

$P[0 \leq t \leq 5] = -[e^{-0.0413 \times 5} - e^{-0.0413 \times 0}] = 0.0405$

$P[6 \leq t \leq 10] = -[e^{-0.0413 \times 10} - e^{-0.0413 \times 6}] = 0.1188$

$P[11 \leq t \leq 15] = -[e^{-0.0413 \times 15} - e^{-0.0413 \times 11}] = 0.0966$

$P[16 \leq t \leq 20] = -[e^{-0.0413 \times 20} - e^{-0.0413 \times 16}] = 0.0786$

$P[21 \leq t \leq 25] = -[e^{-0.0413 \times 25} - e^{-0.0413 \times 21}] = 0.064$

$P[26 \leq t \leq 30] = -[e^{-0.0413 \times 30} - e^{-0.0413 \times 26}] = 0.0521$

$P[31 \leq t \leq 35] = -[e^{-0.0413 \times 35} - e^{-0.0413 \times 31}] = 0.0423$

$P[36 \leq t \leq 40] = -[e^{-0.0413 \times 40} - e^{-0.0413 \times 36}] = 0.0345$

$P[41 \leq t \leq 45] = -[e^{-0.0413 \times 45} - e^{-0.0413 \times 36}] = 0.028$

$P[46 \leq t \leq 50] = -[e^{-0.0413 \times 50} - e^{-0.0413 \times 46}] = 0.0228$

$P[51 \leq t \leq 55] = -[e^{-0.0413 \times 55} - e^{-0.0413 \times 51}] = 0.0185$

$P[56 \leq t \leq 60] = -[e^{-0.0413 \times 60} - e^{-0.0413 \times 56}] = 0.0184$

$P[61 \leq t \leq 65] = -[e^{-0.0413 \times 65} - e^{-0.0413 \times 61}] = 0.0122$

Chi-Squarse Calculated

$X^2 = \sum_{i=1}^{n} \frac{(oi-ei)^2}{ei}$

$X^2 = 37.0089$

Therefore

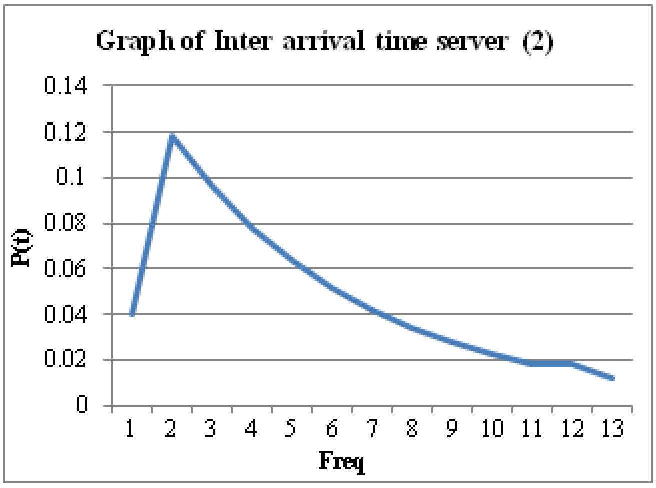
$X^2_{cal} = 37.0089$

$X^2_{k-1-5} = X^2_{13-1-0.05} = X^2_{11(0.05)} = 19.68$

Conclusion

Since $X^2_{cal} = 37.0089 > X^2_{11(0.05)} = 19.68$

We reject H_o and conclude that the inter-arrived time follows exponential distribution.



The fitting of the data shows that the service time and waiting time follows exponential distribution.

7. Data Analysis And Presentation

Data analysis of a queue at Lagos waste management authority (Igando dump site) as a case study, the estimation of the average service time and average arrival rate was computed and their performance measures as well inter-arrival time.

Estimate of the Average Arrival Rate per Day

The average arrival rates per day for each of the three days are as follows:

Server I	Server II
For day one	For day one
31	30
For day two	For day two
32	32
For day three	For day three
30	30

Server 1 (λ1) $= \frac{31+32+30}{3} = 31$

Server 2 (λ2) $= \frac{30+32+30}{3} = 30.6$

$\lambda t = \frac{\lambda 1+\lambda 2}{2} = \frac{31+30.6}{2} = 30.8$

Estimate of the Average service time

The average service time per minute for each of the 3 days are as follows:

Server I	Server II
For day one	For day one
3.09	3.13
For day two	For day two
3.31	3.22
For day three	For day three
4.26	3.93

Server 1 (μ1) $= \frac{3.09+3.31+4.26}{3} = 3.53$

Server 2 (μ2) $= \frac{3.13+3.22+3.93}{3} = 3.43$

$\mu t = \frac{\mu 1+\mu 2}{2} = \frac{3.55+3.43}{2} = 3.49$

Estimate of inter-arrival time

The inter-arrival time between trucks in each queue are as follows:

Server I	Server II
For day one	For day one
19.0	23.8
For day two	For day two
24.5	23.2
For day three	For day three
24.8	24.5

Server 1 $= \frac{19.0+24.5+24.8}{3} = 22.76$

Server 2 $= \frac{23.8+23.2+24.5}{3} = 23.83$

(t) $= \frac{22.76+23.83}{2} = 23.29$

$\frac{1}{t} = 0.043$

Estimate of Performance Measures

$\lambda t = \frac{\lambda 1+\lambda 2}{2} = \frac{31+30.6}{2} = 30.8$

$\mu t = \frac{\mu 1+\mu 2}{2} = \frac{3.55+3.43}{2} = 3.49$

Expected number of trucks waiting in the queue

$$L_q = \left[\frac{1}{(S-1)!}\left(\frac{\lambda}{\mu}\right)^S \frac{\lambda . S\mu}{(S\mu-\lambda)^2}\right] P_o$$

$$L_q = \left[\frac{1}{2-1}\left(\frac{30.8}{3.49}\right)^2 \frac{30.8(3.49)}{(2(3.49)-30.8)^2}\right] 7.8$$

$= (77.88)\ (0.189)\ 7.8 = 114.81$

Expected number of trucks in the system

$L_s = L_q + \frac{\lambda}{\mu}$

$114.81 + \frac{30.8}{3.49}$

$= 114.81 + 8.825$

$L_s = 123.64$

Expected waiting time of a truck in the queue

$W_q = \frac{L_q}{\lambda} = \frac{114.81}{30.8} = 3.73 min$

Expected waiting of a trucks in the system

$W_s = W_q + \frac{1}{\mu} = 3.73 + \frac{1}{3.49} = 3.73 + 0.286 = 4.02$

Probability that the system severs are idle

$P_o = P_{rob}$ [system is empty (idle)] $= 1 - \frac{\lambda}{\mu}$

$= 1 - 8.825$

$= 7.8$

Probability that all severs are simultaneously busy

$$P(n=s) = \frac{1}{S!}\left[\frac{\lambda}{\mu}\right]^S \left[\frac{S\mu}{S\mu-\lambda}\right] P_o$$

$$= \frac{1}{2!}\left[\frac{30.8}{3.49}\right]^2 \left[\frac{2(3.49)}{2(3.49)-30.8}\right] 7.8$$

$= 0.5\ (77.88)\ (0.29)\ 7.8$

$P(n=s) = 88.08$

Traffic Intensity

$$\rho = \frac{\text{Mean of service time}}{\text{Number of service X mean of inter-arrival}}$$

$= \frac{\lambda}{S\mu}$

Recall, mean inter-arrival time (t) = 23.29

inter-arrival rate $= \frac{1}{t} = \frac{1}{23.29} = 0.043$

mean service time = μ = 3.49

service rate $= \frac{1}{\mu} = \frac{1}{3.49} = 0.286$

Note! Traffic intensity is the ratio of the mean service time to the mean inter-arrival time, for an arrival rate λ and service rate μ; it defines the minimum no of servers to cope with the arriving traffic.

$\therefore \frac{0.043}{2(0.286)} = 0.0075$

Probability that an arrival truck will have to wait (system being busy)

$1\text{-}P_o\left(\frac{\lambda}{⍰}\right)$

$1 - 7.8\ (8.83)$

$= 67.874$

8. Summary of Results

Parameter	Symbol	Results
Mean of inter-arrival time	$(t) = \frac{1}{\lambda}$	23.29
Variance	$(t^2) = \frac{1}{\lambda^2}$	542.42
Mean of arrival rate	λ	30.8
Mean of service time	⍰	3.49
Service rate	$\frac{1}{⍰}$	0.286
Variance	$\frac{1}{⍰^2}$	0.082

9. Summary

The queue is an m/m/s with s=2 severs, the traffic intensity was calculated to be 0.0075 since the value is less than 1, it indicates that the arrival rate is within the service capacity of 2 severs. The data analysis shows that the inter-arrival time is exponentially distributed with parameter λ = 30.8 and service time following exponential distribution with parameter ⍰ = 3.49. The expected waiting time of truck in the system was calculated to be 4hrs. The waiting time of truck in the queue is 3hrs.

10. Conclusion

The LAWMA Igando dump site, Lagos State is computerized, since the traffic intensity computed was less than 1, this shows arrival rate is within service capability of two servers. The work rate of the two severs is 67%. The value is expected to keep falling as more idle service point are been engaged. λ = 30.8 indicate that more trucks arrived within an interval time of ⍰ = 3.49 indicate that little time is been used attending to trucks within an average queue length of 9 trucks.

11. Recommendation

Based on the stochastic analysis carried out in LAWMA Igando dump site, Lagos State and the result obtained, the dump site need more tractors to pull down waste so to enable the server been fast in attending to more trucks within a time interval. Hence, the management of the dump site should provide more tractors to enable the site to have fewer queue.

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Appendix

The Data from 26th -28th November, 2014

Day One

S/n	*Arrival time*		*Service time*		*Departure time*		*Waiting time (min)*		*Inter-arrival time*		*Inter/service time*	
1.	5:45	5:49	7:20	7:20	7:23:	7:24	98	96	-	-	3	4
2.	5:54	5:55	7:22	7:23	7:26	7:26	92	91	9	4	4	3
3.	5:58	6:15	7:25	7:27	7:27	7:29	89	74	4	6	2	2
4.	6:16	6:18	7:30	7:33	7:33	7:38	77	80	18	20	3	5
5.	6:19	6:21	7:42	7:45	7:45	7:48	86	87	3	3	3	3
6.	6:21	6:28	7:49	7:53	7:51	7:56	90	88	2	7	3	3
7.	6:30	6:36	7:58	8:10	8:00	8:12	90	90	9	8	2	2
8.	6:50	7:10	10:20	10:22	10:23	10:26	213	196	20	34	3	4
9.	7:15	7:16	10:26	10:29	10:29	10:32	194	196	25	6	3	3
10.	7:19	7:35	10:33	10:35	10:37	10:39	198	184	4	19	4	4
11.	7:37	7:40	10:45	10:47	10:50	10:53	193	193	18	5	5	6
12.	8:15	8:30	12:13	12:14	12:15	12:16	240	226	38	50	2	2
13.	9:12	9:18	12:16	12:17	12:18	12:19	186	181	57	48	2	2
14.	10:36	11:00	12:18	12:20	12:22	12:24	106	84	84	102	4	4
15.	11:04	11:05	12:23	12:27	12:27	12:30	83	85	62	5	4	3
16.	11:08	11:12	2:30	12:32	12:33	12:35	85	83	4	7	3	3
17.	11:20	11:20	3:08	3:09	3:10	3:12	490	488	12	8	2	3
18.	12:45	1:32	3:12	3:13	3:15	3:17	570	105	25	132	3	4
19.	1:59	2:15	3:15	3:18	3:18	3:21	79	56	14	43	3	3
20.	2:50	3:00	3:21	3:25	3:23	3:27	33	27	9	45	2	2
21.	3:08	3:09	5:00	5:02	5:03	5:05	115	116	18	9	3	3
22.	3:15	3:17	5:07	5:10	5:09	5:13	114	116	7	8	2	3
23.	4:10	4:10	5:13	5:18	5:16	5:20	66	70	55	53	3	2
24.	4:11	4:15	5:20	5:24	5:23	5:27	72	72	1	5	3	3
25.	4:15	4:20	5:28	6:22	5:31	6:26	76	126	4	5	3	4
26.	5:08	5:05	6:25	6:27	6:27	6:30	87	85	45	45	2	3
27.	5:16	5:17	6:28	6:31	6:31	6:33	75	76	16	2	3	2
28.	5:18	5:22	6:35	6:38	6:37	6:41	79	79	2	5	2	3
29.	5:23	5:40	6:50	6:54	6:53	6:57	90	77	5	18	3	3
30.	5:42	5:42	6:58	7:14	7:03	7:17	81	96	19	2	5	3
31.	5:43	-	7:15	-	7:19	-	96	-	1	-	4	-

Day Two

S/n	*Arrival time*		*Service time*		*Departure time*		*Waiting time (min)*		*Inter-arrival time*		*Inter/service time*	
1.	5:49	5:53	7:15	7:15	7:18	7:17	89	84	-	4	3	2
2.	6:14	6:26	7:18	7:17	7:21	7:20	67	54	25	33	3	3
3.	6:34	6:38	7:21	7:20	7:24	7:23	50	45	20	12	3	3
4.	6:43	6:50	7:22	7:24	7:27	7:27	44	37	9	12	5	3
5.	7:00	7:06	7:25	7:27	7:30	7:30	30	24	17	16	5	3
6.	7:15	7:19	7:30	7:31	7:32	7:33	17	14	15	13	2	2
7.	7:20	7:21	7:34	7:35	7:37	7:37	17	16	5	2	3	2
8.	7:21	7:23	8:42	8:42	8:45	8:46	84	83	1	2	3	4
9.	7:25	7:32	8:45	8:44	8:49	8:48	84	76	4	9	4	4
10.	7:33	7:34	8:47	8:46	8:50	8:51	77	77	8	2	3	5
11.	7:36	7:40	8:49	8:49	8:53	8:55	77	75	3	6	4	6
12.	8:02	8:07	8:52	8:53	8:55	8:55	53	48	26	27	3	2
13.	9:15	9:17	10:00	10:01	10:3	10:04	48	47	73	70	3	3
14.	9:18	9:20	10:03	10:04	10:07	10:08	49	48	3	3	4	4
15.	9:32	10:07	10:07	10:06	10:10	10:10	38	3	14	47	3	4
16.	10:09	10:9	10:10	10:09	10:14	10:13	5	4	37	2	4	4
17.	10:42	10:43	10:45	10:46	10:48	10:15	6	7	33	34	3	4
18.	10:43	10:45	10:48	10:49	10:51	10:53	8	8	1	2	3	4
19.	10:47	10:52	10:51	10:55	10:54	10:59	7	7	4	7	3	4
20.	10:58	11:10	11:01	11:11	11:04	11:13	6	3	11	18	3	2
21.	11:14	11:15	11:17	11:17	11:20	11:19	6	4	16	5	3	2
22.	11:18	11:42	11:20	11:43	11:23	11:47	5	5	4	27	3	4
23.	11:48	11:56	11:50	11:58	11:54	12:00	6	4	30	14	4	2
24.	12:20	1:42	12:21	1:42	12:25	1:46	5	6	40	104	4	4
25.	1:56	2:22	1:58	3:15	2:01	3:17	5	55	96	42	3	2
26.	2:32	2:50	3:15	3:18	3:19	3:21	47	31	36	28	3	3
27.	3:13	3:15	3:19	3:21	3:22	3:25	9	10	41	25	3	4
28.	3:24	3:32	3:26	3:35	3:29	3:38	5	6	11	17	3	3
29.	4:15	4:18	4:17	4:20	4:20	4:23	5	5	51	46	3	3
30.	5:10	5:12	5:20	5:21	5:23	5:23	13	11	55	54	3	2
31.	5:42	5:55	5:50	6:00	5:54	6:03	12	8	32	43	4	3
32	6:10	6:11	6:15	6:17	6:18	6:20	8	9	28	16	3	3

Day Three

S/n	Arrival Time		Service time		Departure time		Waiting time (min)		Inter-arrival time		Inter/service time	
1.	5:43	5:59	7:12	7:13	7:15	7:16	92	77	-	16	3	3
2.	6:12	6:18	7:15	7:77	7:17	7:20	65	76	29	19	2	3
3.	6:32	6:45	7:17	7:20	7:22	7:22	50	37	20	27	4	2
4.	6:58	7:13	17:22	7:24	7:25	7:27	27	14	26	28	3	3
5.	7:30	7:43	7:32	7:45	7:36	7:48	6	5	32	30	4	3
6.	7:59	8:16	8:00	10:18	8:08	10:23	9	122	29	33	8	5
7.	8:39	8:56	10:19	10:21	10:23	10:26	114	90	40	40	4	4
8.	9:31	9:52	10:22	10:25	10:26	10:29	55	37	52	56	4	3
9.	10:10	10:39	10:26	10:41	10:30	10:44	20	5	39	47	4	9
10.	10:59	11:34	11:02	11:40	11:09	11:49	10	15	49	45	7	5
11.	11:56	12:38	11:58	12:45	12:03	12:50	7	12	57	64	5	4
12.	12:57	1:16	1:15	1:20	1:20	1:24	83	8	61	38	5	5
13.	1:39	2:16	1:40	2:30	1:45	12:35	6	19	42	60	5	5
14.	2:18	2:22	2:31	2:33	2:37	2:38	19	16	39	6	6	4
15.	2:22	2:24	2:35	2:38	2:40	2:42	28	18	12	2	5	4
16.	2:28	2:29	2:40	2:42	2:48	2:46	20	17	6	5	8	3
17.	2:42	3:05	2:43	3:20	2:50	3:23	8	18	14	36	7	6
18.	3:19	3:20	3:30	3:50	3:33	3:56	14	36	37	15	3	5
19.	3:23	3:25	3:52	3:56	3:55	4:00	32	35	4	5	3	4
20.	3:27	3:32	4:10	4:10	4:30	4:14	46	42	4	11	3	4
21.	4:00	4:02	4:13	4:12	4:15	4:16	15	14	33	30	2	4
22.	4:09	4:11	4:15	4:16	4:18	4:19	9	8	9	9	3	3
23.	4:13	4:15	4:20	4:22	4:24	4:26	11	11	4	4	4	4
24.	4:22	4:50	4:30	4:53	4:33	4:57	11	7	9	35	3	4
25.	4:53	5:03	5:00	5:08	5:04	5:11	11	8	31	13	4	3
26.	5:15	5:32	5:33	5:35	5:36	5:38	21	6	22	29	3	3
27.	5:37	5:42	6:15	6:18	6:19	6:21	42	39	22	10	4	3
28.	6:03	6:03	6:20	6:23	6:24	6:26	21	23	26	21	4	3
29.	6:05	6:07	6:24	6:26	6:28	6:29	23	22	2	4	4	3
30.	6:07	6:09	6:28	6:31	6:32	6:35	25	26	2	2	4	4