

Minimizing the Peak-to-Average Power Ratio of OFDM Signals Using Differentially Encoded Subcarriers

G. Srinivasulu¹, S. Venkatakiran²

¹P.G Student, VLSI Design, Dept of ECE, Sri Venkatesa Perumal College of Engineering and Technology, RVS Nagar, Puttur, A.P., India

²Associate Professor, Dept of ECE, Sri Venkatesa Perumal College of Engineering and Technology, RVS Nagar, Puttur, A.P., India

Abstract: *Wireless technologies have gradually become more and more involved in everyday life. An Orthogonal Frequency Division Multiplexing (OFDM) is an attractive multi carrier modulation technique for wireless transmission systems. OFDM has many advantages immunity to impulse interference, robustness to channel fading, high spectral density, resistance to multipath, much lower computational complexity. The major drawback of OFDM is signal suffers a high Peak to Average Power Ratio (PAPR), a high PAPR easily makes the signal peaks move into the non-linear region of the RF power amplifier which causes signal distortion. In this paper we analyze PAPR reduction of OFDM system using a technique that exploits the principle of differential encoding of subcarriers. The effectiveness of proposed technique is evaluated through extensive computer simulations and complementary cumulative distribution function (CCDF) is obtained as a function of number of modulations and subcarriers.*

Keywords: Peak-To-Average Power Ratio (PAPR), OFDM, Differentially encoded subcarriers Selective Mapping (SLM), Quadrature Amplitude Modulation (QAM).

1. Introduction

OFDM, which is also popularly known as simultaneous MFSK, has been widely implemented in high-speed digital communications in delay dispersive environments. Basically it is a Multi-Carrier Modulation (MCM) technique. OFDM was first proposed by Chang, (1966). Chang proposed the principle of transmitting messages simultaneously over multiple carriers in a linear band-limited channel without ISI and ICI. The initial version of OFDM employed a large number of oscillators and coherent demodulators. In 1971, DFT was applied to the modulation and demodulation process by Weinstein and Ebert, (1971). In the year 1980, Peled &

Ruiz introduced the notion of cyclic prefix to maintain frequency orthogonality over the dispersive channel [Andrews et al., 2007]. It is commonly deemed that OFDM is a major technique for beyond-3G wireless multimedia communications. In OFDM technology, the multiple carriers are called subcarriers, and the frequency band occupied by the signal carried by a subcarrier is called a sub-band. OFDM achieves orthogonality in both the time and frequency domains. The most attractive feature of OFDM is its robustness against multipath fading, but with a low complexity; that is, it can reliably transmit data over time-dispersive or frequency selective channels without the need of a complex time domain channel equalizer. OFDM can be efficiently implemented using FFT. The OFDM system based on IFFT/FFT algorithm which has the following major advantages.

- The power amplifier (PA) can have smaller back off because of the reduced PAPR value.
- Lower the transceiver's complexity
- Low spectral re-growth
- Low sensitivity to CFO
- Power constraints of the battery driven handsets

OFDM splits the bit stream into multiple parallel bit streams of reduced bit rate, modulates them using a M-ary modulation, and then transmits each of them on a separate subcarrier. The amplitude spectrum of each modulated subcarrier using PSK or QAM has a sinc shape. At the peak spectrum response of each subcarrier, the spectral responses of all other subcarriers are identically zero. This improves the spectral efficiency. The use of narrow band flat-fading sub channels can effectively resist frequency selective fading. Each modulated subcarrier in the OFDM signal can be an MPSK, MASK, or MQAM signal. Thus, the OFDM signal can be obtained first by 1: N serial-to-parallel (s/p) conversion and then by converting each of the N streams to one of the N subcarriers. Their sum is then unconverted to the RF band. For each subcarrier, there is a complete modulator, which typically consists of a subcarrier oscillator, one (for MASK) or two (for MQAM and MPSK) multipliers, and an adder. A phase shifter is also required for MQAM and MPSK. For pass band modulation, a mixer and a band pass filter are required for up conversion.

In this paper, we propose a low complexity and simple to implement PAPR reduction technique for OFDM systems, where an encoder is applied after the OFDM modulator in the system. The proposed encoder is based on differential encoding principles and largest time-domain signal sample is used as reference in order to carry out operations by the proposed encoder. A positive real multiplier (δ) such that $\delta \neq 1$ is also introduced in the proposed technique to obtain appropriate PAPR level. Information about the reference signal sample and multiplier is transmitted to the receiver as side information. This side information is used by the proposed decoder at the receiver side to compensate the received signal in order to recover the original time-domain signal. To understand the complexity of the proposed technique, we evaluate the computational complexity analytically and compare with the available PAPR reduction

techniques. We notice that the computational complexity of the proposed technique is the lowest among several of the available techniques, thanks to simple encoding operations at the transmitter and receiver. The effectiveness of the proposed technique is investigated through complementary cumulative distribution function (CCDF) of the PAPR. Extensive computer simulations with various system parameters are performed to obtain the numerical results of CCDFs. The numerical result of PAPR threshold with respect to δ is also obtained, where the reduction in PAPR level is observed with the increase in δ . Thus, δ can be adjusted to obtain suitable PAPR level. Moreover, Monte Carlo simulations are evaluated to obtain the average bit error probability of the OFDM system after applying the proposed technique. No change in the error performance has been noticed. Thus, the proposed technique offers reduction in PAPR without signal distortion and performance degradation.

2. OFDM System Model

(a) Basic OFDM system

The input data sequences (X) are passed through serial to parallel converter. In which input sequence is converted into parallel complex symbols of size 'N' i.e. $X = [X_0, X_1, \dots, X_{N-1}]^T$, where 'N' is total number of the below figure shows the basic block diagram of OFDM system

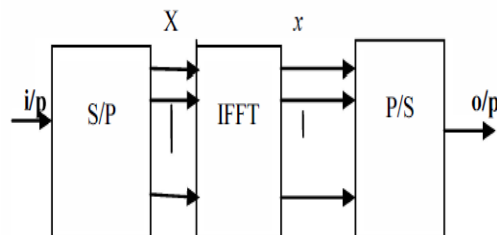


Figure 1: Basic Block Diagram of OFDM System

Sub carriers used for parallel transmission then IFFT is taken for all the symbols. Output is 'x (n)' can be written as

$$x(n) = \text{IFFT}[X(k)] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}, \quad 0 \leq n \leq N-1 \quad \text{----- (1)}$$

(b) Peak Average Power Ratio (PAPR)

When transmitting data from the mobile terminal to the network, a power amplifier is required to boost the outgoing signal to a level high enough to be picked up by the network. The power amplifier is one of the biggest consumers of energy in a device and should thus be as power efficient as possible to increase the operation time of the device on a battery charge. The efficiency of a power amplifier depends on two factors:

The amplifier must be able to amplify the highest peak value of the wave. Due to silicon constraints, the peak value decides over the power consumption of the amplifier.

The peaks of the wave however do not transport any more information than the average power of the signal over time. The transmission speed therefore doesn't depend on the peak power output required for the peak values of the wave but rather on the average power level.

As both power consumption and transmission speed are of importance for designers of mobile devices the power amplifier should consume as little energy as possible. Thus, the lower the difference between the peak power to the average power (PAPR) the longer is the operating time of a mobile device at a certain transmission speed compared to devices that use a modulation schemes with a higher PAPR.

(c) PAPR in OFDM

The transmit signal in multicarrier transmission system (OFDM) can have high peak values in time domain since all the subcarriers are added during IFFT operation. So this system have high peak to average power ratio (PAPR) than single carrier system. This reduces the efficiency of power amplifier and forces the power amplifier to operate in nonlinear region, causes out band radiation that affect signals in adjacent frequencies and in-band distortion this affects the received signals by rotation and attenuation. The PAPR problem is more important in many wireless applications like LTE and other mobile communication systems. Thus, PAPR of OFDM signal can be defined as

$$\text{PAPR} = \frac{\max_{0 \leq n \leq N-1} |x(n)|^2}{E[x(n)^2]} \quad \text{----- (2)}$$

Where $E[x(n)^2]$ is average power of OFDM symbol. Normally PAPR is measured by Complementary Cumulative Distribution Function (CCDF). It is defined as probability that the PAPR of OFDM symbol exceed the threshold value. The CCDF of PAPR can be written as

$$\text{Pr}(\text{PAPR} > \text{PAPR}_0) = 1 - (1 - e^{-\text{PAPR}_0})^N \quad \text{----- (3)}$$

When number of subcarriers increases distribution of complex OFDM signals follows Rayleigh distribution as per the central limit theorem. In PTS PAPR can be reduced with little penalty because of bandwidth efficiency loss. For a given vector multiple OFDM signals are generated and choose the minimum PAPR signal for transmission.

3. OFDM System Model with Proposed Technique

The block diagram of the OFDM system with the proposed differential encoder is shown in Figure 2. The input data is assumed to be a sequence of binary digits from an equally likely and statistically independent data source. These binary digits are then mapped to some constellation points using a digital modulation scheme such as M -ary Quadrature amplitude modulation (M -QAM) or M -ary phase-shift keying (M -PSK). A serial-to-parallel (S/P)

Converter is used to store N constellation points for an OFDM symbol interval. The cyclic prefix (or guard interval) has no effect in PAPR analysis and thus, is not considered for PAPR analysis in this paper. It is well known that the PAPR of the discrete-time version is not precise and hence, at least oversampling factor of $L=4$ has been shown to provide sufficiently accurate PAPR results. These L -times oversampled time-domain signal samples are represented as

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \cdot e^{j2\pi nk/NL}, \quad 0 \leq k \leq NL-1, \quad \text{----- (4)}$$

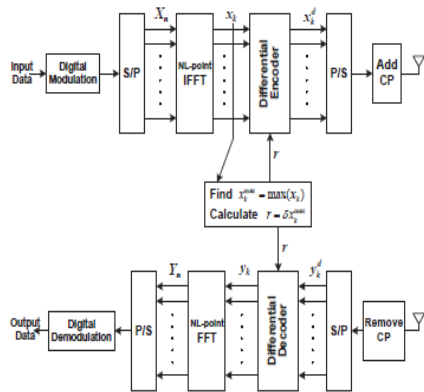


Figure 2: OFDM System Model with Proposed Technique

This leads to the following discrete-time PAPR expression

$$\text{PAPR} = \frac{\max_{0 \leq k \leq NL-1} |x_k|^2}{\mathcal{E}[|x_k|^2]} \quad \text{----- (5)}$$

Where $\mathcal{E}[\cdot]$ is the expectation operator. The proposed differential encoder is applied on the time-domain signal samples and requires a reference sample for encoding. The reference sample (r) in this proposed technique is obtained through the product of the maximum of the time-domain signal sample ($x_{\max k}$) and a specified positive real number (δ). Mathematically,

$$r = \delta x_k^{\max} \quad \text{----- (6)}$$

Where $x_{\max k}$ is computed as

$$x_k^{\max} = \max(x_k) \quad \text{----- (7)}$$

And $\delta > 0$ provided $\delta = 1$. Thus, the operation in the proposed Encoder can be represented mathematically as,

$$x_k^d = r - x_k = \delta x_k^{\max} - x_k \quad \text{----- (8)}$$

Therefore, the new PAPR using the proposed technique can be computed by

$$\text{PAPR} = \frac{\max_{0 \leq k \leq NL-1} |x_k^d|^2}{\mathcal{E}[|x_k^d|^2]} \quad \text{----- (9)}$$

The differential decoder is applied at the receiver side to compensate the encoding operation, as shown in the Figure 2. It performs the following mathematical operation on the received time-domain signal samples.

$$y_k = r - y_k^d \quad \text{----- (10)}$$

4. Simulation Results

In this section, the performance of the proposed technique using computer simulations is presented. OFDM systems with 64, 256 and 1024 subcarriers ($N = 64, 256, 1024$) are considered in this paper as well as different digital modulation schemes such as QPSK, 16-QAM and 64-QAM

are employed without any coding or any other form of diversity. To obtain precise PAPR, the transmit signal is oversampled by a factor of 4 ($L = 4$) and 106 were generated for numerical results. The numerical results are illustrated by complementary cumulative distribution function (CCDF) of PAPR where PAPR is measured with and without the proposed technique. CCDF Points to the probability that an OFDM block surpasses the PAPR threshold level (γ).

Figure 3 illustrates the CCDFs of PAPR of the proposed technique at different δ for 64-subcarrier OFDM system ($N = 64$) with QPSK modulation and oversampling factor of 4 ($L = 4$). The CCDF of PAPR of the original OFDM signal with QPSK and $L = 4$ is also shown in the same figure for comparison. It is shown that the original OFDM signal has a PAPR which exceeds 10.6 dB for less than 0.1 percent of the blocks. With the proposed technique, 0.1 percent PAPR reduces to 7.8 dB at $\delta = 0.5$, resulting in 2.8 dB reduction. When δ increases to 0.95, 2 and 5, 0.1 percent PAPR reduces to 5.8, 3.5 and 1.6 dB, resulting in 4.8, 7.1 and 9 dB reductions, respectively.

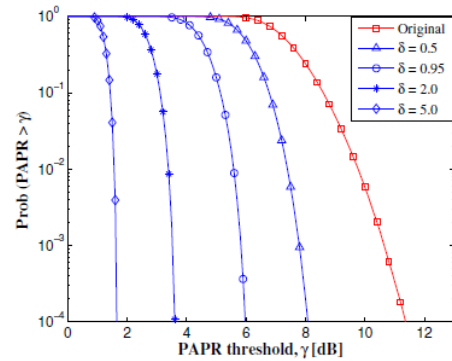


Figure 3: PAPR Distribution for different δ for 64-subcarrier OFDM with QPSK modulation.

Similarly, Figure 4 shows the CCDFs of PAPR of the proposed technique at different δ for 256-subcarrier OFDM system ($N = 256$) with 64-QAM modulation and oversampling factor of 4 ($L = 4$). The CCDF of PAPR of the original OFDM signal with 64-QAM and $L = 4$ is also shown in the same figure for comparison. In contrast to 11.3 dB of PAPR for less than 0.1 percent of the blocks for original OFDM, the proposed technique reduces 0.1 percent PAPR to 7.8, 6.5, 4.2, 2.4 and 1.5 at $\delta = 0.50, 0.75, 1.5, 3$ and 5, respectively, resulting in 3.5, 4.8, 7.1, 8.9 and 9.8 dB reductions.

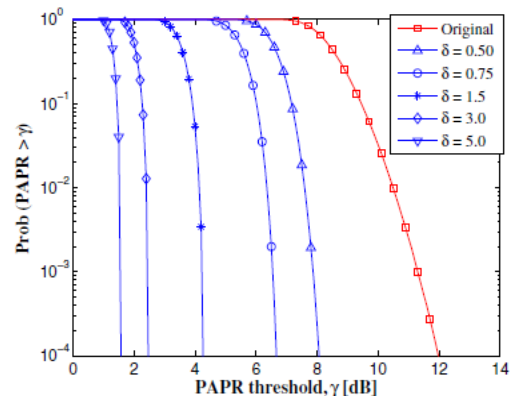


Figure 4: PAPR Distribution for different δ for 256-subcarrier OFDM with 64-QAM modulation

The result of PAPR threshold in dB with respect to δ is shown in Figure 5. It is found that the reduction in PAPR improves with the increase in the value of δ . Thus, PAPR can be lowered by using larger values of δ

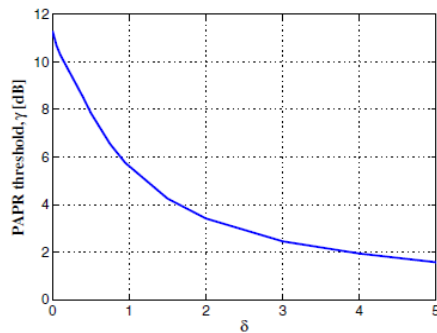


Figure 5: PAPR threshold (γ) versus δ , for 256-subcarrier OFDM with 64-QAM modulation

5. Conclusion

In this paper, we have studied about OFDM and PAPR problem in OFDM, and then we analyzed OFDM with QAM Modulation technique. This paper proposed a low complex and easy-to-implement PAPR reduction technique, which incorporated the proposed differential encoder and differential decoder at the transmitter and receiver side of the OFDM system, respectively. We assumed an ideal communication link between encoder and decoder to exchange information about the reference for encoding and decoding of the OFDM signal. We investigated the performance of the proposed PAPR reduction technique and we carried out extensive computer simulations to obtain numerical results for CCDFs and average bit error probability of the OFDM system with the proposed technique. The numerical results for CCDFs successfully proved PAPR reductions by applying the proposed technique. Thus, the proposed PAPR reduction technique is very effective and significant reductions in PAPR can be achieved, without compromise on error performance and signal distortion.

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