# Longitudinally Rough Short Bearing

P. I. Andharia<sup>1</sup>, Mital Patel<sup>2</sup>

<sup>1</sup>Department of Mathematics, M.K. Bhavnagar University, Bhavnagar, Gujarat, India

Abstract: Here we have studied the performance of short bearing under the presence of magnetic fluid as a lubricant. Bearing surfaces are considered to be longitudinally rough. The roughness is characterized by a stochastic random variable with non-zero mean, variance and skew-ness. The modified Reynold's equation is solved with suitable boundary conditions to obtain the pressure distribution. This expression for pressure distribution is used to calculate the load carrying capacity. The results are presented graphically. It is seen that due to magnetization the performance of bearing system gets improvement. It is also observed that the effect of roughness is negative on the performance of the bearing. The investigation suggests that the negative effect of roughness can be reduced by positive effect of magnetization parameter. It is also observed that the performance gets improve in the case of suitable combination of roughness parameters.

Keywords: Short bearing, Longitudinal roughness, Magnetic fluid, Reynold's equation, Load carrying capacity

## 1. Introduction

The slider bearing is the simplest and continuously encountered among the hydrodynamic bearings. In slider bearing, the film is continuous and non-diverging. These kinds of bearings are designed to support the axial loads. Particular solutions of Reynold's equation for slider bearing with various simple film geometries are described in several books and research papers (Lord Rayleigh [1], Archibald [2]). Prakash and Vij [3] analysed the hydrodynamic lubrication of a plane inclined slider bearing taking various geometries into consideration and given that the quality of being porous decreased the friction and load carrying capacity. Patel and Gupta [4] extended the above analysis of Prakash and Vij [3] by integrating slide velocity and proved that in order to increase the performance of the bearing system the value of the slide parameter deserved to be minimized.

However, bearing surfaces could be roughened through manufacturing process, the wear and the impulsive damage. In order to understanding for the effect of surface roughness Christensen [5-6] developed a stochastic concept and introduced an averaging film model to lubricated surfaces with striated roughness. Many investigators has applied a stochastic method to model the random roughness (Tzeng and Seibel [7], Christensen and Tonder [8-10]). Christensen and Tonder [8-10] presented all inclusive general analysis for surface roughness based on a general probability density function by modifying and developing the method of Tzeng and Seibel [7]. Accordingly many investigators have been carried out to study the effect of surface roughness, such as the works in the hydrodynamic journal bearing by Taranga et.al. [11], the hydrodynamic slider bearings by Christensen and Tonder [12] and the squeeze film spherical bearing by Andharia et al. [13]. In all these studies straight lubricant were used. The use of magnetic fluid as a lubricant modifying the performance of the bearing has splendidly recognized. Agrawal [14] considered the configuration of Prakash and Vij [3] in the presence of a magnetic fluid lubricant and establish its performance better than the one with straight lubricant. Bhat and Deheri [15] extended the analysis of Agrawal [14] by studying a magnetic fluid based

porous composite slider bearing. Bhat and Deheri [16] discussed a general porous slider bearing with squeeze film formed by a magnetic fluid. Patel and Deheri [17] presented behavior of transversely rough magnetic fluid based porous short bearing. Also Andharia et al. [18] has discussed performance of a magnetic fluid based longitudinally rough short bearing. Recently Andharia et al. [22] presented surface roughness effect of transverse patterns on the performance of short bearing.

Here it has been proposed to investigate and examine the performance of longitudinally rough short bearing in the presence of a magnetic fluid as a lubricant.

## 2. Analysis

The geometry and configuration of bearing is shown in Fig. 1, which is infinite in Z-direction.



[Figure 1]

The slider moves with the uniform velocity U in X-direction. The length of bearing L and breadth B is in Z-direction, where B<<L. The pressure gradient  $\partial p/\partial z$  is very larger than pressure gradient  $\partial p/\partial x$ . The maximum and minimum film thicknesses are  $h_1$  and  $h_2$  respectively. The assumptions of usual hydrodynamic lubrication theory are taken into consideration in the development of the analysis.

The lubricant film is supposed to be isoviscous and incompressible and the flow is laminar. The magnetic field is oblique to the stator as in Agrawal [14]. Following discussions carried out by Prajapati [19] regarding the effect of various forms of magnitude of magnetic field is expressed as

$$M^{2} = KB^{2} \left\{ \left(\frac{1}{2} + \frac{z}{B}\right) \sin\left(\frac{1}{2} - \frac{z}{B}\right) + \left(\frac{1}{2} - \frac{z}{B}\right) \sin\left(\frac{1}{2} + \frac{z}{B}\right) \right\} (1)$$

Where B is the breadth of bearing and K is a suitably chosen constant from dimensionless point of view (Bhat and Deheri [16])

The bearing surfaces are assumed to be transversely rough. The thickness h of the lubricant film is given by

$$\mathbf{h} = \mathbf{\bar{h}} + \mathbf{h}_{\mathbf{g}} \tag{2}$$

Where  $\bar{\mathbf{h}}$  is the mean film thickness and  $\mathbf{h}_{g}$  is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces.  $\mathbf{h}_{g}$  is considered to be stochastic in nature and governed by probability density function  $f(\mathbf{h}_{g})$ ,  $-\mathbf{c} \leq \mathbf{h}_{g} \leq \mathbf{c}$ , where  $\mathbf{c}$  is the maximum deviation from the mean film thickness.

The mean  $\alpha$ , the standard deviation  $\sigma$  and the measure of symmetry  $\mathbf{\mathcal{E}}$  the random variable  $h_s$  are defined by the relationship :

$$\alpha = \mathbf{E} \left( \mathbf{h}_{\mathbf{s}} \right) \tag{3}$$

$$\sigma^2 = \mathbf{E} \left[ \left( \mathbf{h}_{\mathbf{s}} - \alpha \right)^2 \right] \tag{4}$$

and

$$\mathbf{\mathcal{E}} = \mathbf{E} \left[ (\mathbf{h}_{\mathbf{g}} - \alpha)^3 \right] \tag{5}$$

Where E is the expectancy operator defined by

$$E(R) = \int_{-c}^{c} f(h_s) dh_s$$
 (6)

Wherein (Tzeng and Saibel [7])

$$f(h_s) = \frac{35}{32c^7} (c^2 - h^2)^3, -c \le h \le c$$
  
= 0, elsewhere (7)

It is easily observed that  $\alpha$ ,  $\sigma$  and  $\boldsymbol{\varepsilon}$  are independent of x.

The standard deviation, the mean and the measure of symmetry play important role. Therefore with the usual notations of hydrodynamic lubrication, the modified Reynold's equation for film pressure (Prajapati [19], Bhat [20], Deheri, Andharia and Patel [21]) is given by

$$\frac{d^2}{dz^2} \left( p - \frac{\mu_0 \overline{\mu} M^2}{2} \right) = 6\mu U \left[ (h) \frac{d}{dx} \left( \frac{1}{n(h)} \right) \right]$$
(8)

Where

$$\begin{split} h &= h_2 \left\{ 1 + m \left( 1 - \frac{X}{L} \right) \right\} \\ [(h) &= h^{-3} [1 - 3\alpha h^{-1} + 6h^{-2} (\alpha^2 + \sigma^2) - 10h^{-3} (\varepsilon + 3\sigma^2 \alpha + \alpha^3)] \end{split}$$

$$n(h) = h^{-1}[1 - \alpha h^{-1} + h^{-2}(\alpha^2 + \sigma^2) - h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)]$$

Where in  $m = \frac{h_1 - h_2}{h_2}$ 

while  $\mu_0$  is the magnetic susceptibility,  $\overline{\mu}$  is the free space permeability and  $\mu$  is the lubricant viscosity.

The associated boundary conditions are

$$p = 0; z = \pm \frac{B}{2} \text{ and } \frac{dp}{dz} = 0; z = 0$$
 (9)

By integrating Eq. 
$$(8)$$
 with respect to z

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(p - \frac{\mu_0 \overline{\mu} M^*}{2}\right) = 6\mu U \left[(h) \frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{1}{n(h)}\right) z + C_1 \right] (10)$$

Where C<sub>1</sub> is a constant. At z = 0;  $\frac{dp}{dz} = 0$ ;  $\frac{d}{dz} (M^2) = 0$  and  $C_1 = 0$ Again by integrating Eq. (10) with respect to z

gain by integrating Eq. (10) with respect to z  

$$p - \frac{\mu_0 \overline{\mu} M^2}{2} = 6\mu U \left[ (h) \frac{d}{dx} \left( \frac{1}{n(h)} \right) \frac{z^2}{2} + C_2 \right]$$
(11)

Where  $C_2$  is a constant.

At 
$$z = \pm \frac{B}{2}$$
;  $p = 0$ ;  $M^2 = 0$  and

$$C_2 = -3\mu U \left[ (h) \frac{d}{dx} \left( \frac{1}{n(h)} \right) \frac{B^2}{4} \right]$$

By Eq. (11), the pressure distribution is  

$$p = \frac{\mu_0 \overline{\mu} M^2}{2} - 3\mu U \left[ (h) \frac{d}{dx} \left( \frac{1}{n(h)} \right) B^2 \left( \frac{1}{4} - \frac{z^2}{B^2} \right) (12)$$
Introducing the following dimensionless quantities

$$Z = \frac{z}{B}, X = \frac{x}{L}, m = \frac{n_1 - n_2}{h_2}, \mu^* = \frac{n_2 \kappa \mu_0 \mu}{\mu U},$$
$$P = \frac{h_2^3}{\mu U B^2}, \overline{\alpha} = \frac{\alpha}{h_2}, \overline{\sigma} = \frac{\sigma}{h_2}, \overline{\epsilon} = \frac{\epsilon}{h_2}, \overline{L} = \frac{L}{h_2}, \overline{B} = \frac{B}{h_2}$$
(13)

The pressure distribution in dimensionless form as

$$P = \frac{\mu^2}{2} \left[ \left( \frac{1}{2} + Z \right) \sin \left( \frac{1}{2} - Z \right) + \left( \frac{1}{2} - Z \right) \sin \left( \frac{1}{2} + Z \right) \right] + \frac{3m}{\overline{L}} \left( \frac{1}{4} - Z^2 \right) G(H)$$
(14)

Where 
$$G(H) =$$
  
 $A_1^{3} - 5\overline{\alpha}A_1^{2} + (15\overline{\alpha}^{2} + 9\overline{\sigma}^{2})A_1 - (35\overline{\alpha}^{3} + 63\overline{\alpha}\overline{\sigma}^{2} + 14\overline{\epsilon})$   
 $+ (50\overline{\alpha}^{4} + 132\overline{\alpha}^{2}\overline{\sigma}^{2} + 18\overline{\sigma}^{4} + 32\overline{\alpha}\overline{\epsilon})A_1^{-1}$   
 $- (54\overline{\alpha}^{5} + 216\overline{\alpha}^{3}\overline{\sigma}^{2} + 54(\overline{\alpha}^{2} + \overline{\sigma}^{2})\overline{\epsilon} + 162\overline{\alpha}\overline{\sigma}^{4})A_1^{-2}$   
 $+ 40(\overline{\alpha}^{3} + 3\overline{\alpha}\overline{\sigma}^{2} + \overline{\epsilon})^{2}A_1^{-3}$   
 $\overline{[A_1^{3} - \overline{\alpha}A_1^{2} + (\overline{\alpha}^{2} + \overline{\sigma}^{2})A_1 - (\overline{\alpha}^{3} + 3\overline{\alpha}\overline{\sigma}^{2} + \overline{\epsilon})]^{2}}$ 

Where in  $A_1 = 1 + m (1-X)$ 

The load carrying capacity of the bearing is given by  $\mathbf{B}$ 

$$w = \int_{\frac{-B}{2}}^{\frac{-}{2}} \int_{0}^{1} p(x, z) dx dz$$
 (15)

Dimensionless load carrying capacity is obtained as

$$W = \frac{\bar{L}}{B} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{1} P \, dX \, dZ$$
(16)

$$= \frac{L}{\overline{B}} 0.15853 \ \mu^* + \frac{3m}{L} Q_1 \tag{17}$$

Where 
$$Q_1 = \int_{\frac{1}{2}}^{\frac{1}{2}} \int_0^1 \left(\frac{1}{4} - Z^2\right) G(H) dx dz$$

## 3. Result and Discussion

It is observed that Eq. (14) and Eq. (17) presents the dimensionless pressure distribution and dimensionless load carrying capacity respectively. These performance characteristics depend on various parameters such as magnetization parameter  $\mu^*$ , length ratio L/h2, breadth ratio

## Volume 4 Issue 7, July 2015 www.ijsr.net

B/h2, aspect ratio m, roughness parameters  $\sigma$ ,  $\alpha$  and  $\mathcal{E}$  etc. Eq. (17) is numerically integrated using Simpson's 1/3 rule for different values of  $\mu^*$ ,  $\sigma$ ,  $\alpha$  and  $\mathcal{E}$ . The results are presented graphically in Figures (2) – (23).

Figures (2) - (7) represent the variation of load carrying capacity with respect to magnetization parameter  $\mu^*$  for various values of L/h2, B/h2,  $\sigma/h2$ ,  $\alpha/h2$ ,  $\epsilon/h2$  and m respectively. These figures suggest that the load carrying capacity increases sharply due to magnetization parameter  $\mu^*.$ 



Figure 2: Variation of load carrying capacity with respect to  $\mu^*$  and L/h2



Figure 3: Variation of load carrying capacity with respect to  $\mu^*$  and B/h2











Figure 6: Variation of load carrying capacity with respect to  $\mu^*$  and  $E/h^2$ 



Figure 7: Variation of load carrying capacity with respect to  $\mu^*$  and m

Figures (8) - (12) represent the variation of load carrying capacity with respect to  $\sigma/h2$  for various values of L/h2, B/h2,  $\alpha/h2$ ,  $\epsilon/h2$  and m respectively. From these figures it is seen that the load carrying capacity increases significantly with  $\sigma/h2$ , but this increase is very less. However, the effect of  $\sigma/h2$  was reported negative in the case of transverse surface roughness [22]. Figure (9) shows that load carrying capacity decreases marginally for increasing value of B/h2. Furthermore, the aspect ratio m has a strong positive effect in the sense of load carrying capacity increases sharply.



Figure 8: Variation of load carrying capacity with respect to  $\sigma/h2$  and L/h2



Figure 9: Variation of load carrying capacity with respect to  $\sigma/h2$  and B/h2



Figure 10: Variation of load carrying capacity with respect to  $\sigma/h2$  and  $\alpha/h2$ 



Figure 11: Variation of load carrying capacity with respect to  $\sigma/h2$  and E/h2



Figure 12: Variation of load carrying capacity with respect to  $\sigma/h2$  and m

Figures (13) - (16) represent the variation of load carrying capacity with respect to  $\alpha/h2$  for various values of L/h2, B/h2, E/h2 and m respectively. From these figures it is clear that the load carrying capacity decreases due to  $\alpha/h2$ . Figure (14) shows that load carrying capacity decreases marginally for increasing value of B/h2.



Figure 13: Variation of load carrying capacity with respect to  $\alpha/h2$  and L/h2



Figure 14: Variation of load carrying capacity with respect to  $\alpha/h2$  and B/h2



Figure 15: Variation of load carrying capacity with respect to  $\alpha/h2$  and  $\epsilon/h2$ 



Figure 16: Variation of load carrying capacity with respect to  $\alpha/h2$  and m

Figures (17) - (19) present the variation of load carrying capacity with respect to  $\mathcal{E}/h2$  for various values of L/h2, B/h2 and m respectively. From this figures it is clear that the load carrying capacity decrease due to  $\mathcal{E}/h2$ . Figure (18) shows that load carrying capacity decreases marginally for increasing value of B/h2.



**Figure 17:** Variation of load carrying capacity with respect to ε/h2 and L/h2



**Figure 18:** Variation of load carrying capacity with respect to E/h2 and B/h2



Figure 19: Variation of load carrying capacity with respect to E/h2 and m

Figures (20) - (21) present the variation of load carrying capacity with respect to L/h2 for various values of B/h2 and m respectively. These figures show the load carrying capacity increases considerably due to L/h2.



**Figure 20:** Variation of load carrying capacity with respect to L/h2 and B/h2



**Figure 21:** Variation of load carrying capacity with respect to L/h2 and m

Figure (22) shows the variation of load carrying capacity with respect to B/h2 and m. From the figure it is clearly shown that load carrying capacity decreases due to B/h2.



Figure 22: Variation of load carrying capacity with respect to B/h2 and m

Figure (23) presents the variation of load carrying capacity with respect to m for various values of B/h2 and m respectively. It is clear that the load carrying capacity increases due to m.



Figure 23: Variation of load carrying capacity with respect to m and B/h2

## 4. Conclusion

The present study indicates that the effect of roughness parameters is trivial. This conditional effect increases with the larger values of  $\sigma/h2$  and L/h2. The results show that the adverse effect of B/h2,  $\alpha/h2$  and  $\epsilon/h2$  can be reduced to a larger extent by the positive effect of magnetization parameter  $\mu^*$  and L/h2, taking an appropriate values of aspect ratio m.

## References

- Lord Rayleigh, "Notes on the Theory of Lubrication", Philosophical Magazino and Journal of Science, Vol. 53, pp. 1-12, 1918.
- [2] F.R. Archibald, "Load Capacity and Time Relation for Squeeze Films", Jour. Basic Engg. Trans., ASME. Sear, Vol. D78, pp. 231-245, 1956.
- [3] J. Prakash and S.K. Vij, "Hydrodynamic Lubrication of Porous Slider", J. Mech. Engg. Sci., Vol. 15, pp. 232-234, 1973.
- [4] K.C. Patel and J.L. Gupta, "Hydrodynamic Lubrication of a Porous Slider Bearing with Slip Velocity", WEAR, Vol. 85, pp. 309-317, 1983.
- [5] H. Christensen, "Stochastic Model for Hydrodynamic Lubrication of Rough Surfaces", Proceedings of the Institutes of Mechanical Engineers, Vol. 184, pp. 1013-1025, 1969-70.
- [6] H. Christensen and K.C. Tonder, "Some Aspects of the

Functional Influence of Rough Surfaces in Lubrication", WEAR, Vol. 17, pp. 149-162, 1971.

2015.

- [7] S.T. Tzeng, E. Saibel, "Surface Roughness Effect on Slider Bearing Lubrication", Trans. ASLE, Vol. 10, pp. 334-340, 1967.
- [8] H. Christensen and K.C. Tonder, "Tribology of Rough Surfaces: Stochastic Models of Hydrodynamic Lubrication", SINTEF Report No. 10/69-18, 1969a.
- [9] H. Christensen and K.C. Tonder, "Tribology of Rough Surfaces: Parametric Study and Comparison of Lubrication Models", SINTEF Report No. 22/69-18, 1969b.
- [10] H. Christensen and K.C. Tonder, "The Hydrodynamic Lubrication of Rough Bearing Surfaces of Finite Width", ASME-ASLE lubrication conference, Paper no. 70-lub-7, 1970.
- [11] R. Taranga, A.S. Sekhar, B.C. Manjumdar, "The Effect of Roughness Parameter on the Performance of Hydrodynamic Journal Bearing With Rough Effects", Tribology Int., Vol. 32, pp. 231-236, 1999.
- [12] H. Christensen and K.C. Tonder, "The Hydrodynamic Lubrication of Rough Bearing Surfaces of Finite Width", ASME Journal of lubrication Technology, Vol. 93, pp. 324-330, 1971.
- [13] P.I. Andharia, G.M. Deheri, and J.L. Gupta, "Effect of Longitudinal Surface Roughness on the Behaviour of Squeeze Film in a Spherical Bearing", International Journal of Applied Mechanics and Engineering, Vol. 6, pp. 885-897, 2001.
- [14] V.K. Agrawal, "Magnetic Fluid Bases Porous Inclined Slider Bearing", WEAR, Vol. 107, pp. 133-139, 1986.
- [15] M.V. Bhat and G.M. Deheri, "Porous Composite Slider Bearing Lubricated with Magnetic Fluid", Japanese Journal of Applied Physics, Vol. 30, pp. 2513-2514, 1991.
- [16] M.V. Bhat and G.M. Deheri, "Porous Slider Bearing with Squeeze Film Formed by a Magnetic Fluid", Pure and Applied mathematika sciences, Vol. 39(1-2), pp. 39-43, 1995.
- [17] Jimit R. Patel and Gunamani Deheri, "Behavior of a Magnetic Fluid Based Rough Short Bearing", i-Scholar, Vol. 1, No. 1, pp. 29-48, 2013.
- [18] P.I. Andharia, G.M. Deheri, S. Mehta, "Performance of a Magnetic Fluid- based Longitudinally Rough Short Bearing", Proceedings of International Conference on Advances in Tribology and Engineering Systems, Springer India, 2014. (Conference proceedings)
- [19] B.L. Prajapati, "On Certain Theoretical Studies in Hydrodynamic and Electro-magneto Hydrodynamic Lubrication", Ph.D. thesis, S.P. University, Vallabh Vidyanagar, 1995.
- [20] M.V. Bhat, "Lubrication with a Magnetic Fluid", Team Spirit (India) Pvt., Ltd, 2003.
- [21] G.M. Deheri, P.I. Andharia and R.M. Patel, "Transversely Rough Slider Bearing with Squeeze Film Formed by a Magnetic Fluid", International Journal of Applied Mechanics and Engineering, Vol. 10.1, pp. 53-76, 2005.
- [22] P.I. Andharia and Mital Patel, "The Surface Roughness Effect of Transverse Patterns on the Performance of Short Bearing", International Journal of Scientific & Engineering Research, Volume 6, Issue 5, pp. 82-87,

## **Author Profile**



**Dr. P.I. Andharia** currently is Assistant Professor in department of Mathematics, M.K. Bhavnagar University, Bhavnagar. He has completed his B.Sc. from M.K. Bhavnagar University in the year 1986. He has completed M.Sc. from M.K. Bhavnagar University

in the year 1988. He has completed Ph.D. from Sardar Patel University, V.V. Nagar in the year 2001. His 20 research papers published, 20 conferences attended and various papers presented at conferences. His Many research papers reviewed for national/international journals. His research interest is Tribology in Applied Mathematics.



**Mital Patel** has completed her B.Sc. Mahtematics from Saurashtra University in the year 2005. She is M.Sc. Mathematics from Saurashtra University in the year 2007. She has completed her M. Phil. from Saurashtra University in the year 2012 and currently Ph.D. in Trikleton from M.K. Physican University

pursuing his Ph.D. in Tribology from M.K. Bhavnagar University. Her research interest is Tribology in Applied Mathematics.

## Volume 4 Issue 7, July 2015 www.ijsr.net