Stress-strain relationship becomes

$$\sigma = E\varepsilon = EBq \qquad (4)$$

Explicit formulations are as shown below
Dynamic Equilibrium equation

$$M^t \ddot{u} + C^t \acute{u} \; + F^t_{int} \; = F^t_{ext} \qquad (5)$$

F^t_{int} =K u is the vector of internal forces resulting from the element stiffness and displacement

F^t_{ext} is the sum of all external forces including frictional forces due to contact.

ü, ú,u are the nodal acceleration, velocity and displacement.
M and C are mass and damping matrices

Central difference time integration was used to update values in increments as in:

$$\ddot{u} = (M^t)^{-1} \; (F^t - F^t_{int}) \qquad (6)$$

$$\acute{u}^{t+\frac{1}{2}} = (M^t)^{-1} \; (F^t - F^t_{int}) \; \Delta t^{t+\frac{1}{2}} + \acute{u}^{t-\frac{1}{2}} \qquad (7)$$

$$u^{t+1} = u^t + \acute{u}^{t+\frac{1}{2}} \; \Delta t^{t+1} \qquad (8)$$

There is no iteration within this increments.t denotes the step time.

Penalty method is used for contact detection in this contact bending problem so that no convergence problem arises within contact module algorithm.
Force at contact detection point,

$$F = k_p * g_p \qquad (9)$$

k_p - Penalty stiffness
g_p - Gap function(contact if g_p=0, penetration if g_p<0)
Rigid rollers thus apply force on the beam

3.2 Simulation

MCP100 (Medium Weight Parallel flange of depth 100) sections are selected .This standard dimensions are strictly followed from Indian Standard Medium Weight Beams (ISMB).S235 grade mild steel, S stands for structural steel. The number indicates the minimum yield strength.Poisson's ratio is 0.3, Young's modulus is 210e3 MPa, Yield strength of 300MPa, density of 7.8e-9 tons/mm^3, chemical composition with carbon (0.22%), phosphorous (1.6%), manganese (0.05%), sulphur (0.05%), silicon (0.05%).

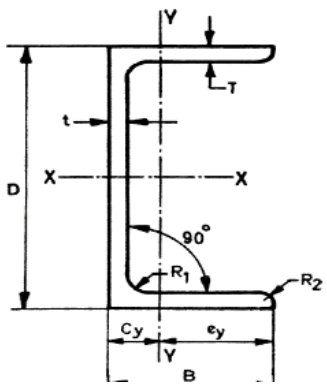

Figure 3: Beam specifications

The specifications shown in figure 3 is quantified in table 1.

Table 1: Beam dimensions

D(mm)	100
B(mm)	50
T(mm)	5
t(mm)	8

Simulation was carried out in Abaqus/Explicit module using three steps. First step dedicated to loading and rolling, second step for unloading. Third for attaining steady state. This was crucial for simulation. 15 variety of other variations has been tried and ruled out due to various reasons like web crippling, uneven loading, high roller speed, disturbances in beam due to stick-slip, etc. C3D8R element with three translational D.O.F was used for structural meshing characterized by regular mesh shown in figure 4.

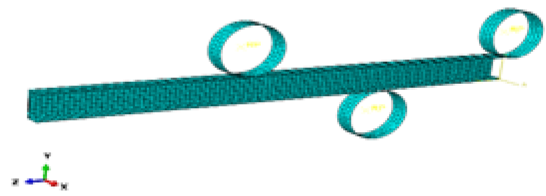

Figure 4: Meshed model

R3D4 elements with three translational and three rotational D.O.F was used for rollers since no stress/strain output from roller elements were required. Brick element chosen for beam consumed more time but justified. Penalty contact was used to define contact with tangential interaction properties. Translation about X axis is arrested in beam surface. Top right roller is arrested in all D.O.F except translation in y direction.Centre roller is arrested about all D.O.F except rotation in x direction.Top left roller is arrested about all D.O.F

4. Results and Discussions

4.1 Theoretical Results

Theoretical model calculate from Timoshenko beam model was based on the assumption that sudden stress reversal happens at the neutral axis. Graph is plotted in Origin pro software with values exported from MS Office.

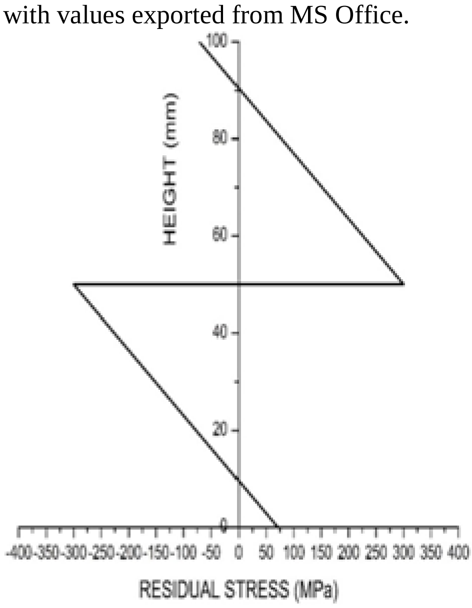

Figure 5: Theoretical Residual Stresses

The maximum tensile and compressive stresses obtained are 300MPa and -300MPa respectively. Conceptual sudden stress reversal is shown at middle and gradual stress reversal at top and bottom flanges are as shown in figure 5.

4.2Numerical Results

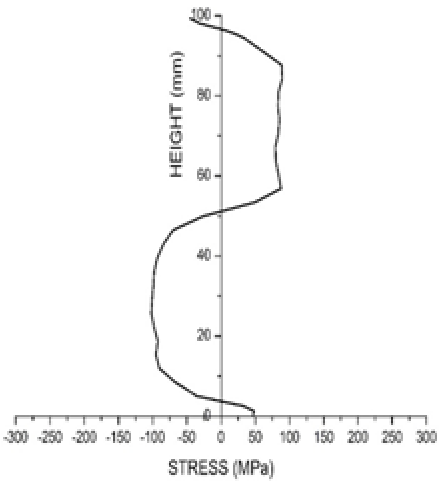

Figure 6: Theoretical Residual Stresses

The numerical results plotted in Origin Pro software using exported values from Abaqus v6.10 is as shown in figure 6.

5. Comparison

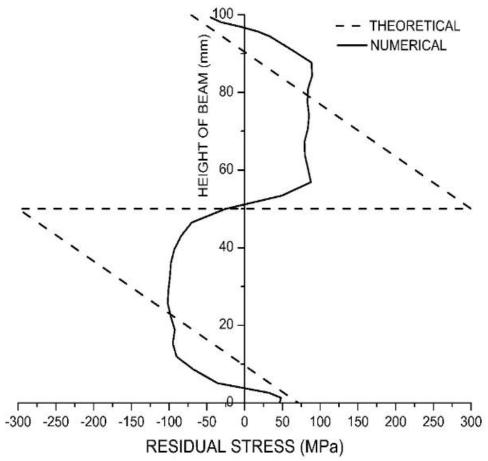

Figure 6: Theoretical Residual Stresses

Theoretical and Numerical results showed moderate coherence. This was due to the fact that extreme values are high which is based on assumption that sudden stress reversal occurs at a single layer or neutral axis which is practically impossible. Stresses can only gradually reverse its direction or value in practical scenario. This explains one of the over simplification in Timoshenko beam model. The top and bottom portion showing almost gradual reversal is shown with matching numerical results.

6. Conclusions

The simulation of C section bending using pyramid roller bending process is successfully conducted without web crippling. Gradual loading with experience and trial and error method was adopted for this.

Simulation is done in explicit module which eliminates any convergence difficulty with contact, different load, and speed variation unlike in static analysis and care has been taken to avoid stick-slip phenomenon.

Obtained numerical results matched the pattern of theoretical residual stresses with major tensile stresses on top and major compressive stresses on bottom with reasonable explanation for differences in extreme values.

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