$$I_{0,z}^{\beta,\eta,\delta} f(z) \frac{\Gamma(p-\eta+1)\Gamma(p+\beta+\delta+1)}{\Gamma(p+1)\Gamma(p-\eta+\delta+1)} z^{\eta} = z^p + \sum_{k=n+p}^{\infty} a_k z^k \phi(k,\eta,\beta,\delta)$$

Where

$$\phi(k,\eta,\beta,\delta) = \frac{\Gamma(k+1)\Gamma(k-\eta+\delta+1)\Gamma(p-\eta+1)\Gamma(p+\beta+\delta+1)}{\Gamma(k-\eta+1)\Gamma(k+\beta+\delta+1)\Gamma(p+1)\Gamma(p-\eta+\delta+1)} \quad \ldots(3.24)$$

Since for $k \geq n+p, \phi(k,\eta,\beta,\delta)$ is a decreasing function of k.

We have

$$\phi(k,\eta,\beta,\delta) \leq \phi(n+p,\eta,\beta,\delta)$$

i.e.

$$\phi(k,\eta,\beta,\delta) \leq \frac{\Gamma(n+P+1)\Gamma(n+p-\eta+\delta+1)\Gamma(p-\eta+1)\Gamma(p+\beta+\delta+1)}{\Gamma(n+p-\eta+1)\Gamma(n+p+\beta+\delta+1)\Gamma(p+1)\Gamma(p-\eta+\delta+1)} \quad \ldots(3.25)$$

on setting

$$G(z) = I_{0,z}^{\beta,\eta,\delta} f(z) \frac{\Gamma(p-\eta+1)\Gamma(p+\beta+\delta+1)}{\Gamma(p+1)\Gamma(p-\eta+\delta+1)} z^{\eta} \quad \ldots(3.26)$$

There for this above equation in view of (3.24), (3.25), reduce in to the following inequality

$$|G(z)| \leq |z|^p + \phi(n+p,\eta,\beta,\delta)|z|^{n+p}$$

$$\sum_{k=n+p}^{\infty} a_k \leq |z|^p + \phi(n+p,\eta,\beta,\delta)|z|^{n+p} a_{n+p} \quad \ldots(3.27)$$

Now from the Lemma – 2 on using (2.6), (2.7), we have

$$\sigma(\gamma,m,n+p) \sum_{k=n+p}^{\infty} a_k \leq \sum_{k=n+p}^{\infty} \sigma(\gamma,m,k) a_k \leq (1-\gamma)\, a_{n+p} \leq \frac{(1-\gamma)}{\sigma(\gamma,m,n+P)} \quad \ldots(3.28)$$

The result in (3.27) in view of (3.28) takes the following form

$$|G(z)| \geq |z|^p + \phi(n+p,\eta,\beta,\delta) \frac{(1-\gamma)}{\sigma(\gamma,m,n+P)} |z|^{n+p}$$

$$m \geq 0, -1 \leq \gamma < 1$$

Now on using the result in (3.25) and (3.26), we at once arise at the derived result in (3.7) for the function defined in (3.8).

4. Special Cases

(1) If in (3.1) and (3.3), we take n = p = 1, then these results reduce to the following known results [2, p. 2811, eqns. (3.7), (3.8)] i.e.

$$\left|I_{0,z}^{\beta,\eta,\delta} f(z)\right| \geq \frac{\Gamma(2-\eta+\delta)}{\Gamma(2-\eta)\Gamma(2+\beta+\delta)} |z|^{1-\eta} \Big[1 -$$

2−η+δμα+1−γ2+β+δ(2−η)1+μα$m+1z$…(4.1)

$$m = n, m, n \in N_0$$

$$\left|I_{0,z}^{\beta,\eta,\delta} f(z)\right| \leq \frac{\Gamma(2-\eta+\delta)}{\Gamma(2-\eta)\Gamma(2+\beta+\delta)} |z|^{1-\eta} \left[1 + \frac{(2-\eta+\delta)\mu(\alpha+(1-\gamma))}{(2+\beta+\delta)(2-\eta)(1+\mu\alpha)(m+1)} |z|\right] \ldots(4.2)$$

$$m = n, m, n \in N_0$$

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