Let,

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and define the transformation

x = T (z) such that

$$x_1 = z_1 \quad (5)$$

$$x_2 = b\sqrt{z_2} - c\sqrt{z_1} \quad (6)$$

The inverse transformation $z = T^{-1}(x)$ is such that

$$z_1 = x_1 \quad (7)$$

$$z_2 = \left(\frac{c\sqrt{x_1}+x_2}{b}\right)^2 \quad (8)$$

It can be checked that we can write the dynamic model of coupled tank system in eqn (5) and (6) can be written as:

$$\dot{x}_1 = x_2 \quad (9)$$

$$\dot{x}_2 = \frac{b\dot{z}_2}{2\sqrt{z_2}} - \frac{c\dot{z}_1}{2\sqrt{z_1}} \quad (10)$$

Substitute the values of $\dot{z}_1$ and $\dot{z}_2$ in eqn (10), we get

$$\dot{x}_2 = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2 + \frac{ab}{2\sqrt{z_2}}u \quad (11)$$

Where the values of z_1 and z_2 in above equation are function of x_1 and x_2 as given in eqn (7) and (8).
Hence dynamic model of the coupled tank system can be written as:

$$\dot{x}_1 = x_2 \quad (12)$$

$$\dot{x}_2 = f + \phi u \quad (13)$$

$$y = x_1 \quad (14)$$

Where, $f = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2$

$$\phi = \frac{ab}{2\sqrt{z_2}}$$

Table 1: Parameters of Coupled Tank System

Parameters	*Value*
$A_1, A_2\ (cm^2)$	154
$a_2, a_{12}\ (cm^2)$	0.5
β_{12}	1.5315195
β_2	0.6820043
g (cm^2/sec)	981

4. Sliding Mode Controller Design

The idea behind SMC is to choose a sliding surface along which the system can slide to its desired final value.

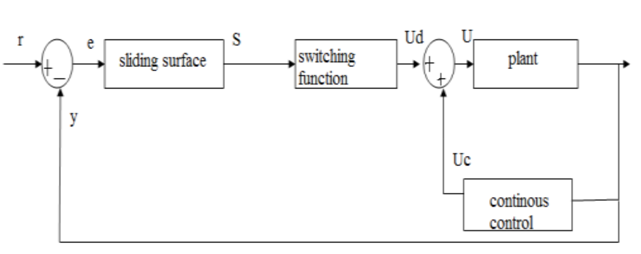

Figure 3: Closed loop schematic diagram of SMC

For designing SMC, firstly the sliding surface select and then design a suitable control law, so that the control variable is being driven to its reference value. The structure of SMC law U (t) is based on two main parts: a continuous part $U_C(t)$ and a discontinuous part $U_D(t)$.
ie, $U(t) = U_C(t) + U_D(t)$

Where, $U_C(t)$ is the dominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. It is a function of reference value and controlled variable. The discontinuous part $U_D(t)$ of SMC comprise a non-linear element that contains the switching element of the control law. This part of the controller is discontinuous across the sliding surface. The objective is to make the error and derivative of error equal to zero. As the system error is defined as the difference between actual height and desired height, mathematically

$$e = x_1 - x_{1(des)}$$

The expression for the n^{th} order sliding function is given by:

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e$$

where n is the order of the system and $\lambda > 0$ is the slope of the sliding surface.

$$S(t) = \dot{e} + \lambda e$$

In order to reduce the chattering problem associated with Standard Sliding Mode Controller (SSMC), an **Integral Sliding Mode Controller (ISMC)** is used. The sliding surface s (t) for integral sliding mode controller is presented by Slotine and Li [7],

$$S = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e\, dt$$

$$S(t) = \dot{e} + 2\lambda e + \lambda^2 \int_0^t e\, dt$$

$$S(t) = \dot{x}_1 + 2\lambda\,(x_1 - x_{1(des)}) + \lambda^2 \int_0^t (x_1 - x_{1(des)})\, dt \quad (15)$$

Stability Condition:
Consider a candidate Lyapunov function,

$$V = \frac{1}{2}S^2$$

From Lyapunov theorem we know that if $\dot{V}$ is a negative definite, the system trajectory will be driven and attracted towards the sliding surface and remains sliding on it until the origin is reached asymptoptically.

$$\dot{V} = S\dot{S}$$

A sufficient condition for the stability of the system is,

$$\dot{S} \leq -|W| \leq 0$$

Where W is a positive constant. This equation is called reaching condition or sliding condition.
On taking derivative of eqn (15) w.r.t time,

$$\dot{s} = [f + \phi u] + 2\lambda[x_2] + \lambda^2[x_1 - x_{1(des)}]$$

On putting $\dot{s} = 0$, we get the continuous part of control law as:

$$U_C(t) = \frac{1}{\phi}\left[-f - 2\lambda x_2 - \lambda^2[x_1 - x_{1(des)}]\right]$$

The basic discontinuous control law of SMC is given by,

$U_D(t)$ = -Ksgn(S)

Where the parameter K is a constant manual tuning parameter.
Therefore the integral sliding mode controller is,

$U(t) = U_C(t) + U_D(t)$

U (t) =

$$\frac{-\left\{\frac{bc}{2}\left[\frac{b\sqrt{x_1}}{c\sqrt{x_1+x_2}}-\frac{c\sqrt{x_1+x_2}}{b\sqrt{x_1}}\right]+\frac{c^2}{2}-b^2\right\}-2\lambda x_2-\lambda^2\left[x_1-x_{1(des)}\right]}{\frac{ab^2}{2\left[c\sqrt{x_1+x_2}\right]}} - K\mathrm{sgn}(S)$$

Where, λ and K are strictly positive constant. This asymptotically stabilize the output of the system y (t) = $x_1(t) = h_2(t)$ to its desired value.

5. Results and Discussion

The response of the system for different operating level and set point tracking performance of the system with designed Integral Sliding Mode Control (ISMC) are observed.

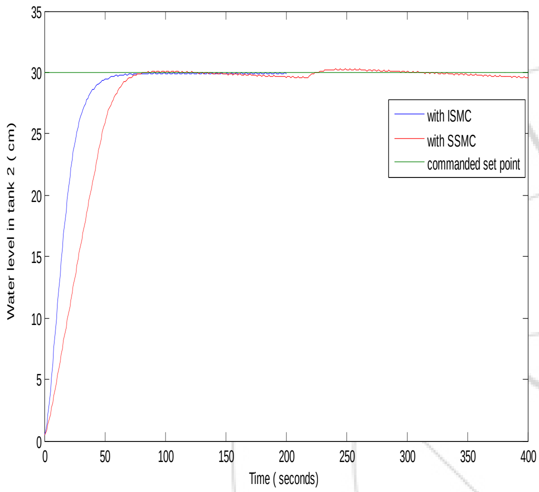

Figure 4: Response of the system with Integral Sliding Mode Control(ISMC) and Standard Sliding Mode Control(SSMC)

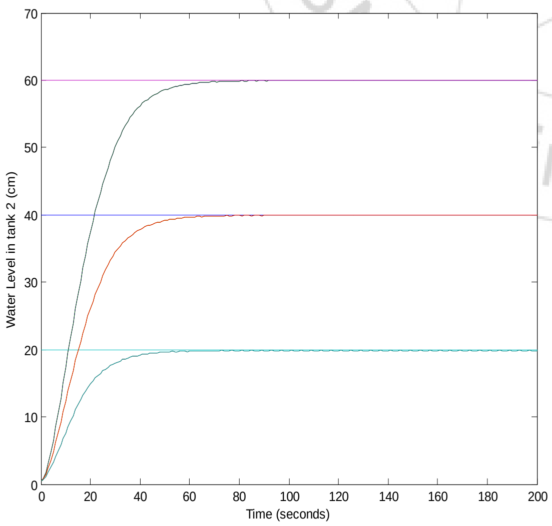

Figure 5: Response of the system with Integral Sliding Mode Control for different operating level

In figure 4, the system response with Standard Sliding Mode Control (SSMC) and Integral Sliding Mode Control (ISMC) is shown. The chattering problem associated with Standard Sliding Mode Control can be reduced by using Integral Sliding Mode Control. In figure 5, the response of the system with Integral Sliding Mode Control for different operating level is shown. The system achieves consistent performance and maintains the desired transient response characteristic throughout all operating points [at 20 cm, 40 cm, 60 cm] without overshoot.

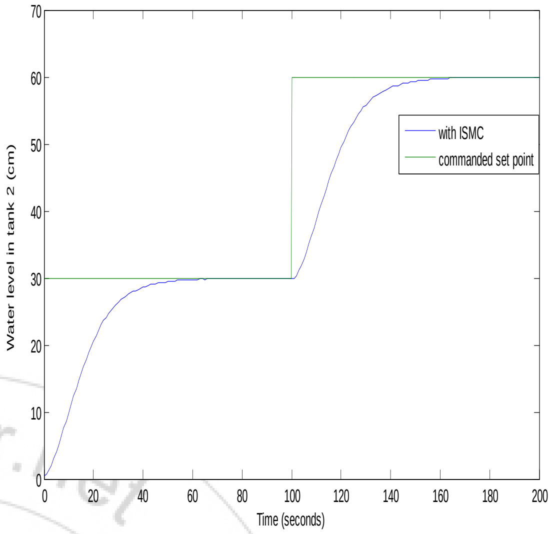

Figure 6: Set point tracking performance of the system with Integral Sliding Mode Control (ISMC)

The set point tracking test consist of changing the set point consecutively during the operation. The set point change is done at 100 second by a magnitude of 30 cm height in water level and the tracking performance of the system with Integral Sliding Mode Control is shown in figure 6. The response of the system shown in figure 6 clearly indicate how the controller takes the action for the given set point. The disturbance rejection of the controlled system shown in figure 7 and the response shows confirms the controller action even in the presence of disturbance (load disturbance) so as to reach the desired set point.

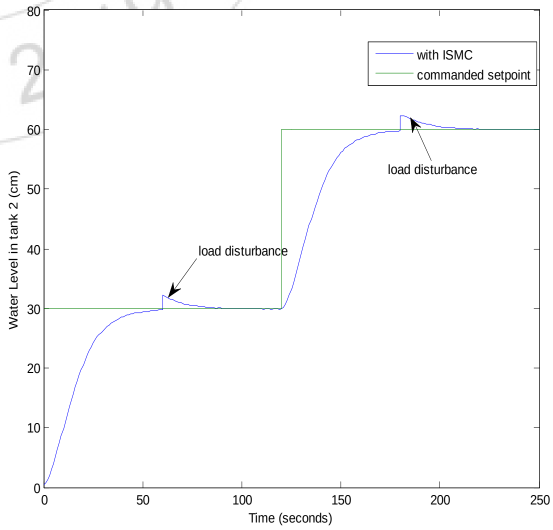

Figure 7: Disturbance rejection of the controlled system

6. Conclusion

The non-linear control designs using Standard Sliding Mode Control (SSMC) for the application of level control of coupled tank system is used. In order to reduce the chattering problem associated with Standard Sliding Mode Control (SSMC), an Integral Sliding Mode Control (ISMC) is used. It can be shown that the Integral Sliding Mode Control (ISMC) can cope with the coupled tank non-linear characteristics at all operating points. The designed non-linear controllers are able to sustain the desired transient response throughout the set point changes without significant overshoot, maximally fast and with high degree of accuracy and also the designed controller allows good disturbance rejection, thereby maintaining best dynamic performance.

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Author Profile

Jiffy Anna John was born in Kerala, India in 21/01/1991. She received B.Tech degree in Electrical and Electronics Engineering from Baselios Mathews II College of Engineering, Kollam, Kerala in 2013.Currently, she is pursuing her M Tech degree in Industrial Instrumentation & Control from TKM College of Engineering, Kollam, India. Her research interests includes Process Control and Control Systems. (jiffyannajohn@gmail.com)