All Path Algorithm for CPM Networks

Kuldeep Singh Kushwah[1], Sanjay Tiwari[2]

[1]ME CTM Student, MITS Gwalior

[2]Associate Professor, MITS Gwalior

Abstract: ***Time Cost Trade-off refers to expedite some selected activities in a project to complete it an earlier deadline than its normal project duration. Mathematical formulation is presented for TCT in this paper. Paper discusses in detail the procedure of exploring all paths in project network. All paths need to be explored in TCT analysis as the constraint is that time wise all path length shall not exceed desired due date duration.***

Keywords: CPM

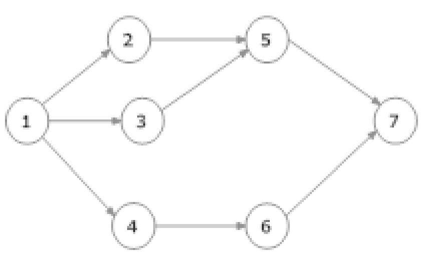
1. Introduction

A Construction project consists of several interdependent activities. Some of these activities run sequentially while other may run in parallel. Project network techniques are commonly used for planning and control of these projects. CPM and PERT are two such commonly used network techniques. A CPM network may be represented either in Activity-on-Arrow or Activity (AoA) on Node (AON) network. CPM assumes fixed duration for each activity and this is the severe limitation. The project duration is obtained by summing the durations of all activities along the critical path (time-wise longest path) and project cost is sum of cost of all the activities of the project. If the project is to be completed earlier than its normal completion time some of the activities on critical path are to be speeded up by employing more resources and / or using alternate technologies etc. this in turn increases the cost of the activities and thus the project cost.CPM fails to make decisions in such situations. Time Cost Trade-off (TCT) technique need to be employed to enhance the capability of CPM in making logical decisions about which activities and to what extent shall be expedited to achieve the set project deadline so as to minimize the cost of expedition. To meet the earlier deadline one of the prime constraint is that length (time wise) of all paths shall be less than or equal to specified due date duration. Thus all paths in a network are to be explored which is a difficult task as the problem in literature is known as NP-hard.The paper presents an 'All path Algorithm' for exploring all paths from first activity to last activity in a project network which is basically the so called Directed Acyclic Graph (DAG) in Graph Theory.

2. Backtracking

Backtracking is a straight ford algorithmic technique which can be used to solve a variety of problems. Here we present an approach to solve the time-cost trade-off problem employing backtracking as one of its component. We use backtracking to enumerate all the paths from source to sink in the activity network which is a (Directed Acyclic Graph) DAG. Enumerating all parts helps us in formulating constraints representing network relationships.

We explore the graph using its adjacency list in a "depth-first" manner i.e. once we have arrived at a vertex, we explore all possible paths from this vertex to the sink and then we "backtrack" to the predecessor of this vertex to explore other possible paths. This process is repeated until all possible paths (from source to sink) have been discovered. After all paths have been traced, we can easily formulate the network constraints.



Project network

1 → 2 → 5 → 7

1 → 3 → 5 → 7

1 → 4 → 6 → 7

All paths in the example network

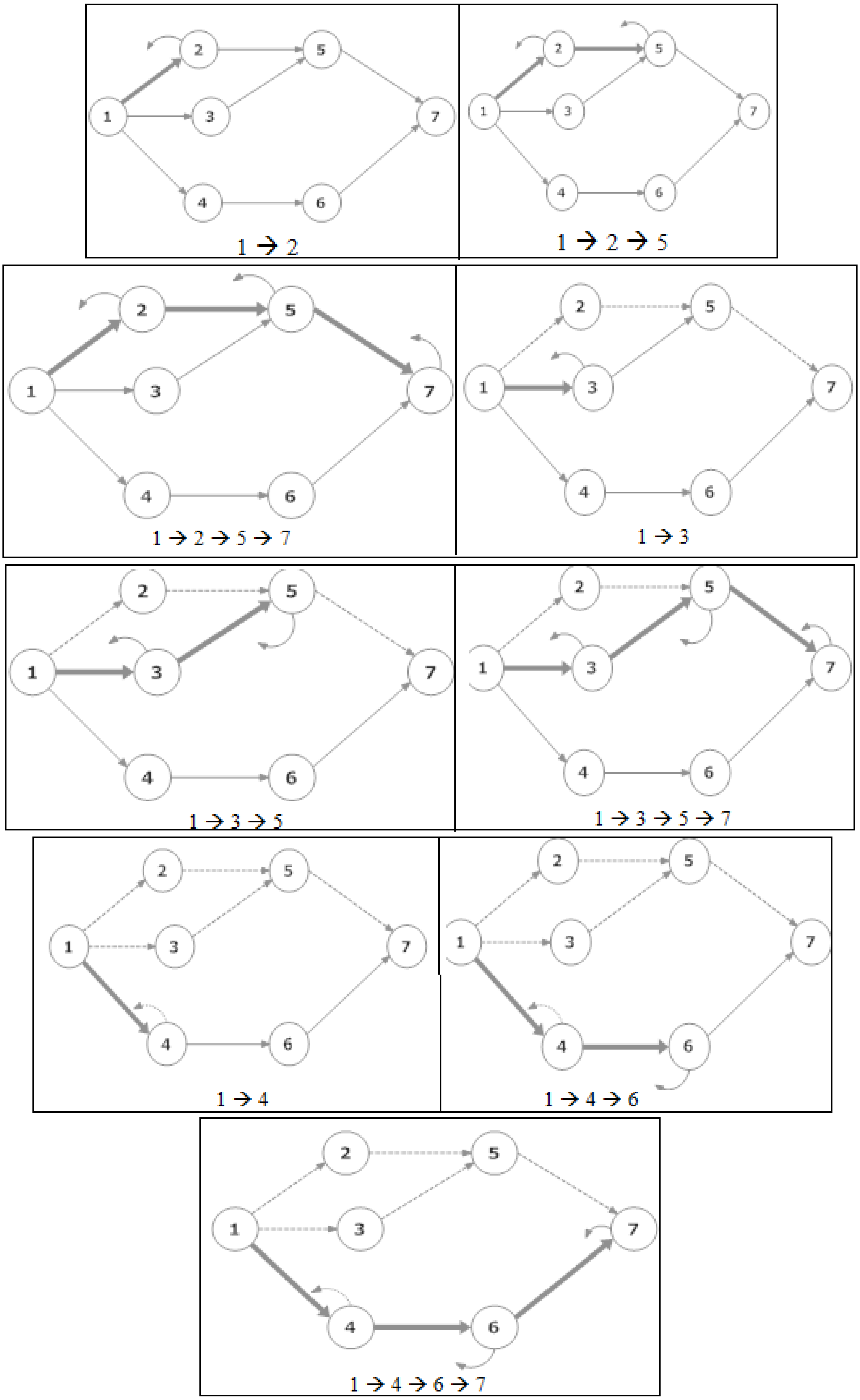


Figure 1: Backtracking to explore all paths

3. Integer Programming Model

The TCT decision refer to select activity option for each activity in such a manner that total cost of project for a specified duration is least. Mathematically,

Objective function: Minimize $\sum_{i=1}^{n} C_i$

Subject to:
Network Constrains:

$$D_1 + D_2 + D_5 + D_7 \leq D$$

$$D_1 + D_3 + D_5 + D_7 \leq D$$
$$D_1 + D_4 + D_5 + D_7 \leq D$$
$$D_{ic} \leq D_i \leq D_{iN}$$

Where, D_i *is the duration of activity i*,

D_{iN} is the normal duration of activity i,

D_{ic} is the crash duration of activity i, and

D is the specified deadline.

The problem we have is a discrete time-cost trade-off problem. For a given activity, we have a set of available options, out of which our objective is to choose the optimal one. This is done with the help of integer programming. We introduce a Boolean variable which is either 0 or 1. For example, X_{ij} stores the $0/1$ value for the j^{th} option of the i^{th} activity. If $X_{ij} = 1, j^{th}$ option of the i^{th} option of the activity is selected as our optimal choice otherwise not. For each activity *i*, the sum of the variables X_{ij} should be one because only one option is selected as the optimal option for the activity *i*. So besides network constraints, we have following constraints too.

$$\sum_{j=1}^{O_i} C_{ij} X_{ij} = C_i, \text{for all activity } i \quad (1)$$

$$\sum_{j=1}^{O_i} D_{ij} X_{ij} = D_i, \text{for all activity } i \quad (2)$$

$$\sum_{j=1}^{O_i} X_{ij} \geq 0, \text{for all } X_{ij} \text{ are integers} \quad (3)$$

i.e.
$\sum_{j=1}^{oi} X_{ij}$=1; for all activity i

$\forall X_{ij} \geq 0$; $\forall X_{ij} \leq 1$; $\forall X_{ij}$ are integer

Parameters:
O_i: number of modes for activity i

C_{ij} = the cost of option j, activity i;
D_{ij} = the duration of option j, activity i;
C_i=cost of activity i;
D_i = duration of option i,
D= project deadline duration;
n=total number of actual activities;
X_{ij} = assigned variable for option j, activity i,
={1, if activity is assigned to mode k
0, otherwise}.

4. Summary

A mathematical model forTCT technique is presented as integer programming model where objective function is to minimize expedition cost. The objective to be achieved subject to some constraints and foremost among them is that all path lengths shall be less than or equal to specified deadline duration. Backtracking has been suggested foe exploring all paths.

References

[1] Ahuja, V., and Thiruvengadam, V. (2004). "Project scheduling and monitoring: current research status." Construction Innovation, 4(1), 19-31.
[2] Butcher, W.S., (1967), "Dynamic programming for project cost-time curves", Journal of the Construction Division,Proceedings of the ASCE93,pp.59-73.
[3] Demeulemeester,E., Herroelen,W.,andElmaghraby,S.E.,(1996),"Optimal procedures for the discrete time/cost trade-off problem in project networks", EuropeanJournalofOperationalResearch,88,pp. 50–68.
[4] Franco, E. G., Zurita, F. L., Delgadillo, G. M., 2006 " A genetic algorithm for the resource constrained project scheduling problem." Report, School of Industrial Engineering, Universidad de La Sabana.
[5] Goncalves, J. F., Mendes, J. J. M., Resende, M. G. C., 2006, " A genetic algorithm for the resource-constrained multi-project scheduling problem." AT&T Labs Research Technical Report: TD-668LM4, Revised January 20.
[6] Robinson, D.R. (1975). "A dynamic programming solution to cost-time trade-off for CPM." Mgmt. Sci., 22(2), 158-166.