On solving the problem considering as parametric linear problem let we reach at $k^{th}$ iteration, the vectors variable X is expressed in terms of a vector of nonnegative integer parameters Z. Hence the above problem can be reconsidered as follows:

$$(4)\ Minimize\ Z^k = L^k + C^k y^k$$
$$subject\ to \quad X = T^k + Q^k y^k$$
$$S = P^k + R^k y^k$$

Where all integer constants $L^k \geq 0$; $C^k \geq 0$ and S stands for all slack variables. Initially the objective function is equal to $L^k$ and $y^k = 0\ whenever\ T^k \geq 0\ , P^k \geq 0$ proceeds to provide the optimal solution in finite number of steps.

## 4. Algorithm

In the following we describe an algorithm to solve the Fuzzy Transportation problem. The capacity of each type of vehicle is fuzzy in nature therefore represented by fuzzy number in the above types of problems. The proposed solution algorithm can be summarized in the following steps:

Step-1: First of all formulate the transportation problem in the form of fuzzy linear programming problem as (1).

Step-2: Apply α-cut to the Trapezoidal fuzzy parameter $\tilde{N}_{ij}$ .

Step-3: Set the certain degree $\alpha=\alpha^*$ into [0, 1] we can initiate taking α=0.

Step-4: Determine the α-cut interval and proceed to determine its inverse interval.

Step-5: Choose $\tilde{n}_{ij} \in {}^c (N_{ij})_\propto$ for that $\alpha^*$.

Step-6: Use the Greenberg Algorithm to solve the problem as explained at (3) – (4) and find the integer solution of the problem.

Step-7: Set $\alpha=(\alpha^* + \varepsilon)$ into [0, 1].

Step-8: Repeat again the process Step-1 to 7 until the interval [0, 1] is fully exhausted then, stops.

Step-9: Compare all the solution for different value of $\alpha^*$ and find the optimum value of the objective function.

## 5. Example

Let us take a numerical example to explain the proposed model. Let we have Three destination and four warehouses. The vehicle capacity for each route is given in the form of Trapezoidal fuzzy number. The problem is defined as follows-

| **Destination → Source↓** | **$D_1$** | **$D_2$** | **$D_3$** | **Available** |
|---|---|---|---|---|
| **$S_1$** | **$C_{11}$=3** $\tilde{N}_{11}$ = [2 4 5 7] | **$C_{11}$=6** $\tilde{N}_{11}$ = [0 3 4 7] | **$C_{11}$=1** $\tilde{N}_{11}$ = [3 4 5 7] | **$S_1$=170** |
| **$S_2$** | **$C_{12}$=6** $\tilde{N}_{12}$ = **[1 5 7 10]** | **$C_{12}$=5** $\tilde{N}_{12}$ = **[0 3 5 6]** | **$C_{12}$=3** $\tilde{N}_{12}$ = **[1 6 7 9]** | **$S_2$=60** |
| **$S_3$** | **$C_{13}$=3** $\tilde{N}_{13}$ = [1 2 5 7] | **$C_{13}$=8** $\tilde{N}_{13}$ = [1 3 4 6] | **$C_{13}$=10** $\tilde{N}_{13}$ = [0 2 3 5] | **$S_3$=35** |
| **$S_4$** | **$C_{14}$=4** $\tilde{N}_{14}$ = [2 4 5 7] | **$C_{14}$=15** $\tilde{N}_{14}$ = [0 3 4 7] | **$C_{14}$=6** $\tilde{N}_{14}$ = [3 4 5 7] | **$S_4$=60** |
| **DEMAND** | **140** | **175** | **10** | **320** |

The unsatisfied demand penalty of each destination is-

$D_1$=5, $D_2$=6, $D_3$=4;

The calculation of the example is conduct by the help of MATLAB and by doing all iteration work we find that for α=1, we find the Integer solution of the given problem As follows-

$$X = \begin{bmatrix} 50 & 110 & 8 \\ 6 & 54 & 0 \\ 32 & 0 & 2 \\ 50 & 8 & 0 \end{bmatrix}$$

Numbers of round of each vehicle from source to destinations are as follows-

$$X = \begin{bmatrix} S\backslash D & D_1 & D_2 & D_3 \\ S_1 & 10 & 27 & 2 \\ S_2 & 1 & 11 & 0 \\ S_3 & 6 & 0 & 0 \\ S_4 & 10 & 2 & 0 \end{bmatrix}$$

Hence the Transportation Cost is Rs. 343

The Unsatisfied demands of the destination are-

$D_1$=2, $D_2$=2, $D_3$=2;

Hence the Penalty cost of each destinations-

$D_1$=10, $D_2$=12, $D_3$=8;

Hence the Total Cost Bear by the destinations is equal to-

Total Cost = Transportation cost + Penalty Cost

= 343+20=363.

## 6. Conclusion

In this paper Trapezoidal fuzzy set concept has been used with Greenberg's Algorithm to evaluate the Integer solution of the parametric form of the Transportation Linear Programming problem. The model can be used by managers to optimize the transportation cost items of own company. The problem is the modified version of Osman and Saad. In the future the other properties Fuzzy sets and other parameters can be included as fuzzy sets such as demand and supply.

## References

[1] Ferguson A. R. & Dantzig G.B., "The allocation of aircraft to routes- An example of linear programming under certain demand" Management Science, vol. 3, pp. 45-73. 1956.

[2] Greenberg's H., "Integer Programming" Addison-Wesley, Reading, Mass.

[3] Kumar Praveen, "Weighted Job Shop Scheduling Model Under Uncertainty", Pure And Applied Mathematika Sciences, vol. LXXIII, No.1-2, March-2011.

[4] Kumar Praveen, "A Generalized Fuzzy Johnson Algorithm" accepted in International Journal of Technology Enhancements and Emerging Engineering Research, for publication on Vol-3(8) 2015.

[5] Osmaan S. Mohamed, Saad M. Omar & other "Solving a special class of large scale fuzzy multi-objective integer linear programming problems" Fuzzy Sets and Systems vol. 107, pp. 289-297, 1999.

[6] Saad M Omer "On the Integer solution of the generalized transportation problem under fuzzy environment", OPSEARCH vol. 42 no. 3, pp. 238-251, 2005.

[7] Stefan Chanas & Dorota Kuchta, "Fuzzy integer transportation problem", Fuzzy Sets and System vol. 98, pp. 291-198, 1998.
[8] Zadeh L. & Bellman R., "Decision making in a fuzzy environment", Management Science, vol. 17 pp. 141-164, 1965.
[9] Zimmermann H.J., Fuzzy set theory & Its Application, Boston: Kluwer Academic Publishers, 1991.

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