Some New Results on K-even Sequential Harmonious Labeling of Graphs

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Abstract: Singh and Varkey introduced the odd sequential graphs. Gayathri and Hemalatha introduced even sequential harmonious labeling of graphs and also k-even sequential harmonious labeling of graphs. Here, we investigate some new results on k-even sequential harmonious labeling of graphs. In this paper, we have shown that the graphs $P_n^3$, Alternate triangular snakes and Alternate quadrilateral snakes are k-even sequential harmonious graphs.

Keywords: Path, Alternate triangular snake, Alternate quadrilateral snake

1. Introduction

All the graphs in this paper are finite, simple and undirected. The symbols $V(G)$ and $E(G)$ denote the vertex set and the edge set of a graph $G$.

The cardinality of the vertex set is called the order of $G$. The cardinality of the edge set is called the size of G. A graph with $p$ vertices and $q$ edges is called a $(p,q)$ graph.

In [1], Gayathri and Hemalatha says that a labeling is an even sequential harmonious labeling if there exist an injection $f$ from the vertex set $V$ to $\{0, 1, 2, ..., 2q\}$ such that the induced mapping $f^+$ from the edge set $E$ to $\{2, 4, 6, ..., 2q\}$ defined by

$$f^+(u,v) = \begin{cases} f(u) + f(v), & \text{if } f(u) + f(v) \text{ is even} \\ f(u) + f(v) - 1, & \text{if } f(u) + f(v) \text{ is odd and distinct} \end{cases}$$

A graph $G$ is said to be an even sequential harmonious graph if it admits an even sequential harmonious labeling.

Here, we have introduced k-even sequential harmonious labeling by extending the above definition for any integer $k \geq 1$. We say that a labeling is a k-even sequential harmonious labeling if there exist an injection $f$ from the vertex set $V$ to $\{k-1, k, k+1, ..., k+2q-1\}$ such that the induced mapping $f^+$ from the edge set $E$ to $\{2k, 2k+2, 2k+4, ..., 2k+2q-2\}$ defined by

$$f^+(u,v) = \begin{cases} f(u) + f(v), & \text{if } f(u) + f(v) \text{ is even} \\ f(u) + f(v) + 1, & \text{if } f(u) + f(v) \text{ is odd and distinct} \end{cases}$$

A graph $G$ is said to be a $k$-even sequential harmonious graph if it admits a $k$-even sequential harmonious labeling.

In this paper, we investigate some new results on k-even sequential harmonious labeling of graphs. Throughout this paper, $k$ denote any positive integer $\geq 1$. For brevity, we use k-ESHL for k-even sequential harmonious labeling.

2. Main Results

Definition: 2.1

By a graph $P_n^3$, we mean the graph obtained from $P_n$ joining each pair of vertices at distance 3 in $P_n$.

Theorem: 2.2

The graph $P_n^3$, $(n \geq 4)$ is a $k$-even sequential harmonious graph.

Proof:

Let the vertices of $P_n^3$ be $\{v_i: 1 \leq i \leq n\}$ and the edges of $P_n^3$ be $\{v_i v_{i+1}: 1 \leq i \leq n-1\} \cup \{v_i v_{i+3}: 1 \leq i \leq n-3\}$. Which are denoted in Fig.2.2(a).

A graph $G$ is said to be an even sequential harmonious graph if it admits an even sequential harmonious labeling.

Here, we first, label the vertices of $P_n^3$ as follows,

Define $f: V(P_n^3) \rightarrow \{k-1, k, k+1, ..., k+2q-1\}$ by

$$f(v_i) = 2(i-1)-1+k, 1 \leq i \leq n.$$  

Then the induced edge labels are

$$f^+(v_i v_{i+1}) = 4i-4+2k, 1 \leq i \leq n-1.$$  

$$f^+(v_i v_{i+3}) = 4i+2k-2, 1 \leq i \leq n-3.$$  

Clearly, the edge labels are even and distinct,

$$f^+(E) = \{2k, 2k+2, 2k+4, ..., 2k+2q-2\}.$$  

Hence, the graph $P_n^3$, $(n \geq 4)$ is a $k$-even sequential harmonious graph.

2-ESHL of $P_9^3$ is shown in Fig.2.2(b).
**Definition: 2.3**

An Alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to new vertex $v_i$.

That is, every alternate edge of a path is replaced by a cycle $C_3$.

**Theorem: 2.4**

Alternate triangular snakes is a $k$-even sequential harmonious graph.

**Proof:**

Let $A(T_n)$ be a alternate triangular snake.

Let the vertices of $A(T_n)$ be $\{v_i: 1 \leq i \leq n\}$ \cup $\{u_{2i-1}: 1 \leq i \leq n/2, \text{if } n \text{ is even and } 1 \leq i \leq (n-1)/2, \text{if } n \text{ is odd}\}$ \cup $\{w_{2i-1}: 1 \leq i \leq (n-2)/2, \text{if } n \text{ is even and } 1 \leq i \leq (n-1)/2, \text{if } n \text{ is odd}\}$. Which are denoted in Fig.2.4(a).

We first, label the vertices of $A(T_n)$ as follows,

Define $f : V(A(T_n)) \to \{k-1, k, k+1, \ldots, k+2q-1\}$ by

$$f(v_i) = 3i+1-k, \quad 1 \leq i \leq n$$

$$f(u_{2i-1}) = 6i-6+2k, \quad 1 \leq i \leq n/2$$

Then the induced edge labels are

$$f^{-1}(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}.$$

Hence, the graph $A(T_n)$ is a $k$-even sequential harmonious graph.

**Definition: 2.5**

An Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to new vertex $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$.

That is, every alternate edge of a path is replaced by a cycle $C_4$.

**Theorem: 2.6**

Alternate quadrilateral snakes is a $k$-even sequential harmonious graph.

**Proof:**

Let $A(Q_n)$ be a alternate quadrilateral snake.

Let the vertices of $A(Q_n)$ be $\{v_i: 1 \leq i \leq n\}$ \cup $\{u_i: 1 \leq i \leq n, \text{if } n \text{ is even and } 1 \leq i \leq n-1, \text{if } n \text{ is odd}\}$ \cup $\{w_i: 1 \leq i \leq (n-2)/2, \text{if } n \text{ is even and } 1 \leq i \leq (n-1)/2, \text{if } n \text{ is odd}\}$. Which are denoted in Fig.2.6(a).

We first, label the vertices of $A(Q_n)$ as follows,

Define $f : V(A(Q_n)) \to \{k-1, k, k+1, \ldots, k+2q-1\}$ by

$$f(u_i) = 4i-k, \quad 1 \leq i \leq n$$

Then the induced edge labels are

$$f^{-1}(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}.$$

Hence, the graph $A(Q_n)$ is a $k$-even sequential harmonious graph.
Clearly, the edge labels are even and distinct,

\[ f'(E) = \{2k, 2k+2, 2k+4, \ldots, 2k+2q-2\}. \]

Hence, the graph \(A(Q_n)\) is a k-even sequential harmonious graph.

2-ESHL of \(A(Q_6)\) is shown in Fig. 2.6(b)

![Figure 2.6(b): 2-ESHL of \(A(Q_6)\)](image)

References


