International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

M^X/G/1 Queue with Negative Arrival and Breakdown

M. I. Afthab Begum[1], P. Fijy Jose[2], S. Bama[3]

[1]Professor, Department of Mathematics, Avinashilingam University, Tamil Nadu, India

[2]Research Scholar, Department of Mathematics, Avinashilingam University, Tamil Nadu, India

[3]Research Scholar, Department of Mathematics, Avinashilingam University, Tamil Nadu, India

Abstract: ***The author considers a batch arrival queueing system in which two types of customers (namely positive and negative) arrive. The arrival of a negative customer to a queueing system removes one positive customer who is in service if any. A cycle starts whenever the system empties and the server is turned off and stays idle in the system. The idle period ends as soon as a batch of positive customers arrives for services. The positive customer who is in the head of queue will join the service immediately. The arrival of negative customers causes the server to breakdown. Whenever the server fails, it is sent to repair facility. The arriving negative customers during the idle period are assumed to be lost. Immediately after the server is fixed the customer waiting in the head of the queue is taken up for service. In the present paper, the author analyzes steady state behavior of the system size distribution of M^X/G/1 queue with negative customers. The probability generating function (PGF) of the stationary queue length are presented and performance measures are obtained. Numerical examples to illustrate the effect of the parameters are also provided.***

Keywords: M^X/G/1, Negative customer, Break down, Probability generating functions

1. Introduction

Over the last two decades, queueing systems with negative arrivals have been studied extensively and applied to computer networks, communication systems and manufacturing systems. The arrival of a negative customer to a queueing system causes one positive customer to be removed if any is present in service. Positive customers enter a queue and receive service as ordinary queueing network customers. A negative customer will vanish if it arrives to an empty queue, and it will reduce by one the number of positive customers in the system. Negative customers cannot accumulate in a queue and do not receive service. Positive customers which leave a queue to enter another queue can become negative or remain positive. For instance, in computer networks, if a virus enters a node, one or more files may be infected and the system manager may have to go through a number of backup to recover the infected files. In some cases, they may not be recoverable. A virus may originate from outside the network, e.g. by an electronic mail. A virus cannot be modeled as a customer, since it does not require service.

2. Literature Survey

The notion of negative customers was introduced by Gelenbe (1989)[8]. The M/M/1 queue with negative customers was investigated by Harrison and Pitel (1993)[10], who extended it, to the M/G/1 queue (1996)[11]. Yang and Chae(2001)[16] extended it to the GI/M/1 queue. Applications of negative arrivals to queueing networks can be found in a survey by Artalejo (2000)[1]. Negative customers can be divided into various cases based on the assumption of the killing mechanism. Two typical removal disciplines considered in the literature are the removal of a customer at the end (RCE) and the removal of a customer at the head (RCH)[] by Harrison and Pitel (1993,1996)[10],[11]. In addition to the RCE and RCH disciplines, Chakka and Harrison (2001)[5] addressed the RCE-immune discipline. Under the RCE-immune case, negative arrivals have no effect if there is only one ordinary customer in the system. Park et al. (2009)[15] in their article, considered the Geo/G/1 queue with either RCH- type negative customers or disasters. Their aim is to widen the applicability of the Geo/G/1 queue to the operation of real systems. In addition, besides examining the queue length distribution, they looked at the system sojourn time distribution in the discrete- time queue. It surely helps to analyze the various queueing models in digital computer and communication systems, including mobile and Broad-Integrated Services Digital Networks (B-ISDN)based on Asynchronous Transfer Mode (ATM) technology, because these protocols are operated in terms of “slot” units, which represent equal time intervals (see Bruneel and Kim(1993))[4].Since the introduction of the concept of negative customers by Gelenbe (1989)[8], research on queueing systems with negative arrivals has been greatly motivated by some practical applications such as computer, neural networks, manufacturing systems and communication networks. For queues with negative customers, readers may refer to Gelenbe (1991)[9], Chao et al. (1999)[6], Artalejo (2000)[1] and Liand Zhao (2004)[14], and references therein. There is a lot of research on queueing system with negative arrivals. But most of these contributions considered continuous-time queueing model: Boucherie and Boxma (1995)[3], Jain and Sigman (1996)[13], Bayer and Boxma (1996)[2], all of them investigated the same M/G/1 model but with the different killing strategies for negative customers; Harrison et al. (2000)[12] considered the M/M/1 G-queues with breakdowns and repair; Yang and Chae (2001)[16] considered GI/M/1 model by using embedded Markov chain method. However, the GI/G/1 model is difficult to analyze. Later Zhou (2005)[17] considered GI/G/1 with negative arrival in discrete-time and the killing strategy is RCH (removal of customer at the head) and justified it by means of an example. Discrete-time queueing models are particularly appropriate to describe the various queueing related phenomena in digital computer and communication systems, including mobile B-ISDN networks based on ATM technology, due to the packetized nature of these transport protocols. The analysis of this model will be reduced to an equivalent discrete-time GI/G/1 queueing model without negative arrival.

In the present work the authors consider a $M^X/G/1$ queueing model with positive and negative arrivals.The positive customers arrive in batches following compound Poisson process and the negative customers arrive according to Poisson process. The arrival of negative customers will remove the customer in service (if any). The arrival of negative customer, to the empty system is considered as lost.The steady state behavior of the model is analysed using supplementary variable technique.Numerical computations are provided to study the effect of parameters on system performance measures which validates the analytical results.

3. Methods

Mathematical Analysis of the System

3.1Model Description

The System has the following specifications:

Idle Period

A cycle starts, whenever the system empties and the server is turned off and stays idle in the system. The idle period ends as soon as a batch of customers arrives for service. The time during which the server stays idle in the system is called idle period.

Arrival Pattern

Single server queueing system with two classes of customer‟s namely positive and negative customers with server breakdowns is considered.The positive customers arrive in batches in accordance with the time homogeneous Poisson process with group arrival rate λ. The batch size X is a random variable with probability distribution $\Pr(X=k)=g_k$, k=1, 2, 3… i.e.) the probability that a batch of k units arrive in an infinitesimal interval (t,t+h) is $\lambda g_k + O(h)$.

The customers who arrive and join the system form a single waiting line based on the order of the batches. It is further assumed that the customers within a batch are pre-ordered for service. The customers are served one by one according to the order in the queue.The negative customers arrive according to Poisson process with arrival rate α.

Busy Period

Busy period starts as soon a batch arrives and the firstcustomer join the service instantaneously. The service time of the customer is a random variable S following General Distribution S(x) with density function s(x).If a negative customer arrives during the busy period then it only removes the customer under service but also causes the server to breakdown. Whenever the server fails, it is sent to repair facility. If an arriving negative customer finds the server is idle then the customer is assumed to be lost.

Repairs

The repair time R is arbitrary distributed and has probability function R(t) and density function r(t). Immediately after the server is fixed it starts to serve the customer from the waiting line. It is assumed that the server after repair is as good as new. This type of service continues until the system becomes empty again. The system is denoted by $M^X/G/1/$ Negative arrival.

Notation

The following notations are used to discuss the model

N(t)= The system size at time t

λ = Group arrival rate

X= Group size random variable

$\Pr(X=k)=g_k$, k=1, 2, 3…

X(z)= Probability generating function of X.

The notations of random Variables (RV), Cumulative Distribution Function (CDF), Probability Density Function (PDF), Laplace-Stieltjes Transform (LST) and its k^{th} moments are listed below

Table 1

	RV	CDF	PDF	LST	k^{th} moment
Service time	S	S(x)	s(x)	$S^*(\theta)$	$E(S^k)$
Repair time	R	R(y)	r(y)	$R^*(y)$	$E(R^k)$

Where $F^*(\theta)=\int_0^\infty e^{-\theta x} f(x)dx=\int_0^\infty e^{-\theta x} d(F(x))$

Let $S^0(t)$, $R^0(t)$ denote the remaining service time and repair time at time t. Further the server states are denoted by the RV Y (t) at time t i.e. Y (t) = 0, 1, 2 respectively denotes, the server is in idle, busy, repair mode. The supplementary variables are introduced in order to obtain a Bivariate Markov Process $\{N(t),\delta(t)\}$ where N (t) denotes the system size random variable and $\delta(t)=0, S^0(t)$, $R^0(t)$ according as Y (t) = 0,1,2 respectively

Let $R_n(t)=\Pr\{N(t)=n, Y(t)=0\}$

$P_n(x,t)\,dt=\Pr\{N(t)=n,\ x\le S^0(t)\le x+dt,\ Y(t)=1\}$, $n\ge 1$

$B_n(x,t)\,dt=\Pr\{N(t)=n,\ x\le R^0(t)\le x+dt,\ Y(t)=2\}$, $n\ge 0$

Then, $R_n(t)$ denote the probability that there are n customers in the system at time t.

$P_n(x,t)$ denotes the probability that there are n customers in the system at time and the server is busy and the remaining service time x lies in the interval $[x, x+\Delta t]$

$B_n(x,t)$ denotes the probability that there are n customers in the system at time and the server is under repair and the remaining service time xlies in the interval $[x, x+\Delta t]$

International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Further $P_n(0)$, $B_n(0)$ denote the probability that there are n customers in the system at the termination of service time and repair times respectively.
Assuming that at steady state, probabilities are independent of time t, we have

$$\lim_{t\to\infty} P_n(x,t) = P_n(x)$$

$$\lim_{t\to\infty} B_n(x,t) = B_n(x),$$

$$\lim_{t\to\infty} \frac{\partial}{\partial y} B_n(x,t) = \frac{d}{dy} B_n(x)$$

$$\lim_{t\to\infty} R_n(t) = R_n$$

$$\lim_{t\to\infty} (\frac{\partial}{\partial t} P_n(x,t) = \frac{\partial}{\partial t} B_n(x,t)) = R_n$$

3.2 The System Size Distribution

The system size distribution following the argument of Cox (1955)[7], the steady state equations are obtained for queueing system using supplementary variable technique,

IdleState

$$\lambda R_0 = P_1(0) + B_0(0)$$

Busy State

$$-\frac{d}{dx} P_n(x) = -(\lambda+\alpha) P_n(x) + P_{n+1}(0) S(x)$$

$$+\lambda \sum_{k=1}^{n-1} P_{n-k}(x)\, g_k + R_0 \lambda g_n S(x) + B_n(0) S(x) \;\; n \geq 1$$

Breakdown state

$$-\frac{d}{dx} B_n(x) = -\lambda B_n(x) + \lambda \sum_{k=1}^{n} B_{n-k}(x)\, g_k$$

$$+\alpha \int_0^\infty P_{n+1}(w) dw r(x) \;\; n \geq 1$$

$$-\frac{d}{dx} B_0(x) = -\lambda B_0(x) + \alpha \int_0^\infty P_1(w) dw r(x)$$

The L.S.T of the steady state equations are obtained by using the definition of Laplace- Stieltjes Transformation and its properties. The L.S.T of the density functions are defined in previous table. The remaining notations of the L.S.T are listed below

Table 2

Probability Distribution	L.S.T
$P_n(x)$	$P_n^*(\theta) = \int_0^\infty e^{-\theta x} P_n(x)\, dx$
$B_n(x)$	$B_n^*(\theta)$

Thus the L.S.T of the equations with respect to x and y are given by,

$$\theta P_n^*(\theta) - P_n(0) = (\lambda+\alpha) P_n^*(\theta) - P_{n+1}(0) S^*(\theta)$$

$$-\lambda \sum_{k=1}^{n-1} P_{n-k}^*(\theta) g_k - \lambda R_0 g_n S^*(\theta) - B_n(0) S^*(\theta) \;\; n \geq 1$$

(1)

$$\theta B_n^*(\theta) - B_n(0) = \lambda B_n^*(\theta) - \lambda \sum_{k=1}^{n} B_{n-k}^*(\theta)\, g_k$$

$$-\alpha \int_0^\infty P_{n+1}(w) R^*(\theta) \;\; n \geq 1$$

$$\theta B_n^*(\theta) - B_n(0) = \lambda B_n^*(\theta) - \lambda \sum_{k=1}^{n} B_{n-k}^*(\theta)\, g_k$$

$$-\alpha P_{n+1}^*(0) R^*(\theta) \;\; n \geq 1 \quad (2)$$

$$\theta\, B_0^*(\theta) - B_0(0) = \lambda B_0^*(\theta) - \alpha \int_0^\infty P_1(w) R^*(\theta)$$

$n \geq 1$

$$\theta\, B_0^*(\theta) - B_0(0) = \lambda B_0^*(\theta) - \alpha P_1^*(0) R^*(\theta)$$

$n \geq 1$ (3)

4. Discussions

4.1 Probability Generating Functions

Now to obtain the partial PGFs of the number of customers in the system, the following partial PGFs are defined

$$P^*(z,\theta) = \sum_{n=1}^{\infty} P_n^*(\theta)\, z^n, \; P(z,0) = \sum_{n=1}^{\infty} P_n(0)\, z^n$$

$$B^*(z,\theta) = \sum_{n=1}^{\infty} B_n^*(\theta)\, z^n, \; B(z,0) = \sum_{n=1}^{\infty} B_n(0)\, z^n$$

Multiplying the corresponding equations by suitable powers of z and adding, the partial PGFS are obtained through some algebraic manipulations. The identity $\sum_{n=2}^{\infty} z^n \sum_{k=1}^{n-1} a_{n-k} b_k = \left(\sum_{n=1}^{\infty} a_n z^n\right)\left(\sum_{n=1}^{\infty} b_n z^n\right)$ is used to derive the PGFs.

Multiplying the equation (2) and (3) by suitable powers of z and then adding $n = 0$ to ∞ we get,

$$\theta \sum_{n=0}^{\infty} B_n^*(\theta) z^n - \sum_{n=0}^{\infty} B_n(0) z^n = \lambda \sum_{n=0}^{\infty} B_n^*(\theta) z^n$$

$$-\lambda \sum_{n=0}^{\infty} z^n \sum_{k=1}^{n} B_{n-k}^*(\theta)\, g_k - \alpha \sum_{n=0}^{\infty} P_{n+1}^*(0) z^n R^*(\theta)$$

$$\theta B^*(z,\theta) - B(z,0) = \lambda B^*(z,\theta) - \lambda X(z) B^*(z,\theta)$$

$$-\frac{\alpha}{z} \sum_{n=1}^{\infty} P_n^*(0) z^n R^*(\theta) \quad (4)$$

$$B^*(z,\theta)[\theta - w_x(z)] = B(z,0) - \frac{\alpha}{z} P^*(z,0) R^*(\theta)$$

Where $w_X(z) = \lambda(1 - X(z))$

At $\theta = w_X(z)$ equation (4) implies,

$$B(z,0)=\frac{\alpha}{z}P^{*}(z,0)R^{*}(w_X(z)) \quad (5)$$

Substituting the value of $B^{*}(z,\theta)$

$$B^{*}(z,\theta)=\frac{\frac{\alpha}{z}P^{*}(z,0)\left[R^{*}(w_X(z))-R^{*}(\theta)\right]}{\theta-w_X(z)} \quad (6)$$

At $\theta=0$, $R^{*}(\theta)=1$ we get

$$B^{*}(z,0)=\frac{\frac{\alpha}{z}P^{*}(z,0)(1-R^{*}(w_X(z)))}{w_X(z)} \quad (7)$$

Similarly multiplying the equation (1) by suitable powers of z adding over $n\geq 1$ we get,

$$\theta P^{*}(z,\theta)-P(z,0)=(\lambda+\alpha)P^{*}(z,\theta)-\lambda X(z)P^{*}(z,\theta)$$

$$-\frac{S^{*}(\theta)}{z}[P(z,0)-P_1(0)z]-\lambda R_0X(z)S^{*}(\theta)$$

$$-[B(z,0)-B_0(0)]S^{*}(\theta)$$

$$P^{*}(z,\theta)[\theta-g_\alpha(w_X(z))]=P(z,0)\left[\frac{z-S^{*}(\theta)}{z}\right]$$

$$-\frac{\alpha}{z}P^{*}(z,0)R^{*}(w_X(z))S^{*}(\theta)+R_0w_X(z)S^{*}(\theta) \quad (8)$$

Where $g_\alpha(w_X(z))=w_X(z)+\alpha$

At $\theta=0$ we get

$$P^{*}(z,0)\left[\frac{\alpha}{z}R^{*}(w_X(z))-g_\alpha(w_X(z))\right]=\frac{P(z,0)}{z}(z-1)$$

$$+R_0w_X(z) \quad (9)$$

$$P^{*}(z,0)=\frac{\frac{P(z,0)}{z}(z-1)+R_0w_X(z)}{\frac{1}{z}[\alpha R^{*}(w_X(z))-zg_\alpha(w_X(z))]} \quad (10)$$

Substituting (10) in (8) we get,

$$P^{*}(z,\theta)[\theta-g_\alpha(w_X(z))]$$

$$=\frac{P(z,0)}{z}\left[z\left(1-S^{*}(\theta)\frac{\alpha R^{*}(w_X(z))}{\alpha R^{*}(w_X(z))-zg_\alpha(w_X(z))}\right)\right.$$

$$\left.-S^{*}(\theta)\left\{1-\frac{\alpha R^{*}(w_X(z))}{\alpha R^{*}(w_X(z))-zg_\alpha(w_X(z))}\right\}\right]$$

$$+R_0w_X(z)S^{*}(\theta)\left[\frac{-zg_\alpha(w_X(z))}{\alpha R^{*}(w_X(z))-zg_\alpha(w_X(z))}\right] \quad (11)$$

At $\theta=g_\alpha(w_X(z))$ in (11) we get

$$\frac{P(z,0)}{z}=\frac{R_0w_X(z)S^{*}(g_\alpha(w_X(z)))g_\alpha(w_X(z)}{g_\alpha(w_X(z)[S^{*}(g_\alpha(w_X(z)))-z]+\alpha R^{*}(w_X(z))[1-S^{*}(g_\alpha(w_X(z)))]} \quad (12)$$

Substituting (12) in (9) we get

$$P^{*}(z,0)=\frac{zR_0w_X(z)[1-S^{*}(g_\alpha(w_X(z)))]}{g_\alpha(w_X(z)[S^{*}(g_\alpha(w_X(z)))-z]+\alpha R^{*}(w_X(z))[1-S^{*}(g_\alpha(w_X(z)))]} \quad (13)$$

The partial PGFs of the system size of the model are listed below

$$P^{*}(z,0)=\frac{zR_0w_X(z)[1-S^{*}(g_\alpha(w_X(z)))]}{g_\alpha(w_X(z)[S^{*}(g_\alpha(w_X(z)))-z]+\alpha R^{*}(w_X(z))[1-S^{*}(g_\alpha(w_X(z)))]} \quad (14)$$

$$B^{*}(z,0)=\frac{\frac{\alpha}{z}P^{*}(z,0)(1-R^{*}(w_X(z)))}{w_X(z)} \quad (15)$$

To derive the total PGF of the system size distribution, the following generating functions are considered.

P_I = Probability when the server is idle state $=R_0$

$P_{comp}(z)=$ ThePGF when server is busy or in breakdown state

$$=P^{*}(z,0)+B^{*}(z,0)$$

$$=\frac{P^{*}(z,0)}{zw_X(z)}[zw_X(z)+\alpha(1-R^{*}(w_X(z))]$$

The total Probability Generating Function (PGF) of system size distribution at steady-state is given by

$$P(z)=P_I(z)+P_{comp}(z)$$

$$=\frac{R_0[z-1][\alpha+w_X(z)S^{*}(g_\alpha(w_X(z)))]}{D(z)} \quad (16)$$

Where $D(z)=g_\alpha(w_X(z)[z-S^{*}(g_\alpha(w_X(z)))]$

$$+\alpha R^{*}(w_X(z))[S^{*}(g_\alpha(w_X(z)))-1] \quad (16.1)$$

R_0 can be evaluated using the normalizing condition $P(1)=1$ thus gives

$$P(1)=1=\lim_{z\to 1}\frac{R_0[z-1][\alpha+w_X(z)S^{*}(g_\alpha(w_X(z))]}{D(z)}$$

$$=R_0\alpha\left(\frac{1}{D'(1)}\right)$$ Where $D'(1)$ is the derivative of $D(z)$ at $z=1$ which is given by $D'(1)=\alpha(1-\rho)$

Where $\rho=\frac{\lambda E(X)}{\alpha}(1-S^{*}(\alpha))(1+\alpha E(R))$

i.e,) $R_0=(1-\rho)$

Substituting for R_0 in equation (14) the PGF of the proposed model is given by

$$P(z)=\frac{(1-\rho)[z-1][\alpha+w_X(z)S^{*}(g_\alpha(w_X(z))]}{D(z)}$$

(17)

Where D(z) is given by the equation (16.1)

1.4 Steady State Condition:

Equation (17) shows that $0<\rho<1$ gives the steady state condition for the model.

5. Performance Measures

In this section, the steady-state system size probabilities and the expected number of customers in the system, when the server is in different states are calculated

5.1 The Server in Busy State

Let P_{Busy} denote the steady-state system size probabilities and L_{Busy} denote the average number of customers, present in the system when the system is in busy state. Then the measures are calculated from the partial PGFs of the system size given in equation (14).

$$P_{Busy}=\lim_{z\to 1}P^{*}(z,0)$$

$$=(1-\rho)\left[\lim_{z\to 1}\frac{-w_X(z)}{D(z)}(1-S^{*}(\alpha))\right]$$

Using L'Hospital Rule, $\lim_{z\to 1}\frac{-w_X(z)}{D(z)}=\frac{\lambda E(X)}{D'(1)}$

$$\frac{d}{dz}S^{*}(g_\alpha(w_X(z)))=-\lambda E(X)S^{*'}(\alpha)$$

By calculation then $\frac{d}{dz}[D(z)]_{z=1}=D'(1)=\alpha(1-\rho)$

Where $\rho=\frac{\lambda E(X)}{\alpha}(1-S^{*}(\alpha))(1+\alpha E(R))$

$$P_{Busy}=\frac{\lambda E(X)}{\alpha}(1-S^{*}(\alpha))\quad(18)$$

$$L_{Busy}=\left[\frac{d}{dz}P^{*}(z,0)\right]_{z=1}$$

$$=P_{Busy}+R_0\left(-\frac{w_X(z)}{D(z)}\right)_{z=1}\lambda E(X)S^{*'}(\alpha)$$

$$+R_0\frac{d}{dz}\left(-\frac{w_X(z)}{D(z)}\right)_{z=1}(1-S^{*}(\alpha))$$

From the equation (14)

(where „'" symbol denote the derivative $S^{*}(\theta)$ with respect to θ)

For the further simplifications the following results are used,

$$\frac{d}{dz}\left[-\frac{w_X(z)}{D(z)}\right]=\frac{D'(1)\lambda E(X(X-1))+\lambda E(X)(-D''(1))}{2(D'(1))^2}$$

$$-D''(1)=\lambda E(X(X-1))(1-S^{*}(\alpha))[1+\alpha E(R)]$$
$$+(\lambda E(X))^2[\alpha(1-S^{*}(\alpha))E(R^2)+2S^{*'}(\alpha)(1+\alpha E(R))]$$
$$+2\lambda E(X)\quad(18.1)$$

$$\frac{d^2}{dz^2}\left[S^{*}(g_\alpha(w_X(z)))\right]=-\lambda E(X(X-1))S^{*'}(\alpha)$$
$$+(\lambda E(X))^2S^{*''}(\alpha)$$

$$\frac{d}{dz}[g_\alpha(w_X(z))]=-\lambda E(X)$$

$$\frac{d^2}{dz^2}[g_\alpha(w_X(z))]=-\lambda E(X(X-1))$$

$$L_{Busy}=P_{Busy}+(\lambda E(X))^2\frac{S^{*'}(\alpha)}{\alpha}$$
$$+\frac{(1-S^{*}(\alpha))}{2\alpha}\left(\lambda E(X(X-1))+\frac{\lambda E(X)(-D''(1))}{(1-\rho)\alpha}\right)\quad(19)$$

5.2 The Server in Breakdown State

The probability that the server is in breakdown state (P_{BR}) and the expected number of customers in the corresponding states (L_{BR}) is obtained.

$$P_{BR}=\lim_{z\to 1}B^{*}(z,0)$$

$$=\alpha P^{*}(1,0)\lim_{z\to 1}\frac{1-R^{*}(w_X(z))}{w_X(z)}$$

$$P_{BR}=\lambda E(X)E(R)(1-S^{*}(\alpha))\quad(20)$$

$$L_{BR}=\left[\frac{d}{dz}B^{*}(z,0)\right]_{z=1}$$

$$=\alpha\left[P_{Busy}\frac{d}{dz}\left(\frac{1-R^{*}(w_X(z))}{z\,w_X(z)}\right)+L_{Busy}E(R)\right]$$

For the further simplifications the following results are used:

$$\frac{d}{dz}\left[\frac{1-R^{*}(w_X(z))}{w_X(z)}\right]=\lambda E(X)\frac{E(R^2)}{2}$$

$$\lim_{z\to 1}\left[\frac{1-R^{*}(w_X(z))}{w_X(z)}\right]=E(R)$$

$$L_{BR}=\alpha\left[\left(\lambda E(X)\frac{E(R^2)}{2}-E(R)\right)P_{Busy}+E(R)L_{Busy}\right]$$

(21)

2.3 Mean System Size

The expected number of customers in the system is given by

$$L=L_{BR}+L_{Busy}$$

After the simplification it is found that,

$$L=\frac{(-D''(1))}{2\alpha(1-\rho)}+\lambda E(X)\left(-\frac{S^{*}(\alpha)}{\alpha}\right)\quad(22)$$

Where $(-D''(1))$ is given in the equation (18.1).

6. Particular Cases:

i) M^X/M/1 Model with Negative arrival

The generating function and mean queue length for the Markovianqueueing model with M^X/M/1 queue with negative arrivals and breakdowns can be obtained by suitable substitution of $S^*(g_\alpha(w_X(z)))$ and $R^*(w_X(z))$.

If the service time and the repair time follow exponential distributions with parameters μ and γ respectively, then

$$S^*(g_\alpha(w_X(z))) = \frac{\mu}{\mu + g_\alpha(w_X(z))}$$

$$R^*(w_X(z))) = \frac{\gamma}{\gamma + w_X(z)}.$$

Substituting for $S^*(g_\alpha(w_X(z)))$ and $R^*(w_X(z)))$ in equations (17) and (22) the generating function $P^-_{M^X/M/1}(z)$ and mean queue length $L^-_{M^X/M/1}$ for the Markovian model are given by

$$P^-_{M^X/M/1}(z) = \frac{(1-\rho)(z-1)(\mu+\alpha)g_\gamma(w_X(z))}{g_\gamma(w_X(z))[\mu(z-1)+zg_\alpha(w_X(z))]-\alpha\gamma}$$

$$L^-_{M^X/M/1} = \frac{(-D''(1))}{2\alpha(1-\rho)} + \lambda E(X)\left(-\frac{\mu}{\alpha(\mu+\alpha)}\right)$$

$$-D''(1) = \lambda E(X(X-1))(\frac{\alpha}{\mu+\alpha})(1+\alpha E(R)) + 2\lambda E(X)$$

$$+(\lambda E(X))^2[\frac{\alpha^2}{\mu+\alpha}E(R^2) + 2S^{*'}(\alpha)(1+\alpha E(R))]$$

ii) M/G/1 Model with Negative arrival

The steady state results of single arrival M/G/1 model with negative arrivals can be obtained by taking $X(z) = z$ and hence $w(z) = \lambda(1-z)$ and $E(X) = 1$ in equations (17) and (22).

$$P(z) = \frac{(1-\rho)[z-1][\alpha + w(z)S^*(g_\alpha(w(z)))]}{g_\alpha(w(z))[z - S^*(g_\alpha(w(z)))] + \alpha R^*(w(z))[S^*(g_\alpha(w(z))) - 1]}$$

$$L = \frac{(-D''(1))}{2\alpha(1-\rho)} + \left(-\frac{S^*(\alpha)}{\alpha}\right)$$

Where $(-D''(1))$ is obtained from the equation (18.1) by taking $E(X) = 1$.

iii) Geo/G/1 queue with negative arrival

If the batch size X follows Geometric distribution (Geo(p))[Decapitated Geometric] then

$P_k = \Pr(X = k) = (1-p)p^{k-1}$ and

$$X(z) = \sum_{k=1}^{\infty} P_k z^k = \frac{(1-p)z}{1-pz}$$

Subtuting for $X(z)$ in equation (17), we get the corresponding generating function for the model Geo/G/1 queue with negative arrivals.

7. Numerical Analysis

The results of some numerical examples are illustrated in the following tables 1 and 2 for M^X/G/1and M^X/M/1queueing models with negative arrivals. The batch size random variable is assumed to follow Geometric Distribution (1-p) p^{k-1}.In each table the system size probabilities, mean queue length (L) and expected waiting time (E(W)) corresponding to different negative arrival rates are presented. It is shown that the mean system size and expected waiting time increase as negative arrival rate α. The service time S and Repair time R respectively follow 2-stage hyper exponential distribution ($15a(e^{-15t}) + (1-a)12(e^{-12t})$)and 3-Erlang distribution with parameter β=1 in table 1.The parametric values are given in the corresponding tables.For the table 2,S and R follow exponential distributions with parameters 15 and 1 respectively for M^X/M/**1** model.

Table 1 ($p = 0.75, a = 0.32, \lambda = 1$)

α	ρ	P_{Busy}	P_{BR}	L_{Busy}	L_{BR}	L	E(W)
1.25	0.63	0.28	0.35	3.61	5.11	8.727	2.181
1.5	0.69	0.27	0.41	4.33	7.20	11.541	2.885
1.75	0.75	0.27	0.47	5.35	10.17	15.530	3.882
2	0.80	0.26	0.53	6.90	14.71	21.620	5.405

Table 2 ($p = 0.75, a = 1, \lambda = 1, E(R) = 1$)

α	ρ	P_{Busy}	P_{BR}	L_{Busy}	L_{BR}	L	E(W)
1.25	0.55	0.24	0.30	2.48	3.62	6.114	1.528
1.5	0.60	0.24	0.36	2.83	4.85	7.692	1.923
1.75	0.65	0.23	0.41	3.26	6.41	9.681	2.420
2	0.70	0.23	0.47	3.82	8.43	12.266	3.066

The values of Tables 1 and 2are used for the graphical representations inFigures 1and 2.

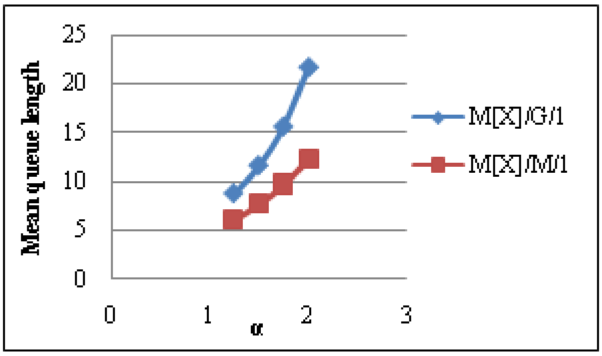

Figure 1:α Vs L

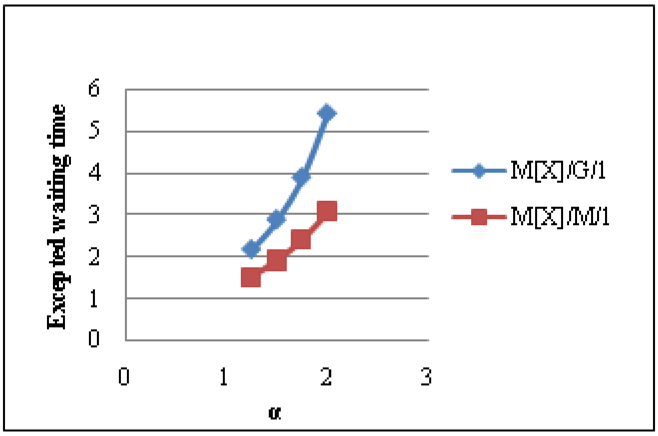

Figure 2:α Vs E(W)

8. Conclusion

In this paper the author considered a non- Markovian queueing system with two types of arrivals. The positive customers arrive in batches of random size following compound Poisson process and the negative customers arrive according to Poisson process. The arrival of negative customers will remove the customer in service (if any). The arrival of negative customer, to the empty system is considered as lost. The system is studied under steady state and various steady state solutions are obtained. Park et al. (2009) [15] analysed a Geo/G/1 queue with negative customers. The author compared the results of present paper with that of Park et al. (2009)[15].

9. Future Scope

Through numerical values we have shown that how the various parameters present in the system affect the mean queue length. This helps us to minimize the queue length of these type of models.

References

[1] J.Artalejo, “G-networks: a versatile approach for work removal in queueing networks”, Eur. J. Oper. Res. 126,233-249,2000.

[2] N. Bayer and O.J. Boxma, “Wiener-Hopf analysis of an M/G/1 queue with negative customers and of a related class of random walks”, Queueing Systems,23, 301-316,1996.

[3] R.J. Bouchericand O.J. Boxma, “The workload in the M/G/1 queue with work removal”, Probab. Eng. Inform. Sci. 10, 261-277,1995.

[4] H. Bruneeland B. Kim, “Discrete-Time Models for Communication Systems Including ATM”, Kluwer Academic Publishers: Boston, Dordrecht, and London,1993.

[5] R. Chakka and P.G. Harrison, “A Markov modulated multi-server queue with negative customers - The MMCPP/GE/c/L G-queue”, Acta Informatica, 37, 881-919,2001.

[6] X. Chao., M. Miyazawa and M. Pinedo, “Queueing Networks: Customers, signals and product form solutions” (John Wiley and Sons, Chichester),1999.

[7] D.R. Cox, “The analysis of non-Markovian stochastic processes by the inclusion of supplementary variables”, Proceedings of the Cambridge Philosophical Society, 51, 433-441,1955.

[8] E. Gelenbe, “Random neural networks with negative and positive signals and product form solution”, Neural Comput., 11, 502-510, 1989.

[9] E.Gelenbe, “Product-form networks with negative and positive customers”, J. Appl. Prob. 28, 655-663,1991.

[10] P.G. Harrison and E. Pitel, “Sojourn times in single-server queues with negative customers”, J. Appl. Prob., 30, 943-963,1993.

[11] P.G. Harrison and E. Pitel, “The M/G/1 queue with negative customers”, Adv. Appl. Prob., 28, 540-566, 1996.

[12] P.G. Harrison, N.M. Patel and E. Pitel,“Reliability modeling using G-queues”, Eur.J. Oper.Res, 273-287,2000.

[13] G. Jain, and K. Sigman , “A Pollaczeck-Khinchine formula for M/G/1 queues with disasters”, J. Appl. Prob.33,1191-1200,1996.

[14] Q.L. Li and Y.Q. Zhao, “A MAP/G/1 queue with negative customers”, Queueing Systems 47, 5-43,2004.

[15] H.M. Park, W.S. Yang and K.C. Chae, “The Geo/G/1 queue with negative customers and disasters”, Stochastic Models, 25,673-688,2009.

[16] W.S. Yang and K.C. Chae, “A note on the GI/M/1 queue with Poisson negative arrivals”. J. Appl. Prob., 38, 1081-1085,2001.

[17] W.H. Zhou,, “Performance analysis of discrete-time queue GI/G/1 with negative arrivals”, Applied Mathematics and Computation,170,1349-1355,2005.