

An Interpolation Technique for Channel Estimation in OFDM Systems

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Abstract: *In this paper we investigate the bit error rate (BER) performance of orthogonal frequency division multiplexing (OFDM) wireless communication system with the implementation of the LS-Interpolation-based on pilot symbol-assisted channel estimation algorithm over the frequency selective multi-path Rayleigh fading channel. The Least square (LS) method is used for estimation of channel at the pilot frequencies, using different interpolation techniques such as spline cubic interpolation, cubic interpolation, low-pass interpolation and FFT interpolation, linear interpolation are employed to interpolate the channel at data frequencies. In signal mapping, the OFDM system incorporates M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM) digital modulation techniques. Matlab simulations are carried out to analyze the performance of the developed OFDM system with the employment of pilot based channel estimation algorithms for various digital modulations in Rayleigh fading channel. A comparative analysis of the proposed interpolation with conventional one dimensional interpolation techniques shows its performance.*

Keywords: Channel estimation, Orthogonal frequency division multiplexing (OFDM), LS estimation, Interpolation techniques.

1. Introduction

Applications of orthogonal frequency-division multiplexing (OFDM) to wireless and mobile communications are currently under study. Although multicarrier transmission has several considerable drawbacks (such as high peak to average ratio and strict requirements on carrier synchronization), its advantages in lessening the severe effects of frequency selective fading without complex equalization are very attractive features. In order to obtain the high spectral efficiencies required by future data wireless systems, it is necessary to employ multilevel modulation with nonconstant amplitude (e.g., 16QAM). This implies the need for coherent receivers that are capable to track the variations of the fading channel.

The channel estimation for the block-type-pilot arrangement can be based on Least Square (LS) or Minimum Mean Square Error (MMSE) method. Contemporary research shows that the MMSE estimator can provide 5-10 dB gain in signal to noise ratio (SNR) for the same mean square error of channel estimation over LS estimator. The MMSE method uses the channel statistical properties including the channel autocorrelation matrix and the noise variance. However, in practical wireless environments the channel statistical properties cannot be measured accurately. On top of that, computational complexity of the MMSE estimator is relatively higher than that of the LS estimator. Therefore, in practical OFDM systems the LS method is widely implemented due to its low complexity and minimum requirements of knowing the channel state information. Nevertheless, the LS estimator suffers from inherent Additive White Gaussian Noise (AWGN) and Inter Carrier Interference (ICI), which subsequently results in degradation on the receiver performance [8]. As a matter of fact, estimator design can be viewed as a trade-off between

achieving a desired level of performance and maintaining a low complexity.

Following the above background and problem statements, one of the major aims of the paper is to gain a thorough understanding of existing LS and MMSE algorithms, followed by proposing a revised algorithm both for LS and MMSE to minimize their existing limitations. It is worthwhile to mention that the paper is an extended version of the previous research carried out in. While, the former had not incorporated the issue of minimizing the computational overhead associated with the existing methods, the current study develops low complexity algorithms to yield optimal or near-optimal designs for the estimators. This study begins by conducting a comprehensive performance evaluation of LS and MMSE algorithms for the OFDM system.

Subsequently, we propose a new estimator, which is based on conventional LS algorithm and named as Simplified Least Square (SLS) estimator in our study. The proposed estimator mainly utilizes a modified weighting matrix and establishes no dependency on the original transmitted signal. The SLS can reduce the noise and the interference significantly by performing autocorrelation operation between the derived weighting factor and the channel attenuation.

In this paper, an effective interpolation technique is presented for estimating the channel frequency response at data subcarriers for comb-type pilot insertion scheme. The proposed interpolation technique is based on reduction of the interpolation error and channel noise inherent in the LS estimation. The proposed interpolation is based on a concept of concentrated channel response in the time domain and distributed channel response in the frequency domain. In the first step of the proposed interpolation technique, the computation of the frequency response of the multipath

fading channel using one dimensional interpolation techniques is carried out. After the process of estimating the channel frequency response at all tones, the conversion of the estimated channel response from frequency domain into time domain is performed through the operation of IFFT, then zeros are padded at insignificant subcarrier positions that contains only noise. Finally, the time domain channel impulse response is converted into frequency domain through the operation of the Fast Fourier Transform (FFT). BER performance comparison of the proposed interpolation technique reveals its performance improvement over the conventional 1-D interpolation techniques. The performance improvement of the proposed interpolation algorithm is at the cost of the computational complexity associated with FFT and IFFT.

The rest of the paper is organized as follows: Section 2 explains the OFDM system model. Channel estimation algorithm and the proposed interpolation technique are discussed in Section 3 and Section 4 respectively. Section 5 describes the simulation results and finally, the paper is concluded in Section 6.

2. OFDM System Model

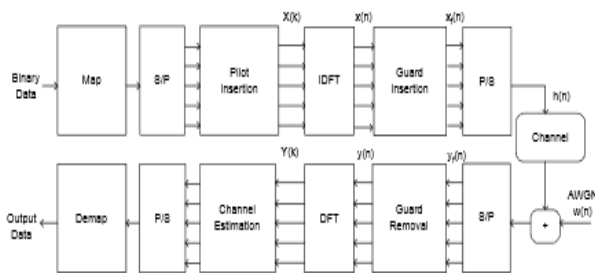


Figure 1: Baseband OFDM System

The OFDM system based on pilot channel estimation is given in Figure 1. The binary information is first grouped and mapped according to the modulation in "signal mapper". After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length N $\{X(k)\}$ into time domain signal $\{x(n)\}$ with the following equation

$$x(n) = \text{IDFT}\{X(k)\} \quad n = 0, 1, 2, \dots, N-1$$

$$= \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi kn}{N}}$$

Where N is the DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follows:

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, \dots, -1 \\ x(n), & n = 0, 1, \dots, N-1 \end{cases}$$

where N_g is the length of the guard interval.

The transmitted signal $x_f(n)$ will pass through the frequency selective time varying fading channel with additive noise. The received signal is given by

$$y_f(n) = x_f(n) \otimes h(n) + w(n)$$

Where $w(n)$ is additive white gaussian noise and $h(n)$ is the channel impulse response. The channel response h can be represented by

$$h(n) = \sum_{i=0}^{r-1} h_i e^{j \frac{2\pi}{N} f_{D_i} T n} \delta(n - \tau_i) \quad 0 \leq n \leq N-1$$

Where r is the total number of propagation paths, h_i is the complex impulse response of the i th path, f_{D_i} is the i th path Doppler frequency shift, τ_i is delay spread index, T is the sample period and τ_i is the i th path delay normalized by the sampling time. At the receiver, after passing to discrete domain through A/D and low pass filter, guard time is removed:

$$y(n) = \begin{cases} y_f(n) & \text{for } -N_g \leq n \leq N-1 \\ y_f(n+N_g) & n = 0, 1, \dots, N-1 \end{cases}$$

Then $y(n)$ is sent to DFT block for the following operation:

$$Y(k) = \text{DFT}\{y(n)\} \quad k = 0, 1, 2, \dots, N-1$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi kn}{N}}$$

Assuming there is no ISI, relation of the resulting $Y(k)$ to $H(k) = \text{DFT}\{h(n)\}$, $I(k)$ that is ICI because of Doppler frequency and $W(k) = \text{DFT}\{w(n)\}$, with the following equation

$$Y(k) = X(k)H(k) + I(k) + W(k) \quad k = 0, 1, \dots, N-1$$

Where

$$H(k) = \sum_{i=0}^{r-1} h_i e^{j \pi f_{D_i} T} \frac{\sin(\pi f_{D_i} T)}{\pi f_{D_i} T} e^{-j \frac{2\pi \tau_i}{N} k}$$

$$I(k) = \sum_{i=0}^{r-1} \sum_{\substack{K=0, \\ K \neq k}}^{N-1} \frac{h_i X(K)}{N} \frac{1 - e^{j 2\pi (f_{D_i} T - k + K)}}{1 - e^{j \frac{2\pi}{N} (f_{D_i} T - k + K)}} e^{-j \frac{2\pi \tau_i}{N} K}$$

Following DFT block, the pilot signals are extracted and the estimated channel $H_e(k)$ for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by

$$X_e = \frac{Y(k)}{H_e(k)} \quad k = 0, 1, \dots, N-1$$

Then the binary information data is obtained back in "signal demapper" block.

3. Channel Estimation Algorithm

One dimensional (1D) Channel estimation in OFDM has two common types i.e. block-type and comb-type; based upon the arrangement of pilots. Block-type channel estimation is used

for slow fading channels while comb-type is best suited for fast fading channels. Arrangement of pilots for comb-type and block-type channel estimation is shown in fig.2. A comb-type channel estimation has been used because of the use of the fast fading Rayleigh channel for performance analysis of the OFDM system. Equi-spaced pilot insertion is adopted because of optimum performance. The channel frequency response at pilot subcarrier is estimated by using MMSE estimator because of its superior performance as compared to least square (LS) estimator.

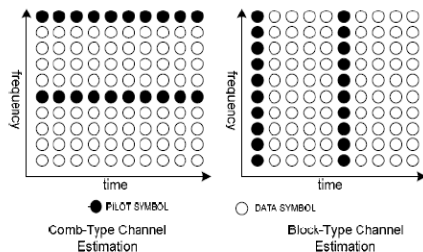


Figure 2: Arrangement of Pilots

A. Channel Estimation at Pilot Frequencies

In comb-type pilot based channel estimation, the pilot signals are uniformly inserted into $X(k)$ according to the following equation:

$$X(k) = X(mL+l)$$

$$= \begin{cases} x_p(m), l = 0 \\ \text{inf data } l = 1, \dots, L-1 \end{cases}$$

Since LS estimate is susceptible to noise and ICI, MMSE is thought about while compromising complexity. Since MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator is suggested in. In this simplified version, the inverse is only need to be calculated once. In [13], the complexity is further reduced with a low-rank approximation by using singular value decomposition.

B. LS Estimator

The LS estimator for the cyclic impulse response g minimizes

$$(xFg)(y - xFg)^H$$

and generates the channel attenuation as bellow

$$h_{LS} = FQ_{LS}F^H x^H y$$

$$Q_{LS} = (F^H x^H Fx)^{-1} \text{ and } (y - xFg)^{(H)}$$

are the conjugate transpose operations. Hence,

$$h_{LS} = x^{-1}y$$

where h_{LS} is the channel attenuation for LS

4. Proposed Interpolation Technique

The proposed interpolation technique is based on reduction of the interpolation error and channel noise associated with LS estimation. The LS estimator does not consider the channel noise during the estimation process and thus, its performance degrades. The conceptual view of the LS estimator is illustrated in Fig. 3. The nature of the Rayleigh fading channel in time domain is concentrated while distributed in frequency domain. The sample spaced channel has all the fading impulses at integer multiples of the system sampling rate and there is no energy leakage between the channel taps. The distribution of channel energy in time domain and frequency domain is shown in Fig. 4. The channel frequency response computed at data subcarriers by using 1-D interpolation techniques is subject to interpolation error.

The proposed interpolation technique illustrated in Fig. 5 consists of the following steps:

- Interpolate the channel frequency response estimate by LS estimator at the pilot subcarriers using one dimensional interpolation technique and then, take the N-point IFFT.
- Pad N-L (where L = Channel order) zeros at the end of the channel frequency response computed after the interpolation step.
- Finally, take the N-point FFT of the zero padded sequence to yield the channel frequency response at all subcarriers.

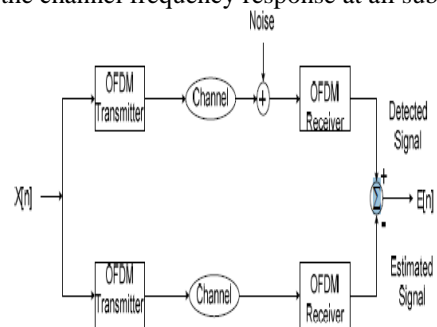


Figure 3: Conceptual View of LS estimator the

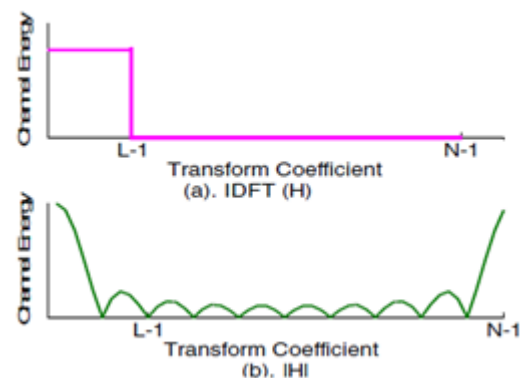


Figure 4: Channel Energy Distribution in time and frequency domain

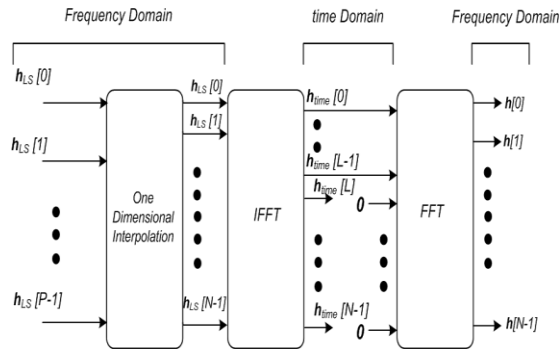


Figure 5: Proposed Interpolation Technique

5. Simulation Results

Figs. 6 & 7 illustrate the performance comparison curves of the uncoded OFDM system over Rayleigh fading channel for QPSK and BPSK modulation schemes respectively. It is clear from the Figs. 6 & 7 that the performance of the proposed interpolation technique using different one dimensional interpolation techniques is better than one-dimensional interpolation techniques. This performance improvement of the proposed interpolation over one-dimensional is because of the noise cancellation after estimating the channel frequency vector for all subcarriers. The channel frequency response computed after interpolation technique is converted into time domain and then, zeros are padded and finally, the zero padded sequence is converted into the frequency domain. The estimated frequency response has a low interpolation error as compared to the channel frequency response estimated by using onedimensional interpolation techniques.

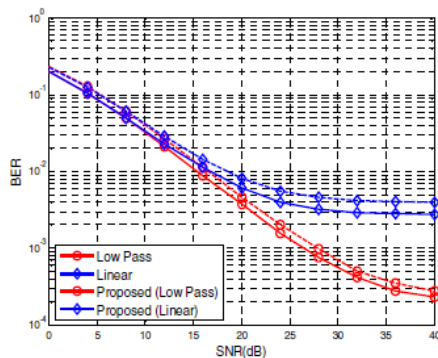


Figure 6: Performance comparison of BPSK Modulated OFDM for different Interpolation techniques

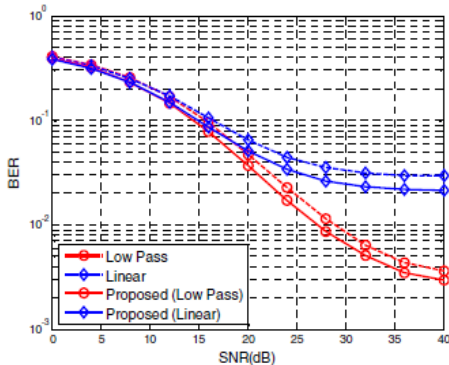


Figure 7: Performance comparison of 16-QAM Modulated OFDM for different Interpolation techniques.

The performance of uncoded OFDM with different modulation schemes for proposed interpolation technique using Low pass interpolation is shown in Fig. 8. It is clear from the Fig. 8 that the performance degradation for higher modulation scheme occurs. This degradation in BER performance is due to the nearby positioning of the constellation points for higher order modulation schemes.

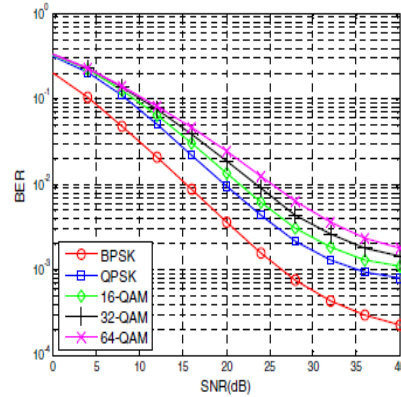


Figure 8: Performance comparison of the OFDM System with Different Modulation Schemes for Proposed Interpolation technique using Low pass interpolation

6. Conclusion

In this paper we investigated the BER performance of BPSK and QAM-modulated OFDM wireless communication systems with the implementation of LS-Interpolation-based comb-type pilot symbol-assisted channel estimation algorithm over frequency selective multi-path Rayleigh fading channel. In channel estimation, the OFDM system employed Least square estimator for the estimation of channel at pilot frequencies while different interpolation techniques are used to interpolate the channel at data frequencies. Simulation results show that the proposed OFDM system with LS channel estimator achieves good error rate performance under the BPSK and QAM modulation schemes over Rayleigh fading channel. The proposed Interpolation technique using Low-pass interpolation performs better in channel frequency response estimation than other studied interpolation algorithms and the BER performance of OFDM system with comb pilot-assisted channel estimation is less affected by Doppler frequency.

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