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# Analysis of Complex Valued Recurrent Neural Network with Time-Delays

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**Abstract:** ***We have presented a complex valued recurrent neural network given by a set of non-linear differential equation. In this paper, activation dynamics of complex valued neural networks are studied on general time scales. Besides presenting conditions guaranteeing the existence of a unique equilibrium pattern, its global exponential stability is discussed. Some numerical examples for different time scales are given in order to highlight the results. The equation is shown to possess a unique equilibrium point beside having bounded solution. The equation is shown to possess a unique equilibrium point besides having a bounded solution.***

**Keywords:** Complex valued recurrent neural network, unique equilibrium contraction mapping, stability, activation function.

## 1. Introduction

The focus of the research in recurrent neural network. Now is to study more accurately the wide spread applications like signal processing pattern recognition and other engineering applications [1-12]. Here complex signals are involved therefore it is better to use complex valued recurrent neural network. Many authors [13-15] has presented complex valued recurrent neural networks.

In real valued recurrent neural network we usually consider their activation function to be smooth and bounded. But in complex domain according to Liouville"s theorem. If f(z) is bounded and analytic at all z$\in$c, then f(z) is a constant function. Therefore in complex recurrent neural network it is more challenging to define an activation function.

## 2. Preliminaries

In this paper we consider the following valued recurrent neural network with time delays given by Hu and Wang [16]

$$\dot{z}(t) = -D(z)+Af(z(t))+B(g(z(t-\tau))) + u_z \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$(\dot{x}+i\dot{y})(t) = -D(x+iy)+A((x+iy)t)+B(g(x+iy)(t-\tau))+u_{(x+iy)}$$

Let z=x+iy, where „i" denotes the imaginary unit that is $i=\sqrt{-1}$, f(z) can be expressed as by separating into its real and imaginary parts as:

$$\dot{x} = -Dx+A_1\emptyset(x)+B_1(\xi(x))+u_x$$

In equation (1), $z=(z_1\ z_2\ z_3\ z_4\ldots\ldots\ldots z_n)^T \in C^n$ is the state vector

$D=diag(d_1,d_2,d_3,\ldots\ldots\ldots\ldots d_n) \in R^{N\times N}$ with $d_j>0$ (j=1,2,3……n) is the self feedback connection weight matrix,

$A=(a_{jk})_{n\times n} \in c^{n\times n}$ and

$B=(b_{jk})_{n\times n} \in c^{n\times n}$

are respectively, the connection weight matrix with time and without time delays.

$$( f(z(t))=(f_1(z_1(t)),f_2(z_2(t)),\ldots\ldots\ldots f_n(z_n(t)))^T : c^n\rightarrow c^n$$

And $g(z(t-\tau)) =(g_1(z_1(t-\tau_1)),g_2(z_2(t-\tau_2)),\ldots\ldots\ldots g_n(z_n(t-\tau_n)))^T : c^n\rightarrow c^n$

Are the vector valued activation functions with and without time delays, whose element consists of complex valued non-linear function $\tau_j$ (j=1,2,3,……,n) are constant time delays $u=(u_1,u_2,\ldots..,u_n)\epsilon c^n$ is the exrnal input vector.

The purpose this brief article is to investigate the existence and the stability characteristic of the equilibria of network of the form

$$\frac{dr(t)}{dt} = -D\, r(t)+A_1\, r(t-\tau_1)+u_r$$

$$\frac{ds(t)}{dt} = -D\, s(t)+A_2\, s(t-\tau_2)+u_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

## 3. UIQUENESS

<u>Theorem</u>: - Assume the following

1) The connection weights $A_1,A_2$ are real and constant.
2) $u_1,u_2$ are real and constant.
3) The gain parameter $\lambda$, μ and the time delays $\sigma_1$ and $\sigma_2$ are non-negative constants.
4) There exits a number c$\epsilon$(0,1) such the neural gains and the connection weights satisfy.

$$IA_1I\leq c<1$$

$$IB_1I\leq C<1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

There exists an unique equilibrium point (r*,s*) of satisfying.

$$r^* = A_1r^*+u_r$$

$$s^* = B_1s^*+u_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

<u>PROOF</u>:-We have from (3) that an arbitrary solution satisfying the following differential inequalities

$$-D\, r(t) - \alpha \leq \frac{dr(t)}{dt} \leq -D\, r(t)+\alpha$$

$$-D\, s(t) - \beta \leq \frac{ds(t)}{dt} \leq -D\, s(t)+\beta$$

Where,

$\alpha=Ia_1I+Iu_rI$

$\beta$=I$a_2$I+I$u_s$I …………………(6)

Of the system (2) has equilibrium, then such as equilibrium is a fixed point of the mapping

F : $\Omega \rightarrow R^{2n}$

Defined by

F(x,y) = $F_1$ (x,y)

$F_2$ (x,y) …………………..(7)

Where

$F_1(x,y)=A_1\emptyset(x)+B_1\xi(x)+u_r$

$F_2(x,y)=A_2\varphi(x)+B_2\eta(x)+u_s$ …………………(8)

We note from (7) and (8) that if (x,y) and (X,Y) are any two points of $\Omega$ then

$\|F(x,y)-F(X,Y)\|$ =I{$A_1\emptyset$(x)- $A_1\emptyset$(X)}+{$B_1$ $\varphi$(y)- $B_1$ $\varphi$(Y)}I ……………..(9)

$\leq A_1\emptyset^x(x-X)+B_1\ \varphi^y(y-Y)$ ……………(10)

$\leq$I$A_1$I Ix-XI + I$B_1$I Iy-YI

$\leq$c[Ix-XI + Iy-YI] ……………..(11)

$\leq \|(x,y)-(X,Y)\|$ ……………..(12)

In deriving (10) and subsequent enequalities, we have used the facts that $\emptyset^x$ lies between x and X, $\varphi^y$ lies between y and Y. The mapping F is continuous and F($\Omega$)$\subset$ $\Omega$. It follows from (11) and c<1. That is F contracicts on $\Omega$. By as well as known contraction mapping principle. We conclude that there exists a unique point say (r*,s*) such that

F(r*,s*)=(r*,s*) ………………(13)

And this completed the proof.

Thus there exists a unique pattern and memory associated with each set of the external inputs $u_r$ and $u_s$ where the matrix A,B are fixed in the next section we derive conditions for the global asymptotic stability of the unique equilibrium (r*,s*) of (3).

## 4. Global Stability

**Theorem:** Complex valued recurrent neural network (1) globally exponentially stable if the condition (1) is hold.

**Proof:** If the condition in theorem (1) holds. Then complex valued recurrent neural network (1) has unique equilibrium point, suppose that the equilibrium point (r*,s*) of (3) is globally asymptotically stable.

One can derivative that the derivations x,y are governed by:

$\frac{dx(t)}{dt}$ = -D x(t) +$A_1$ $\emptyset^x$(t) x (t-$\sigma_1$)

$\frac{dy(t)}{dt}$ = -D y(t) + $A_2$ $\varphi^y$ (t) y(t- $\sigma_2$) …………………(14)

In which $\emptyset^x$(t) lies between r* and (t- $\sigma_1$) and $\varphi^y$ lies between s* and (t- $\sigma_2$). We note that

$\emptyset^x(\omega)$= 1-$\emptyset(\omega) \leq 1$

And we have from (14) that

$\frac{dx(t)}{dt}$ = -D x(t) +I$A_1$I Ix (t- $\sigma_1$)I

$\frac{dy(t)}{dt}$ = -D y(t) + I$A_2$I Iy (t- $\sigma_2$)I ……………………(15)

Consider a Lyapunov function V(t) = V(x,y)(t). defined by

V(t)=ID x(t)I + ID y(t)I+I$A_1$I $\int_{t-\sigma 1}^{t} x(m)\,dm$ +I$A_2$I $\int_{t-\sigma 2}^{t} y(m)$dm …………(16)

Calculating the upper right derivative D*V along the solution of (14)

D*V $\leq$ -ID x(t)I + I$A_1$I Ix(t-$\sigma_1$) I – ID y(t)I + I$A_2$I Iy(t- $\sigma_2$)I + I$A_1$I [ Ix(t)-x(t- $\sigma_1$)I]

+ I$A_2$I [Iy(t)-y(t- $\sigma_2$)I] ………….(17)

$\leq$ (-1+I$A_1$I) Ix(t)I + (-1+I$A_2$I) I y(t)I (by eq'n (4))

$\leq$ -(1-C) [ Ix(t) + y(t)I ] …………….(18)

It is consequences of (18) that

V(x,y)(t) + (1-C)$\int_{t0}^{t}[x(m)+y(m)]$dm $\leq$ V(x,y)($t_0$) ……………..(19)

It follows from (16) and (19) that is bounded on (0,$\infty$). The boundedness of V on (0, $\infty$). We note from (15) that x,y are bounded on (0,$\infty$) and implies that x,y are uniformly continuous on (0,$\infty$). A consequences of (19) is that

[ Ix(t)I + Iy(t)I ] $\in$ $L_1$ (0,$\infty$) ……………(20)

The uniformly continuous [ Ix(t)I + Iy(t)I ] on (0,$\infty$) together with (20) implies by Barabalat's

[ Ix(t)I + Iy(t)I ] $\rightarrow \infty$ as t$\rightarrow \infty$ ……………(21)

Thus it follows x(t) $\rightarrow$ 0,y(t ) $\rightarrow$ 0 as t$\rightarrow \infty$

Hence r(t) $\rightarrow$ r*, s(t) $\rightarrow$ s* as t$\rightarrow \infty$

## 5. Conclusion

In the existing sections we have investigated a complex valued recurrent neurals network. This model is shown to be stable. Some new conditions were derived to ascertain the existence and uniqueness of the equilibrium point. The theoretical outcomes obtained are capable for the design and use of complex valued recurrent neural network.

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