Mathematical Models in Terms of Balanced Signed Fuzzy Graphs with Generalized Modus Ponens Method

Dr. G. Nirmala[1], S. Prabavathi[2]

[1] Principal, Research Advisor & Convenor (Mathematics), Govt. Arts College, Melur, Madurai-625 203, Tamilnadu, India

[2] Research Scholar, Dept. of Mathematics, K.N. Govt. Arts College, Thanjavur-613007, Tamil Nadu, India

Abstract: ***Starting from an input fuzzy set and an IF-THEN rule, implementations of the generalized modus ponens in a fuzzy set theoretical frame work allow the derivation of an output fuzzy set. The main goal of this work is how the generalized modus ponens method, can be used in the mathematical models in terms of balanced signed fuzzy graphs and also we define generalized modus ponens, signed fuzzy graph, signed degree of fuzzy graph.***

Keywords: Generalized modus ponens, Mathematical model, signed fuzzy graph, signed degree of fuzzy graph.

1. Introduction

The first publications in fuzzy set theory by Zadeh [1965] and Goguen[1967,1969]. Zadeh [1965,P.339] writes, "The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual frame-work which parallels in many respects the frame-work used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a frame-work provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables".

A mathematical model is a simplified version of the world that is used to study key characteristics of that world. Charles Lave and James March describe a model as follows: "A model is a simplified picture of the real world. It has some of the characteristics of the real world, but not all of them. It is a set of interrelated guess about the world. Like all pictures, a model is simpler than the phenomena it is supposed to represent or explain".

Fuzzy graph theory was introduced by Azriel Rosenfield in 1975. Though it is very young, it has been growing fast and has numerous application in various fields. The concept of signed fuzzy graph is used if the relationship between each pair of nodes is symmetric. With the help of signed fuzzy graph, the interpersonal relationships between groups of individuals can be represented. The simplest approach to study such a group of individuals is to draw a fuzzy graph in which the individuals are the nodes and in which there is an edge for "$\sigma(u)$" to node "$\sigma(v)$" if $\sigma(u)$ is in some relation to $\sigma(v)$. This relationship may be like or dislike, associate with or avoids and so on. Two different relationship can be included in a fuzzy graph by using two different signs, i.e. positive (+) and negative (-) to distinguish them. Then the presence of an edge means that there is a relationship between the nodes and the indication of a (+) sign represents a positive relation such as like, agree etc. and the (-) sign represents the other relation such as dislike, disagree, hate etc. which lead to existence of signed fuzzy graph.

2. Basic Preliminaries

2.1 Definition

If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:
$\tilde{A}=\{(x, \mu_{\tilde{A}}(x))/x\epsilon x\}$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ that maps X to the membership space M (when M contains only the two points 0 to 1, $\tilde{A}$ is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a non-fuzzy set). The range of the membership function is a subset of the non-negative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

2.1 Example

A realtor wants to classify the Restaurant he offers to his clients. One indication of comfort of these Restaurant is the number of rooms in it.

Let X={1,2,3,4,…,10} be the set of available types of Restaurant described by

x=number of rooms in a Restaurant.
Then the fuzzy set "comfortable type of restaurant for a four-person family" may be described as

$\tilde{A}=\{(1,.2),(2,.5),(3,.8),(4,1),(5,.7),(6,.3)\}$

NOTE: A fuzzy set is denoted by an ordered set of pairs, the first denotes the element and the second degree of membership.

2.2 Definition: Mathematical model a self-contained set of formulas and/or equations based on an approximate quantitative description of real phenomena and created in the hope that the behavior it predicts will be consistent with the real behavior on which it is based.

2.2 Example : One technique of solving the 'world problem' in algebra. Suppose the age of a mother is four times the age of her daughter and we are told that after five years the age of the mother will be three times the age of the daughter. We have to find their ages.
Let x be the age of the mother and y be the age of the daughter. Then the data of the problem gives

$x=4y, \quad x+5 = 3(y+5) \longrightarrow (1)$

Giving x=40, y=10.

The two equations of (1) given a mathematical model of the biological situation, so that the biological problem of ages is reduced to the mathematical problem of the solution of a system of two algebraic equations. The solution of the equations is finally interpreted biologically to give the ages of the mother and the daughter.

2.3 Definition: A fuzzy graph(f-graph) is a pair G: (σ, μ) where σ is a fuzzy subset of a set S and μ *is a* fuzzy relation on σ. It is assumed that S is finite and nonempty, μ is reflexive and symmetric. Thus if G : (σ, μ) is a fuzzy graph, then $\sigma : S \to [0,1]$ and $\mu : S \times S \to [0,1]$ is such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, $v \in S$. Also, we denote the underlying graph of the fuzzy graph G: (σ, μ) by G*: (σ^*, μ^*) where $\sigma^* = \{u \in S : \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in S \times S : \mu(u,v) > 0\}$. In examples, if σ is not specified, it is chosen suitably. Also G : (σ, μ) is called a trivial fuzzy graph if G*: (σ^*, μ^*) is trivial. That is σ^* is a singleton set.

2.3 Example

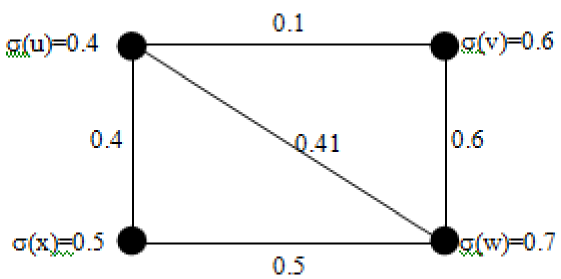

Figure 1: Fuzzy Graph

Let G : (σ,μ) be with $\sigma^* = \{u,v,w,x\}$. Let $\sigma(u) = 0.4$, $\sigma(v) = 0.6$, $\sigma(w) = 0.7$, $\sigma(x) = 0.5$ and $\mu(u,v) = 0.1$, $\mu(v,w) = 0.6$, $\mu(w,x) = 0.5$, $\mu(x,u) = 0.4$ and $\mu(u,w) = 0.41$.

2.4 Definition: Let G : (σ,μ) be a fuzzy graph. A signed fuzzy graph is a fuzzy graph in which every edge is labeled with a '+ sign' or a '−sign'. An edge $\mu(u,v)$ labeled with a '+ sign' is called a positive edge, and is denoted by $\mu^+(u,v)$. An edge $\mu(u,v)$ labelled with a '− sign' is called a negative edge, and is denoted by $\mu^-(u,v)$.

2.4 Example:

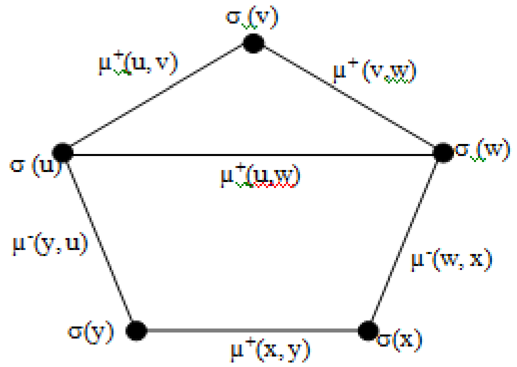

Figure 2: Signed Fuzzy Graph

2.5 Definition: A signed fuzzy graph is balanced if every cycle has an even number of '- signs'. Otherwise, it is unbalanced signed fuzzy graph.

2.5 Example

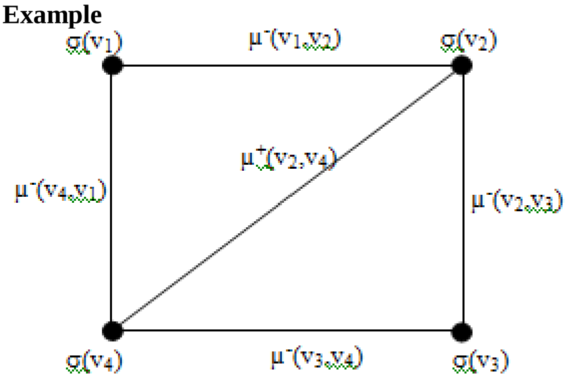

Figure 3: Balanced Signed Fuzzy Graph

2.6 Definition: In a signed fuzzy graph G:(σ,μ), the positive degree of a vertex $\sigma(u)$ is $deg^+[\sigma(u)] = |\{\mu(u,v):\mu^+(u,v) \in E\}|$, the negative degree of a vertex $\sigma(u)$ is $deg^-[\sigma(u)] = |\{\mu(u,v):\mu^-(u,v) \in E\}|$, the signed fuzzy degree of $\sigma(u)$ is $sdeg[\sigma(u)] = deg^+[\sigma(u)] - deg^-[\sigma(u)]$ and the degree of $\sigma(u)$ is $deg[\sigma(u)] = deg^+[\sigma(u)] + deg^-[\sigma(u)]$.

Example 2.6:

$sdeg[\sigma(u)] = deg^+[\sigma(u)] - deg^-[\sigma(u)]$
$Sdeg\ [\sigma(u)] = 0.6 + 0.5 = 1.1$
$Sdeg\ [\sigma(v)] = 0.6 - 0.4 = 0.2$
$Sdeg\ [\sigma(w)] = 0.5 - 0.4 = 0.1$

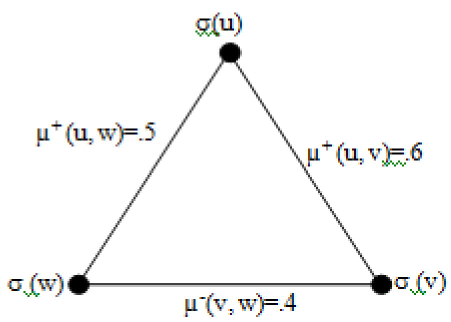

Figure 4: Signed degree of fuzzy graph

2.7 Definition: A vertex having no incident edge is called an isolated vertex. In other words, isolated vertices are vertices with zero degree.

2.7 Example:

Here σ(w) & σ(y) are isolated vertices.

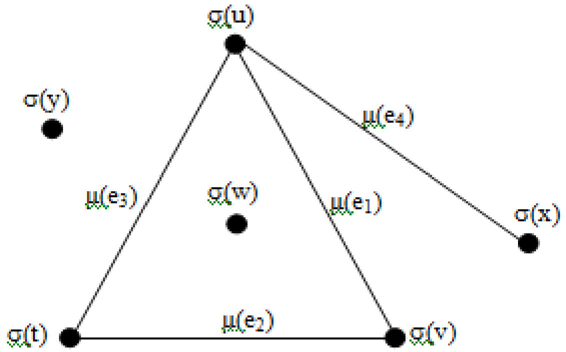

Figure 5 G

2.8 Definition: We consider fuzzy rule a better solution would be to use an alternative derivation rule, called generalized modus ponens.
Fact : x is A'
Rule : if x is a A then y is B
Result : y is B'

Where A' is close to A and B' is close to B. When A, B, A' and B' are fuzzy sets of appropriate universes, the foregoing inference procedure is called approximate reasoning or fuzzy reasoning; it is also called Generalized Modus Ponens (GMP for short).

3. Mathematical Models in Terms of Balanced Signed Fuzzy Graph Using Generalized Modus Pones Method

3.1 If G:(σ,μ) is a fuzzy signed graph with |V| = σ(v), |E| = $\mu(v_i,v_{i+1})$, then $k = \sum_{v\epsilon V} Sdeg[\sigma(v)] \equiv 2\sum\mu(v_i,v_{i+1})(mod4)$, $\sum\mu^+(v_i,v_{i+1}) = \frac{1}{4}(2\sum\mu(v_i,v_{i+1}) + k)$ and $\sum\mu^-(v_i,v_{i+1}) = \frac{1}{4}(2\sum\mu(v_i,v_{i+1}) - k)$.

3.2 Lemma: For any signed fuzzy graph G:(σ,μ) without isolated vertices, $\sum_{v\epsilon V} | sdeg[\sigma(v)]+2n_0 | \leq 2\sum\mu(v_i,v_{i+1})$.

Proof: We can prove this lemma, mathematical models in terms of balanced signed fuzzy graph with generalization modus ponens method.

A signed (or an algebraic) fuzzy graph is one in which every edge has a positive or negative sign associated with it. Thus the **Figure 6** is a signed fuzzy graph.

Let positive sign denote friendship and negative sign denote enemity.

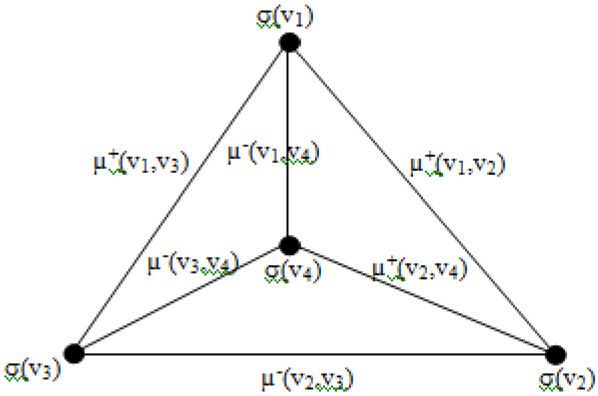

Figure 6: Signed Fuzzy Graph

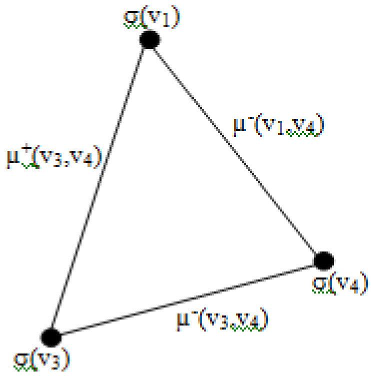

Figure A: $\sigma(v_1)$ is a friend of $\sigma(v_3)$, but $\sigma(v_1)$ & $\sigma(v_3)$ are both jointly enemies of $\sigma(v_4)$.

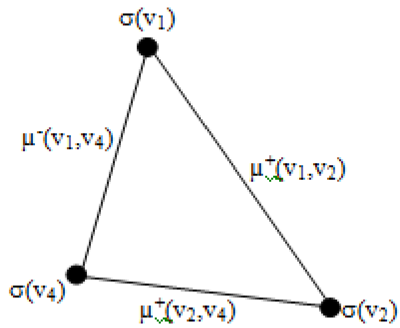

Figure B: $\sigma(v_2)$ is a friend of both $\sigma(v_1)$ and $\sigma(v_4)$, but $\sigma(v_1)$ and $\sigma(v_4)$ are enemies.

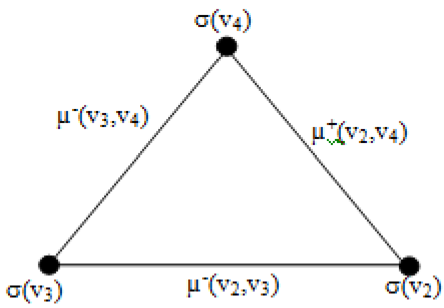

Figure C : $\sigma(v_4)$ is a friend of $\sigma(v_2)$, but $\sigma(v_4)$ & $\sigma(v_2)$ are both jointly enemies of $\sigma(v_3)$.

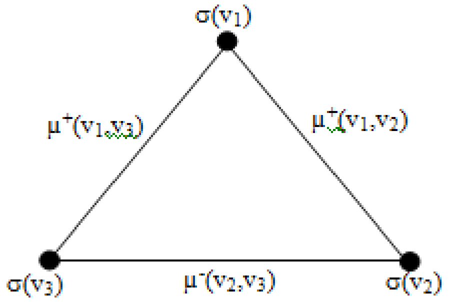

Figure D: $\sigma(v_1)$ is a friend of both $\sigma(v_2)$ and $\sigma(v_3)$, but $\sigma(v_2)$ and $\sigma(v_3)$ are enemies.

The **Fig: A** and **Fig: C** are represents normal behavior and are said to be balanced, while the **Fig:B** and **Fig:D** represent unbalanced situations since if $\sigma(v_2)$ is a friend of both $\sigma(v_1)$ and $\sigma(v_4)$, but $\sigma(v_1)$ and $\sigma(v_4)$ are enemies, this creates a tension in the system. So, the given signed fuzzy graph is unbalanced **(Fig6).**

Given signed fuzzy graph G: (σ,μ) without isolated vertices. First each

$| \text{sdeg}[\sigma(v)] | = | \deg^+[\sigma(v)] - \deg^-[\sigma(v)] |$
$\leq \deg^+[\sigma(v)] + \deg^-[\sigma(v)]$

In a signed fuzzy graph G: (σ,μ) with $|V| = \sigma(v)$, $|E| = \sum\mu(v_i,v_{i+1})$, we denote $\mu^+ (v_i,v_{i+1})$ the number of positive edges of G and $\mu^-(v_i,v_{i+1})$ denote the number of negative edges of G respectively .

Further, $\sigma(n_0)$ denote the number of vertices with positive signed fuzzy degrees.

Since G has no isolated vertices, $2 \leq \deg^+[\sigma(v)] + \deg^-[\sigma(v)]$ when $\text{sdeg}\sigma(v) = 0$.

Thus,
$\sum_{v\epsilon V}| \text{sdeg}[\sigma(v)] | + 2\sigma(n_0) \leq \sum_{v\epsilon V}(\deg^+[\sigma(v)] + \deg^-[\sigma(v)])$
$= 2\sum\mu^+(v_i,v_{i+1}) + 2\sum\mu^-(v_i,v_{i+1})$
$\therefore \sum_{v\epsilon V}| \text{sdeg}[\sigma(v)] | + 2\sigma(n_0) = 2\sum\mu(v_i,v_{i+1})$.

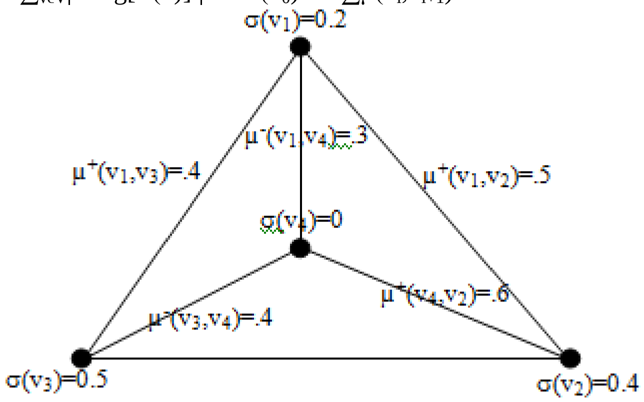

Figure 6 G

RULE1:
Fact : x is 5.2
Rule : if x is $\sum_{v\epsilon V}| \text{sdeg}[\sigma(u)] |$ THEN y is $2\sigma(n_0)$
Result : y is 0

EXAMPLE:
$| \text{sdeg}[\sigma(v_1)] | = \deg^+[\sigma(v_1)] + \deg^-[\sigma(v_1)]$
$= 0.5 + 0.3 + 0.4 = 1.2$
Similarly,
$| \text{sdeg}\ \sigma(v_2) | = 0.5 + 0.6 + 0.4 = 1.5$
$| \text{sdeg}\ \sigma(v_3) | = 0.4 + 0.4 + 0.4 = 1.2$
$| \text{sdeg}\ \sigma(v_4) | = 0.4 + 0.3 + 0.6 = 1.3$
$\sum_{v\epsilon V}| \text{sdeg}[\sigma(v_1)] | = 1.2 + 1.5 + 1.2 + 1.3$
$= 5.2$

$\sigma(n_o) = 0$ ($\because$ the number of vertices with zero fuzzy signed degrees)

RULE 2:
Fact : x is 3
Rule : If x is $2\sum\mu^+(v_i,v_{i+1})$ Then y is $2\sum\mu^-(v_i,v_{i+1})$
Result : y is 2.2

EXAMPLE:
$2\sum\mu^+(v_i,v_{i+1}) = 2(.5+.6+.4) = 2(1.5) = 3$
$2\sum\mu^-(v_i,v_{i+1}) = 2(.3+.4+.4) = 2(1.1) = 2.2$

RULE3:
Fact : x is 5.2
Rule : If x is $\sum_{v\epsilon V}| \text{sdeg}[\sigma(v)] | + 2\sigma(n_0)$ THEN y is $2\sum\mu(v_i,v_{i+1})$
Result : y is 5.2

EXAMPLE:
$\sum_{v\epsilon V}| \text{sdeg}[\sigma(v)] | + 2\sigma(n_0) = 5.2 + 0 = 5.2$
$2\sum\mu(v_i,v_{i+1}) = 3 + 2.2 = 5.2$

Hence proved

4. Conclusion

In this paper we have introduced the signed fuzzy graph, signed degree of fuzzy graph and generalized modus ponens method and we have easily obtained balanced signed fuzzy graphs by using mathematical model with generalized modus ponens method.

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Author Profile

Dr. G. Nirmala, Principal, Research Advisor & Convenor (Mathematics), Govt. Arts College, Melur, Madurai-625 203, Tamilnadu, India

Mrs. S. Prabavathi, Research Scholar, Dept. of Mathematics, K.N. Govt. Arts College, Thanjavur-613007, Tamil Nadu, India.