

# Multiphase Level Set Method for Image Segmentation in the Presence of Intensity Inhomogeneity with the Help of Brain MRI Images

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**Abstract:** *Image segmentation, in real world images is a considerable challenge in the presence of Intensity Inhomogeneity. The widely used Image segmentation algorithms are region based and they depend on homogeneity of the image intensities in the region of interest. This method often fails to provide accurate segmentation results due to the non uniformity of intensities. This paper proposes a novel region –based method for image segmentation, which is able to deal with intensity non uniformities in the segmentation of brain MRI images. First we derive a local intensity clustering criterion function for the image intensities, based on the model of images with intensity Inhomogeneity. This local clustering criterion function is integrated with respect to the neighbourhood centre to give a global clustering criterion function. In a multiphase level set formulation this criterion function defines an energy in terms of the level set functions that represents a partition of the image domain. The multiphase level set method is mainly used to represent triple junctions in the image. A bias field which accounts for the intensity non uniformity is identified. Therefore, by minimizing the energy our method can simultaneously segment the image and estimate the bias field. For intensity non uniformity correction the above estimated bias field is used. In the presence of intensity inhomogeneity our method has been validated on brain MRI images of various modalities, with desirable performance. Experimental results shows that our method is robust to contour initialization, faster and more accurate than existing methods*

**Keywords:** Level set, image segmentation, bias field, intensity Inhomogeneity, MR image

## 1. Introduction

In real world images intensity inhomogeneity occurs due to various factors such as illumination and imperfection of imaging devices, due to noise etc complicates many problems in processing the image. In fact, image segmentation is a considerably difficult task for images with intensity non uniformity. This non uniformity occurs due to overlaps between the range of the intensities in the regions to be segmented. So this is impossible to segment these regions based on pixel intensities. Based on the homogeneity of image intensity, there are so many image segmentation algorithms [1], [4] and these algorithms are not suitable for images with intensity Inhomogeneity. The intensity inhomogeneity can be modelled as a smooth and spatially varying field, multiplied by the constant true signal in the image. Bias field estimation is the procedure to estimate the bias field and restore the true signals by eliminating the effect of intensity non uniformity

The level set method, used as a numerical technique for tracking interfaces and shapes [2] applied increasingly to image segmentation in the past [3][4]. Contours or surfaces are represented as a zero level set of a higher dimensional function in the level set method usually called a level set function. With the level set method the image segmentation problem can be solved based on mathematical theories which include calculus of variations and partial differential equations. The mathematical computations involving curves and surfaces can be performed on a fixed Cartesian grid without having to parameterise these objects is the advantage of level set method. With the help of level set method Deformable models can be easily represented. The level set method is able to represent curves and surfaces with complex topology and change their topology in a logical way.

Image segmentation is an important image processing technique and it seems everywhere if we want to analyse what inside the image. Image segmentation is used to separate an image into several “meaningful” parts. Segmentation is mainly done on one of the two basic properties of intensity values. The properties are discontinuity and similarity. Similarity (homogeneity) in images is to partition an image into regions that are similar according to predefined criteria in images. Homogeneity refers to the uniformity of images. Discontinuity can be stated as partition of an image is based upon sharp changes in intensity. Most of the existing segmentation methods rely on the segmentation of homogeneous images. The accuracy of segmentation is determined by the essential success or failure of computerized analysis procedure. The considerable challenge in the field of medical image processing is to segment medical images in the presence of intensity inhomogeneity.

Image segmentation can be categorised in to two major classes: Region based models [4][5] and Edge based models [3][6][7]. In region oriented methods, pixels are assigned to a region if they are adjacent and homogeneous to the region. In edge oriented methods, pixels to be found must have a minimal gradient value. In the region based model identify region descriptor to guide the motion of the active contour in each region of interest. The most difficult task in this method is to define a region descriptor for intensity Inhomogeneity images. Based on the assumption of intensity homogeneity these models work [1][4][8]. An example for region based model is Piecewise constant model [1][4][8]. Based on the Piece wise smooth model Mumford and Shah [11] proposed the level set method [9][10]. The above method can be able to segment images with intensity Inhomogeneity, but the

disadvantage is, that they are computationally expensive and sensitive to the initialisation of contour [5]. The other, Edge based model use edge information for image segmentation, can applied to images with intensity Inhomogeneity. But these methods suffer from boundary leakage problems and quite sensitive to initial conditions

The main application of Image Segmentation is in the field of medical images. Medical resonance imaging (MRI) is the most important Para clinical measure for assessing and monitoring the pathologic changes implicated in the onset and progression of Multiple Sclerosis. Conventional MRI sequences only provide an incomplete picture of the degree of inflammation and underlying neurodegenerative changes in this disease. Segmented neural structures can be applied in the reconstruction of piecewise homogeneous digital head models. Segmented MRI can be applied directly in forming conductivity models consisting of volume elements, such as finite difference element models

In this paper a multiphase level set method for image segmentation is proposed for the segmentation of Brain MRI images. Firstly we derive a local intensity clustering property and then define a local clustering criterion function for the intensities in a neighbourhood of each point. The global criterion function is formed by integrating the local clustering criterion functions with respect to the neighbourhood centre. This criterion defines energy in terms of a level set function in a level set formulation. An interleaved process of level set evolution and bias field estimation is achieved by minimising this energy functional. The main application of our method is used for segmentation and bias correction of MRI images

## 2. Methodology

### A. Statistical image model with intensity in homogeneity

We formulate the following commonly used model to describe the images with intensity in homogeneity. The intensity in homogeneity is considered as a component of an image. The multiplicative model of intensity in homogeneity is attributed in this paper. An observed image  $I$  from an imaging device can be modelled as

$$I = bJ + n \quad [1]$$

Where  $I$  is the measured image,  $b$  is the bias field that accounts for intensity in homogeneity which is spatially variant,  $J$  is the true image which is assumed to be piece wise constant and  $n$  is the Gaussian distribution with zero mean variants. The bias field  $b$  is considered to be slowly varying

### B. Intensity clustering property.

Intensity in homogeneities in real world images lead to overlapped between the distributions of intensities in the regions of image. So it is impossible to segment these regions directly based on pixel intensities. There for base on the image model in [1], we are able to derive a usable property of local intensities which can referred as local intensity clustering property. For each position in the image domain, we consider a circular neighbourhood centre on the image with a radius. So the intensities in the set form a cluster with cluster centre  $m_i$ , which can be considered as samples drawn from a Gaussian distribution. Naturally  $N$  clusters are well separated.

### C. Energy Functional Formulation

The intensities in the neighbourhood can be classified in to  $N$  clusters with  $c$  enters  $m_i$  is specified in the local intensity clustering property. The standard K-mean clustering is used to classify the local intensities. The k-mean clustering algorithm is an iterative algorithm to minimize the clustering criterion [12], which can be written in continuous form where  $m_i$  is cluster centre if the  $i$ -th clusters. The member ship function  $u_i$  of the region  $\Omega_i$  is to be determined, where  $u_i(x) = 1$  for  $x \in \Omega_i$  and  $u_i(x) = 0$  for  $x$  not an element of  $\Omega$

$$\epsilon_y = \sum_{i=1}^N \int_{\Omega_i \cap \Omega_y} k(y-x) |I(x) - b(y)c_i|^2 dx \quad [2]$$

Where  $k(y-x)$  is introduce as a non negative window function also called Kernel function. With the window function clustering functional can be rewritten as

$$\epsilon_y = \sum_{i=1}^N \int_{\Omega_i} k(y-x) |I(x) - b(y)c_i|^2 dx \quad [3]$$

we can rewrite the energy  $\epsilon(\phi, c, b)$  in the following form:

$$\epsilon(\phi, c, b) = \int \sum_{i=1}^N e_i(x) M_i(\phi(x)) dx \quad [4]$$

Where  $e_i$  is the function defined by

$$e_i(x) = \int K(y-x) |I(x) - b(y)c_i|^2 dy \quad [5]$$

The functions  $e_i$  can be computed using the following equivalent expression:

$$e_i(x) = I^2 1_k - 2c_i I(b * K) + c_i^2 (b^2 * K) \quad [6]$$

where  $*$  is the convolution operation, and  $I_k$  is the function defined by  $I_k(x) = \int K(y-x) dy$ , which is equal to constant 1 everywhere except near the boundary of the image domain  $\Omega$ . By minimising this energy functional with respect to the regions  $\Omega_1, \dots, \Omega_N$ , constant  $c_1, \dots, c_N$  and bias field  $b$  the Kernel is flexible here we are using a truncated Gaussian uniform function such that  $\int k(u) = 1$ . The scale parameter of the Gaussian function is  $\sigma$ . According to the degree of intensity in homogeneity the radius should be selected the bias field  $b$  various faster for more localised intensity in homogeneity, so the approximation is valid only for a smaller neighbourhood.

### D. Multiphase level set formulation.

From the expression of  $\epsilon$  it is difficult to derive a solution to the energy minimisation problem. The energy  $\epsilon$  is converted to a level set formulation by representing the disjoint regions  $\Omega_1, \dots, \Omega_N$  with a number of level set functions and a regularisation term on this level set functions. A level set function is a function that takes positive and negative signs which can be used to represent the partition of the image domain  $\Omega$  in to two disjoint  $\Omega_1$  and  $\Omega_2$ .

$$\Omega_1 = \{ x : \phi(x) > 0 \} \text{ and } \Omega_2 = \{ x : \phi(x) < 0 \}$$

We use multiple level set functions to represent regions  $\Omega_i$  where  $i=1 \dots N$ . The level set formulation of the energy  $\epsilon$  for  $N > 2$  is called the multi phase formulation. In this paper we are considering the value of  $N=3$ . Therefore, we use two level set functions  $\phi_1$  and  $\phi_2$  to define the membership function  $M_1(\phi_1, \phi_2) = H(\phi_1)H(\phi_2)$ ,  $M_2(\phi_1, \phi_2) = H(\phi_1)H(1 - \phi_2)$ ,  $M_3(\phi_1, \phi_2) = H(1 - \phi_1)$ . These

level set functions  $\phi_1 \dots \dots \phi_k$  is represented by a vector valued function  $\vec{\phi} = (\phi_1 \dots \dots \phi_k)$ . Thus the membership function  $M_i(\phi_1(y) \dots \dots \phi_k(\phi))$  can be written as  $M_i(\vec{\phi})$ . The energy  $e_i(x)$  is given by equation [6]. In the multiphase formulation the energy  $\epsilon$  is given by

$$\epsilon(\vec{\phi}, c, b) = \int \sum_{i=1}^N e_i(x) M_i(\vec{\phi}(x)) dx \quad [7]$$

The regularization terms should also be defined

$$F(\vec{\phi}, b, c) \triangleq \epsilon(\vec{\phi}, c, b) + R_p(\vec{\phi}). \quad [8]$$

The gradient flow equation is used to minimize the above energy. By minimizing the above said energy the result of image segmentation and bias estimation is obtained. The energy minimization is achieved by an iterative process: in each iteration, the energy  $F(\vec{\phi}, b, c)$  with respect to  $\vec{\phi}$ ,  $c$  and  $b$  is updated.

1) Energy Minimization with respect to  $\vec{\phi}$

For this  $c$  and  $b$  is fixed. The gâteaux derivative is used for the estimation is given by

$$\frac{\partial \epsilon}{\partial \vec{\phi}} = - \frac{\partial F}{\partial \vec{\phi}} \quad [9]$$

The gradient flow equation is used for solving this is given by

$$\frac{\partial \vec{\phi}}{\partial \tau} = - \delta(\vec{\phi})(e_1 - e_2) + \nu \delta(\vec{\phi}) \operatorname{div} \left( \frac{\nabla \vec{\phi}}{|\nabla \vec{\phi}|} \right) + \mu \operatorname{div}(d_p(|\nabla \vec{\phi}|) \nabla \vec{\phi}) \quad [10]$$

where  $\nabla$  is the gradient operator,  $\operatorname{div}(\cdot)$  is the divergence operator

2) Energy Minimization with respect to  $c$

For this,  $\vec{\phi}$  and  $b$  is fixed, compute optimal  $c$  that minimizes the energy  $\epsilon(\vec{\phi}, c, b)$  denoted by  $\hat{c}_i = (\hat{c}_1, \dots, \hat{c}_N)$ , is given by

$$\hat{c}_i = \frac{\int (b * K) u_i dy}{\int (b^2 * K) u_i dy}, i=1, \dots, N \quad [11]$$

with  $u_i(y) = M_i(\vec{\phi}(y))$

3) Energy Minimization with respect to  $b$

For this calculation,  $\vec{\phi}$  and  $c$  is fixed compute the optimal  $b$  that minimizes the energy  $\epsilon(\vec{\phi}, c, b)$  denoted by  $\hat{b}$ , is given by

$$\hat{b} = \frac{(J^{(1)} * K)}{(J^{(2)} * K)} \quad [12]$$

where  $J^{(1)} = \sum_{i=1}^N c_i u_i$  and  $J^{(2)} = \sum_{i=1}^N c_i^2 u_i$ . Note that the convolutions with a kernel function  $k$  in (12) confirms the slowly varying property of the derived optimal estimator  $\hat{b}$  of the bias field.

**E. Numerical Implementation**

The finite difference scheme as for DRLSE is used in the implementation of equation(10). The Heaviside function  $H$  is

changed by smoothed Heaviside function  $H_\epsilon$  which is given by

$$H_\epsilon(x) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \arctan(x/\epsilon) \right] \quad [13]$$

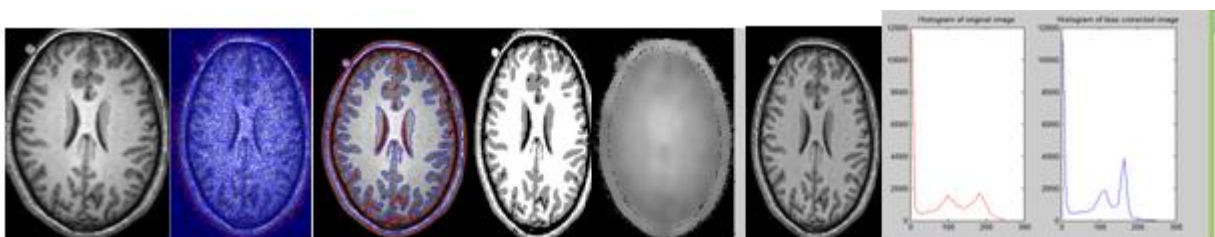
With the value of  $\epsilon=1$ . The derivative of Heaviside, which is the dirac delta function is replaced by the derivative smoothed Heaviside function  $H'_\epsilon$  of is given by

$$\partial_\epsilon(x) = H'_\epsilon(x) = \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + x^2} \quad [14]$$

At each time step the constant  $c$  and bias field  $b$  is updated. The two convolutions  $b * k$  and  $b^2 * k$  for the computation of  $\hat{c}_i$  also appear in the computation of  $\hat{c}_i$  for all  $i=1 \dots N$ . Another two convolutions  $I(J^{(1)} * K)$  and  $J^{(2)} * K$  are computed for the bias field  $b$ . Thus, there are a total of four convolutions to be computed at each time step during the evolution of  $\vec{\phi}$ . A  $w \times w$  mask, with  $w$  being the smallest odd number is constructed as the convolution Kernel  $k$ , such that  $w \geq 4 * \sigma + 1$ , when  $k$  is defined as the Gaussian kernel. For example, given a scale parameter  $\sigma=4$ , the mask size is  $17 \times 17$ . The parameters  $\mu$  and the time step  $\Delta t$ , can be fixed as  $\mu=0.1$  and  $\Delta t = 0.1$ . Our model is not sensitive to the choice of the parameters. The parameter  $\nu$  is usually set to  $0.0001 \times 255^2$  as a default value for most of digital images with intensity range in  $[0, 255]$ . The parameter  $\sigma$  and the size of the neighbourhood (specified by its radius) should be relatively smaller for images with more localized intensity inhomogeneities

**3. Experimental Results**

The scale parameter  $\sigma$  is set to 4 for the experiments. We can apply the value of  $\sigma$  as 4 to 15, the value will be 0.5 pixel for all the different values. Fig 1 shows the 3T MRI image. We are using multiphase level set representation so the three regions can be easily identified. The initial contours can be set inside, outside or across the boundary. The estimated bias field  $\hat{b}$  can be used for intensity Inhomogeneity correction. The bias corrected image is given by  $\text{Image} / \hat{b}$ . The corresponding results of segmentation, bias field estimation and bias correction are shown in figure 1. The results indicates desirable performance of our method in segmentation and bias correction. We use contour based metric for the evaluation of the segmentation result. The subpixel accuracy of the segmentation result is evaluated by this contour based metric. The histogram, of the original image and bias corrected images clearly indicates the difference. The main advantage of our method is robustness to contour initialization.



**Figure 1:** Application of our method to 3T MR images. Column 1: Original image; Column 2: Zero level contours; Column 3: final zero level contours; Column 4: segmentation result; Column 5: Bias field estimation; Column 6: Bias corrected image; Column 7: Histogram of the original image (left) and histogram of bias corrected image (right)

## 4. Conclusion

A multi phase level set method of image segmentation and bias correction is described in this paper. With the help of this method we can represent triple junctions easily. Based on the images with intensity Inhomogeneity a local intensity property is derived, then an energy functional is estimated. The minimization of this energy term helps for segmentation and bias correction of images. The slowly varying property of bias field which is derived from the proposed energy is naturally ensured by the data term. The main advantage of our method is more robust to contour initialization than existing methods. The experimental results have depicted the superior performance of our method in case of accuracy, robustness and efficiency. Our method is mainly applied to MR images.

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