from N, or another way to look at it is that it's a linear combination of the rows of N using coefficients from L, so we can take the rows of N equal to the basis of the row space of matrix M or another matrix with its rows some elements (vectors) in the row space of M not necessary be the basis . If we want to reduce the entities of M and M is an square matrix then according to the proposition, M must be singular.

Example

$$\text{Let } M=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}$$

Then we can do row operation to M so as to get its basis as follows:

$$R_2-\frac{7}{5}R_1\rightarrow R_2$$

$$\begin{bmatrix}5&7&8&3&4\\0&\frac{1}{5}&-\frac{1}{5}&-\frac{1}{5}&-\frac{3}{5}\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}$$

$$R_3-\frac{8}{5}R_1\rightarrow R_3$$

$$\begin{bmatrix}5&7&8&3&4\\0&\frac{1}{5}&-\frac{1}{5}&-\frac{1}{5}&-\frac{3}{5}\\0&-\frac{1}{5}&\frac{1}{5}&\frac{1}{5}&\frac{3}{5}\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}$$

$$R_4-\frac{3}{5}R_1\rightarrow R_4$$

$$\begin{bmatrix}5&7&8&3&4\\0&\frac{1}{5}&-\frac{1}{5}&-\frac{1}{5}&-\frac{3}{5}\\0&-\frac{1}{5}&\frac{1}{5}&\frac{1}{5}&\frac{3}{5}\\0&-\frac{1}{5}&\frac{1}{5}&\frac{1}{5}&\frac{3}{5}\\4&5&7&3&5\end{bmatrix}$$

$$R_5-\frac{4}{5}R_1\rightarrow R_5$$

$$\begin{bmatrix}5&7&8&3&4\\0&\frac{1}{5}&-\frac{1}{5}&-\frac{1}{5}&-\frac{3}{5}\\0&-\frac{1}{5}&\frac{1}{5}&\frac{1}{5}&\frac{3}{5}\\0&-\frac{1}{5}&\frac{1}{5}&\frac{1}{5}&\frac{3}{5}\\0&-\frac{3}{5}&\frac{3}{5}&\frac{3}{5}&\frac{9}{5}\end{bmatrix}$$

$$R_3+R_2\rightarrow R_3\ ,R_4+R_2\rightarrow R_4\ ,R_5+3R_2\rightarrow R_5$$

$$\begin{bmatrix}5&7&8&3&4\\0&\frac{1}{5}&-\frac{1}{5}&-\frac{1}{5}&-\frac{3}{5}\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

Now we can take

$AB=M$ where

$$B=\begin{bmatrix}5&7&8&3&4\\0&1&-1&-1&-3\end{bmatrix}$$

$$A\times\begin{bmatrix}5&7&8&3&4\\0&1&-1&-1&-3\end{bmatrix}=M$$

Multiplying both sides with B^t we get

$$A\times\begin{bmatrix}5&7&8&3&4\\0&1&-1&-1&-3\end{bmatrix}B^t=MB^t$$

$$A\times\begin{bmatrix}5&7&8&3&4\\0&1&-1&-1&-3\end{bmatrix}\begin{bmatrix}5&0\\7&1\\8&-1\\3&-1\\4&-3\end{bmatrix}=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}\begin{bmatrix}5&0\\7&1\\8&-1\\3&-1\\4&-3\end{bmatrix}\Rightarrow$$

$$A\times\begin{bmatrix}163&-16\\-16&12\end{bmatrix}=\begin{bmatrix}163&-16\\225&-20\\264&-28\\101&-12\\140&-20\end{bmatrix}$$

Multiplying both sides with

$$D^{-1}=\begin{bmatrix}163&-16\\-16&12\end{bmatrix}^{-1}=\begin{bmatrix}\frac{3}{425}&\frac{4}{425}\\\frac{4}{425}&\frac{163}{1700}\end{bmatrix}$$

We get

$$A=\begin{bmatrix}163&-16\\225&-20\\264&-28\\101&-12\\140&-20\end{bmatrix}\begin{bmatrix}\frac{3}{425}&\frac{4}{425}\\\frac{4}{425}&\frac{163}{1700}\end{bmatrix}\Rightarrow A=\begin{bmatrix}1&0\\\frac{7}{5}&\frac{1}{5}\\\frac{8}{5}&-\frac{1}{5}\\\frac{3}{5}&-\frac{1}{5}\\\frac{4}{5}&-\frac{3}{5}\end{bmatrix}$$

Now let $r_1=(5,7,8,3,4)$, $r_2=(0,1,-1,-1,-3)$ and let $w_1=r_1+2r_2=(5,9,6,1,-2)$, $w_2=r_1+3r_2=(5,10,5,0,-5)$,so we can form another factorization for M by taking the matrix

$$B=\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}$$

So we will get

$AB=M$

$$\Rightarrow A\times\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}=M$$

Multiplying both sides with B^t , we get

$$A\times\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}=M\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}$$

$$A\times\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}\Rightarrow$$

$$A\times\begin{bmatrix}147&155\\155&175\end{bmatrix}=\begin{bmatrix}131&115\\185&165\\208&180\\77&65\\100&80\end{bmatrix}$$

Multiplying both sides with

$$D^{-1}=\begin{bmatrix}147&155\\155&175\end{bmatrix}^{-1}=\begin{bmatrix}\frac{7}{68}&-\frac{31}{340}\\-\frac{31}{340}&\frac{147}{1700}\end{bmatrix}$$

We get

$$A=\begin{bmatrix}131&115\\185&165\\208&180\\77&65\\100&80\end{bmatrix}\begin{bmatrix}\frac{7}{68}&-\frac{31}{340}\\-\frac{31}{340}&\frac{147}{1700}\end{bmatrix}\Rightarrow A=\begin{bmatrix}3&-2\\3&\frac{-13}{5}\\5&-\frac{17}{5}\\2&-\frac{7}{5}\\3&-\frac{11}{5}\end{bmatrix}$$

Now we note that some entities of A are not integers ,and D is 2×2 matrix so if we choose D such that its determinant equal ± 1 all the entities of A will be integers because D^{-1} will not contains any fraction and this will be done as follows :

Let $r_1=(5,7,8,3,4)$, $r_2=(0,1,-1,-1,-3)$ and let

$w_1=r_1+2r_2=(5,9,6,1,-2)$, $w_2=\frac{1}{5}(r_1+3r_2)=(1,2,1,0,-1)$

,so we can form another factorization for M by taking the matrix

$$B=\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}$$

Then we will get

$AB=M$

$$\Rightarrow A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}=M$$

Multiplying both sides with $\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}$ instead of B^t

$$A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}\times\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}=M\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}$$

$$A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}\times\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}\Rightarrow$$

$$A\times\begin{bmatrix}5&1\\1&0\end{bmatrix}=\begin{bmatrix}5&3\\7&4\\8&5\\3&2\\4&3\end{bmatrix}$$

Multiplying both sides with

$$D^{-1}=\begin{bmatrix}5&1\\1&0\end{bmatrix}^{-1}=\begin{bmatrix}0&1\\1&-5\end{bmatrix}$$

So we get

$$A=\begin{bmatrix}5&3\\7&4\\8&5\\3&2\\4&3\end{bmatrix}\begin{bmatrix}0&1\\1&-5\end{bmatrix}\Rightarrow A=\begin{bmatrix}3&-10\\4&-13\\5&-17\\2&-7\\3&-11\end{bmatrix}$$

We can use also the vectors of the row space for factorizing a rectangular matrix, so if we have a rectangular matrix

$$M=\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}$$

The basis of its row is

$r_1=(2,0,2,-1,1,2,-1)$, $r_2=(0,2,-2,5,-1,0,5)$,if we choose $w_1=\frac{r_1}{2}$, $w_2=\frac{r_2}{2}$ we note that the matrix D will be the identity matrix $D=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and that makes the calculation easy as we see in the following

Then

$$B=\begin{bmatrix}w_1\\w_2\end{bmatrix}=\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}$$

Now from

$$AB=M\Rightarrow A\times\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}=\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}$$

Multiplying both sides with $\begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\\0&0\\0&0\end{bmatrix}$ instead of B^t this will give us in the left side

Then we get

$$D=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$A\times\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}\begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\\0&0\\0&0\end{bmatrix}=\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}\begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\\0&0\\0&0\end{bmatrix}$$

$$\Rightarrow A\times\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}7&3\\11&5\\9&5\\4&2\end{bmatrix}\Rightarrow A=\begin{bmatrix}7&3\\11&5\\9&5\\4&2\end{bmatrix}$$

So

$$\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}=\begin{bmatrix}7&3\\11&5\\9&5\\4&2\end{bmatrix}\times\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}$$

Note that the sum of the entities on the right hand side is 22 instead of 28 ones in the left side

5. Conclusion

The method of using vectors of the row space in factorizing a matrix is an efficient method for reducing the entities of a data set matrix ,although there are many algorithm for matrix factorization ,this a logarithm will be a good method because it is very simple, easy and accurate so it is more use full in reducing the entities of data set matrix.

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