4.21 MSE ($\bar{y}_{PT}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv} C_u C_v]$

$(1-\bar{Y}^2)\, C_v{}^2 + 2\bar{Y}(1-\bar{Y})\rho_{uv} C_u C_v < 0$

$(1+\bar{Y})C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

$\frac{1+\bar{Y}}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.22 With Mean per unit

MSE ($\bar{y}_{PT}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(\bar{Y}^2 - 1)C_u{}^2 + C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

4.31 (MSE) $\bar{y}_{RPT_1}$ < MSE ($\bar{y}_R$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 - 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 - 2\rho_{uv} C_u C_v]$

$(1+2\bar{Y})\, C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(1+2\bar{Y})\, C_v{}^2 < 2\bar{Y}\rho_{uv} C_u C_v$

$\frac{(1+2\bar{Y})}{2\bar{Y}} < \rho\frac{C_u}{C_v}$

4.32 With Mean per unit

MSE $\bar{y}_{RPT_1}$ < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 - 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 < 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v$

$(\bar{Y} - 1)\, C_u{}^2 + (1+\bar{Y})\, C_v{}^2 < 2\bar{Y}\rho_{uv} C_u C_v$

4.41 MSE($\bar{y}_{RPT_2}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 + 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv} C_u C_v]$

$(1+2\bar{Y})\, C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(1+2\bar{Y})\, C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

$\frac{(1+2\bar{Y})}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.42 With Mean per unit

MSE($\bar{y}_{RPT_2}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 + 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 < -2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v$

$(\bar{Y} - 1)\, C_u{}^2 + (1+\bar{Y})\, C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

4.51 MSE($\bar{y}_{RPT_3}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 + 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv} C_u C_v$

$(1-2\bar{Y})\, C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(1-2\bar{Y})\, C_v{}^2 < 2\bar{Y}\rho_{uv} C_u C_v$

$\frac{(1-2\bar{Y})}{2\bar{Y}} < \rho\frac{C_u}{C_v}$

4.52 With Mean per unit

MSE($\bar{y}_{RPT_3}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 + 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 < -2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v$

$(\bar{Y} + 1)\, C_u{}^2 + (1-\bar{Y})\, C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

4.61 MSE($\bar{y}_{RPT_4}$) < MSE ($\bar{y}_R$)

$(\bar{Y}^2 C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 - 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + Cv2-2\rho uvCuCv$

$(1-2\bar{Y})\, C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(1-2\bar{Y})\, C_v{}^2 < -2\bar{Y}\rho_{uv} C_u C_v$

$\frac{(1-2\bar{Y})}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.62 With Mean per unit

MSE $\left(\bar{y}_{RPT_4}\right)$ < MSE ($\bar{y}$)

$(\bar{Y}^2 C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 - 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 < 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v$

$(\bar{Y} + 1)C_u{}^2 + (1-\bar{Y})C_v{}^2 < 2\bar{Y}\rho_{uv} C_u C_v$

5. Empirical Study

To see the performance of the proposed estimators in comparison to other estimators, description of population data are given below:

Population: [Source: Steel and Torrie (1960, p.282)]

Y: Long of leaf burn in sec.,

X_1 :Potassiam Percentage

X_2: Clorine Percentage.

The required population parameters are:

$\bar{Y}$=0.6860, C_y=0.4803, N=30, n =6

ρ_{yx_1} =0.1794, C_{x_1}=0.2295, C_{x_2}=0.7493, $\rho_{x_1x_2}$ =0.4074

$\bar{X}_1$ =4.6537, ρ_{yx_2}=-0.4996 $\bar{X}_2$ =0.8077, g=2

MSE of proposed estimators

1. M ($\bar{y}_{RT}$) =0.13409 , 2.M ($\bar{y}_{PT}$) =0.18836 , 3. M ($\bar{y}_{RPT_1}$) =0.21252

4. M ($\bar{y}_{RPT_2}$) =0.30401, 5.M ($\bar{y}_{RPT_3}$) = 0.10523 , 6. M ($\bar{y}_{RPT_4}$) =0.12227

7. M **($\bar{y}_R$)=0.23478, 8.M ($\bar{y}_P$) =0.51077, 9.M (** $\bar{y}$) =0.23068

Percentage Relative Efficiencies (PRE)

Estimators	R.E over $\bar{y}$	R.E over $\bar{y}_R$	R.E over $\bar{y}_P$
$\bar{y}$	___	101.77	221.41
$\bar{y}_R$	98.25	___	217.55
$\bar{y}_P$	45.16	45.96	___
$\bar{y}_{RT}$	171.97	175.09	___
$\bar{Y}_{PT}$	122.46	___	271.16
$\bar{Y}_{RPT1}$	108.54	110.47	___
$\bar{Y}_{RPT2}$	75.87	___	168.01
$\bar{Y}_{RPT3}$	219.21	___	485.38
$\bar{Y}_{RPT4}$	188.66	192.01	___

6. Conclusion

The proposed estimators are found to be better usual Mean per unit , ratio and product estimators.

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