2.7 Definition: A vertex having no incident edge is called an isolated vertex. In other words, isolated vertices are vertices with zero degree.

2.7 Example:

Here $\sigma(w)$ & $\sigma(y)$ are isolated vertices.

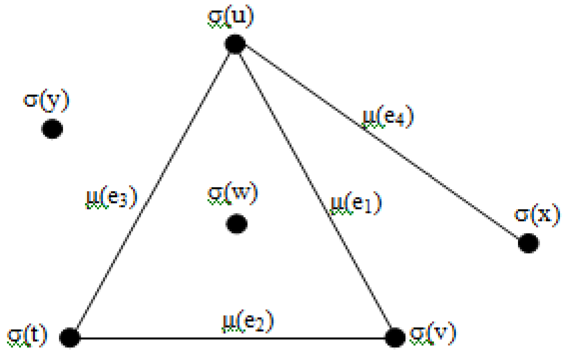

Figure 5 G

2.8 Definition: We consider fuzzy rule a better solution would be to use an alternative derivation rule, called generalized modus ponens.

Fact : x is A'

Rule : if x is a A then y is B

Result : y is B'

Where A' is close to A and B' is close to B. When A, B, A' and B' are fuzzy sets of appropriate universes, the foregoing inference procedure is called approximate reasoning or fuzzy reasoning; it is also called Generalized Modus Ponens (GMP for short).

3. Mathematical Models in Terms of Balanced Signed Fuzzy Graph Using Generalized Modus Pones Method

3.1 If G:(σ,μ) is a fuzzy signed graph with $|V| = \sigma(v)$, $|E| = \mu(v_i,v_{i+1})$, then $k = \sum_{v\epsilon V} Sdeg[\sigma(v)] \equiv 2\sum\mu(v_i,v_{i+1})(\mathrm{mod}4)$, $\sum\mu^+(v_i,v_{i+1}) = \frac{1}{4}(2\sum\mu(v_i,v_{i+1}) + k)$ and $\sum\mu^-(v_i,v_{i+1}) = \frac{1}{4}(2\sum\mu(v_i,v_{i+1}) - k)$.

3.2 Lemma: For any signed fuzzy graph G:(σ,μ) without isolated vertices, $\sum_{v\epsilon V} | sdeg[\sigma(v)]+2n_0 | \leq 2\sum\mu(v_i,v_{i+1})$.

Proof: We can prove this lemma, mathematical models in terms of balanced signed fuzzy graph with generalization modus ponens method.

A signed (or an algebraic) fuzzy graph is one in which every edge has a positive or negative sign associated with it. Thus the **Figure 6** is a signed fuzzy graph.

Let positive sign denote friendship and negative sign denote enemity.

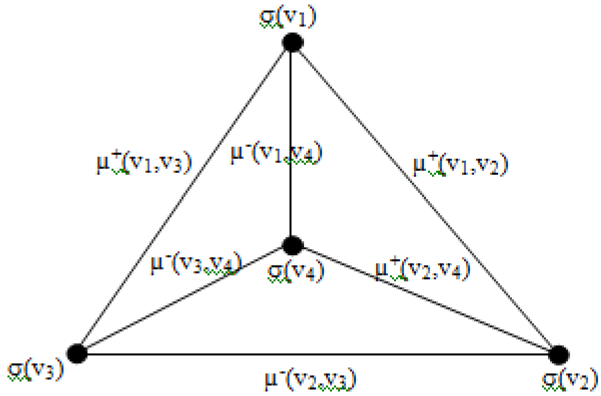

Figure 6: Signed Fuzzy Graph

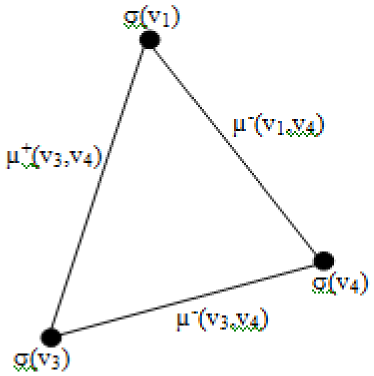

Figure A: $\sigma(v_1)$ is a friend of $\sigma(v_3)$, but $\sigma(v_1)$ & $\sigma(v_3)$ are both jointly enemies of $\sigma(v_4)$.

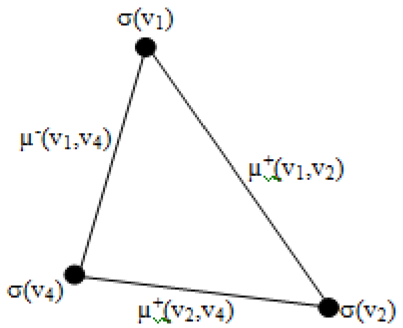

Figure B: $\sigma(v_2)$ is a friend of both $\sigma(v_1)$ and $\sigma(v_4)$, but $\sigma(v_1)$ and $\sigma(v_4)$ are enemies.

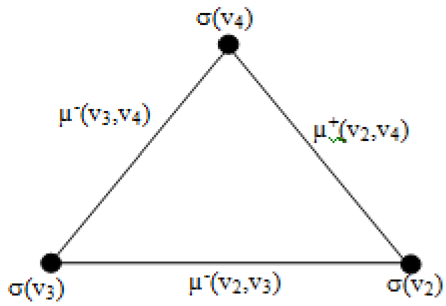

Figure C : $\sigma(v_4)$ is a friend of $\sigma(v_2)$, but $\sigma(v_4)$ & $\sigma(v_2)$ are both jointly enemies of $\sigma(v_3)$.

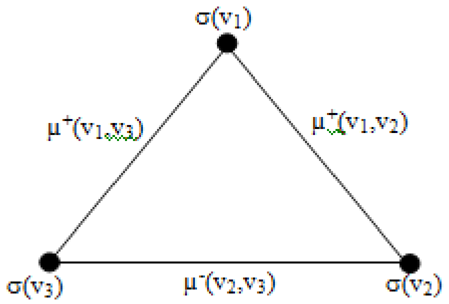

Figure D: $\sigma(v_1)$ is a friend of both $\sigma(v_2)$ and $\sigma(v_3)$, but $\sigma(v_2)$ and $\sigma(v_3)$ are enemies.

The **Fig: A** and **Fig: C** are represents normal behavior and are said to be balanced, while the **Fig:B** and **Fig:D** represent unbalanced situations since if $\sigma(v_2)$ is a friend of both $\sigma(v_1)$ and $\sigma(v_4)$, but $\sigma(v_1)$ and $\sigma(v_4)$ are enemies, this creates a tension in the system. So, the given signed fuzzy graph is unbalanced **(Fig6).**

Given signed fuzzy graph G: (σ,μ) without isolated vertices. First each

$| sdeg[\sigma(v)] | = | deg^+[\sigma(v)] - deg^-[\sigma(v)] |$
$\leq deg^+[\sigma(v)] + deg^-[\sigma(v)]$

In a signed fuzzy graph G: (σ,μ) with $|V| = \sigma(v)$, $|E| = \sum\mu(v_i,v_{i+1})$, we denote $\mu^+ (v_i,v_{i+1})$ the number of positive edges of G and $\mu^-(v_i,v_{i+1})$ denote the number of negative edges of G respectively .

Further, $\sigma(n_0)$ denote the number of vertices with positive signed fuzzy degrees.

Since G has no isolated vertices, $2 \leq deg^+[\sigma(v)] + deg^-[\sigma(v)]$ when $sdeg\sigma(v) = 0$.

Thus,
$\sum_{v\epsilon V}| sdeg[\sigma(v)] | + 2\sigma(n_0) \leq \sum_{v\epsilon V}(deg^+[\sigma(v)] + deg^-[\sigma(v)])$
$= 2\sum\mu^+(v_i,v_{i+1}) + 2\sum\mu^-(v_i,v_{i+1})$
$\therefore \sum_{v\epsilon V}| sdeg[\sigma(v)] | + 2\sigma(n_0) = 2\sum\mu(v_i,v_{i+1})$.

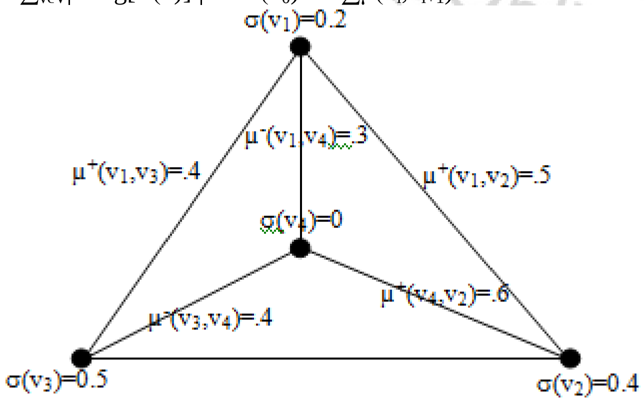

Figure 6 G

RULE1:
Fact : x is 5.2
Rule : if x is $\sum_{v\epsilon V}| sdeg[\sigma(u)] |$ THEN y is $2\sigma(n_0)$
Result : y is 0

EXAMPLE:
$| sdeg[\sigma(v_1)] | = deg^+[\sigma(v_1)] + deg^-[\sigma(v_1)]$
$= 0.5 + 0.3 + 0.4 = 1.2$
Similarly,
$| sdeg\ \sigma(v_2) | = 0.5 + 0.6+ 0.4 = 1.5$
$| sdeg\ \sigma(v_3) | = 0.4 + 0.4 + 0.4 = 1.2$
$| sdeg\ \sigma(v_4) | = 0.4 + 0.3 + 0.6 =1.3$
$\sum_{v\epsilon V}| sdeg[\sigma(v_1)] | = 1.2 + 1.5 + 1.2 + 1.3$
$= 5.2$

$\sigma(n_o) = 0$ ($\because$ the number of vertices with zero fuzzy signed degrees)

RULE 2:
Fact : x is 3
Rule : If x is $2\sum\mu^+(v_i,v_{i+1})$ Then y is $2\sum\mu^-(v_i,v_{i+1})$
Result : y is 2.2

EXAMPLE:
$2\sum\mu^+(v_i,v_{i+1}) = 2(.5+.6+.4) = 2(1.5) = 3$
$2\sum\mu^-(v_i,v_{i+1}) = 2(.3+.4+.4) = 2(1.1) = 2.2$

RULE3:
Fact : x is 5.2
Rule : If x is $\sum_{v\epsilon V}| sdeg[\sigma(v)] | + 2\sigma(n_0)$ THEN y is $2\sum\mu(v_i,v_{i+1})$
Result : y is 5.2

EXAMPLE:
$\sum_{v\epsilon V}| sdeg[\sigma(v)] | + 2\sigma(n_0) = 5.2 + 0 = 5.2$
$2\sum\mu(v_i,v_{i+1}) =3 + 2.2 = 5.2$

Hence proved

4. Conclusion

In this paper we have introduced the signed fuzzy graph, signed degree of fuzzy graph and generalized modus ponens method and we have easily obtained balanced signed fuzzy graphs by using mathematical model with generalized modus ponens method.

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