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A Mathematical Study of Pressure Drop and Shear Stress of Non Newtonian Blood Flow in an Artery with Bell Shaped Stenosis

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Abstract: ***In this paper, a mathematical model is proposed and analysed to study the effects of a bell shaped stenosis on pressure drop and shear stress, in an artery. Thebio rheological aspect is taken into account in the power law fluid representing the non-Newtonian fluid flow model. An analysis has been performed to estimate the effect of power law index and severity of the stenosis on various parameters such as pressure drop and wall shear stress. The analysis shows that as height and length of stenosis increases, not only pressure drop increases but shear stress also increases.The results are compared with the usual Newtonian fluid model.***

Keyword: Power law fluid model, shear stress, pressure drop, stenosisheight.

1. Introduction

Stenosis in arteries of humans is a common occurrence and hemodynamic factors play a significant role in the formation and proliferation of this disease. It is a chronic disease in which thickening, hardening, and loss of elasticity of the arterial walls occurs, leading to impaired blood circulation. The thickening and hardening of the arteries are due to the build-up of calcium deposits in the lumen of the artery causing stenosis. It develops with aging, hypertension, diabetes, hyperlipidemia, and other diseased conditions in the artery. There have been a number of studies using mathematical model for the flow of blood in a stenosed artery,(Young, D. F., 1968; Shukla et. al., 1979; Shukla et. al., 1980;Chaturani and Sany, 1985; Pralhad and Schultz, 1988;Mishra and Chakravarty, 1986;Haldar, 1987;Moshkelani et.al., 2003)etc.

An analysis on the effect of an axially symmetric, time dependent growth into the lumen of a tube of constant cross section through which a Newtonian fluid is steadily flowing is presented by Young, 1968. A theoretical solution of the unsteady-state momentum equation for the startup flow of a power law fluid in circular tubes is presented byPralhad and Schultz, 1988. Shukla et al.,1979, studied the effects of peripheral layer viscosity on physiological characteristics of blood flow through the artery with mild stenosis. It has been shown that the resistance to flow and the wall shear decrease as the peripheral layer viscosity decreases. A two-layered fluid model for blood flow through a stenosed tube has been developed by Pralhad and Schultz, 1988.The model consists of a core (suspension of RBC's) and peripheral plasma layer. Resistance to flow and shear stress have been computed for different stenosis height. Effect of Stenosis on non-newtonian flow of the blood in an artery was studied by Shukla et. al., 1980. It has been shown that the resistance to flow and wall shear stress increases with the size of stenosis. Blood flow through a stenosed artery has been investigated by Chaturani and Sany, 1985. They represented blood flow by a non-Newtonian Herschel –Bulkley equation. It is observed that the wall shear stress and the flow resistance increase in Herschel Bulkley fluid in comparison with corresponding Newtonian fluid.

The non-Newtonian fluid flow in a stenosed coronary bypass is investigated numerically by Chenand Wang, 2006using the carreau Yasuda model for shear thinning behavior of the blood. Results for the non-Newtonian flow, Newtonian flow and the rescaled Newtonian flow are presented.The effects of pulsatility, stenosis and non-Newtonian behavior of blood have been studied by Mandal et. al., 2007. They numerically solved the problem of non-Newtonian and nonlinear pulsatile flow through an irregularly stenosed arterial segment where the non-Newtonian rheology of the flowing blood is characterized by the generalized Power law model. The rate of flow, resistive impedance and the wall shear stress has been calculated.Unsteady response of non-Newtonian blood flow through a stenosed artery in magnetic field was studied by Ikbalet. al., 2008. Results are obtained for the flow velocity, flux and wall shear stress.

2. Assumptions

The following assumptions are made in this chapter:

- The blood is assumed to be homogeneous, incompressible and non-Newtonian fluid. It is assumed that motion of the flowing blood is steady and laminar.
- The radial velocity can be neglected in the artery as the radial velocity in the artery is very small in comparison to the axial velocity (Young, 1968)
- The stenosis developed in the artery is bell shaped axially symmetric and depends upon the axial distance z.
- The maximum height of the stenosis is much less as compared to the length and unobstructed radius of the artery i.e. stenosis is mild.
- There is no external force acting on the flowing blood.

3. Development of the Model

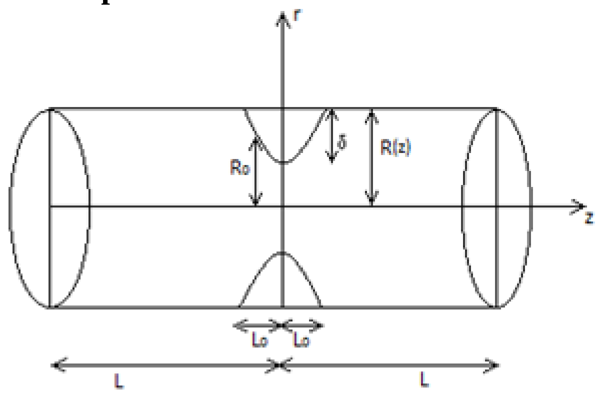

Figure 1: Geometry of a bell shaped stenosis in an artery

The radius of the artery depends upon the geometry of the stenosis and can be written as follows(Srivastava et. al., 2012)

$$R(z) = R_0 \left\{1 - \frac{\delta}{R_0} e^{-\alpha z^2}\right\} \qquad (1)$$

where $\alpha = \dfrac{M^2\beta^2}{R_0^2}$

and δ is the height of the stenosis assumed to be much smaller in comparison to the unobstructed radius R_0 of the artery ($\delta << R_0$). R(z) is the radius of artery in the stenosed region at the axial distance z. M is a parametric constant, β is the relative length of the constriction, defined as the ratio of the radius to the half-length of the stenosis, i.e.,

$$\beta = \frac{R_0}{L_0}$$

Now let us consider the laminar and steady flow of the fluid. When the inertial and entrance effects are neglected, the one dimensional flow equation is given by

$$0 = -\frac{dp}{dz} + \frac{\mu}{r}\frac{d}{dr}\left\{r\left(-\frac{dw}{dr}\right)^n\right\} \qquad (2)$$

Where w is the axial velocity, p is the fluid pressure. The boundary conditions associated with equation (2) are given as follows:

$$\frac{dw}{dr} = 0 \ \text{ at } r=0$$
$$w = 0 \ \text{ at } r = R(z) \qquad (3)$$

4. Method of Solution

Following Shukla et. al. (1979) and solving equation (2) and using equation (3) we get

$$w = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \qquad (4)$$

Also in case of no stenosis $\delta = 0$

$$w' = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left(R_0^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \qquad (5)$$

From the above two results we have

$$\bar{w} = \frac{w}{w'} = \frac{R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}}{R_0^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}} \qquad (6)$$

The constant flux Q is given by

$$Q = \int_0^R 2\pi r w dr = \pi \int_0^R r^2\left(-\frac{dw}{dr}\right)dr$$

which on using equation (2) gives

$$Q = \frac{n\pi}{3n+1}\left(\frac{1}{2\mu}\frac{dP}{dz}\right)^{\frac{1}{n}} R^{\frac{3n+1}{n}} \qquad (7)$$

The pressure gradient can be obtained from equation (7) as follows:

$$\frac{dp}{dz} = -\frac{2\mu}{R^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \qquad (8)$$

Integrating equation (8) along with the condition $p = p_0$ at z=-L and $p = p_L$ at z=L, we have

$$p(z) = -\frac{2\mu}{R_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \int_{-L}^{+L}\left(\frac{R(z)}{R_0}\right)^{-(3n+1)} dz \qquad (9)$$

Or

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \int_{-L}^{+L}\left(\frac{R(z)}{R_0}\right)^{-(3n+1)} dz \qquad (10)$$

Now let us consider three axial zones of flow i.e., inlet ($-L \le z \le L_0$), stenotic ($-L_0 \le z \le L_0$) and outlet region ($L_0 \le z \le L$), then from equation (10) we have

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \left\{\int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0}\left(\frac{R(z)}{R_0}\right)^{-(3n+1)} dz + \int_{L_0}^{L} dz\right\} \qquad (11)$$

Using Equation (2) we have

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left\{1-\frac{\delta}{R_0}e^{-\alpha z^2}\right\}^{-(3n+1)} dz + \int_{L_0}^{L} dz \right\} \tag{12}$$

Or

$$\Delta \bar{p} = \frac{\Delta p R_0^{3n+1}}{4\mu L}\left(\frac{n\pi}{(3n+1)Q}\right)^n = \frac{1}{2L}\left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left\{1-\frac{\delta}{R_0}e^{-\alpha z^2}\right\}^{-(3n+1)} dz + \int_{L_0}^{L} dz \right\} \tag{13}$$

The shearing stress at the wall is given by

$$\tau_w = \mu(r)\left(-\frac{dw}{dr}\Big|_{r=R(z)}\right)^n \tag{14}$$

which on using equation (2) and (3)gives

$$\tau_w = \frac{\mu}{R^{3n}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \tag{15}$$

or

$$\bar{\tau}_w = \frac{\pi R_0^{3n}\tau_w}{\mu Q}\left(\frac{n}{(3n+1)}\right)^n = \left(\frac{R(z)}{R_0}\right)^{-3n} = \left\{1-\frac{\delta}{R_0}e^{-\alpha z^2}\right\}^{-3n} \tag{16}$$

If n=1,results are for the Newtonian case.

5. Discussion and Results

The expression for pressure drop given by equation (13) is plotted in figures (2) - (6). Figure (2) shows the variation of pressure drop with height of stenosis for different values of length of stenosis and a fixed value of n. Similarly figure (3) and (4) shows the variation of pressure drop with height of stenosis for different combinations of n and length of stenosis. It is seen that the pressure drop increases with height of stenosis as the value of n and length of stenosis increases. Further it has been shown that if the length of stenosis increases but the value of n decreases by a greater value then pressure drop decreases. Figure (5) shows the variation of pressure drop with height of stenosis for different values of n and a fixed value of length of stenosis. It has been shown that as the value or n increases the pressure drop increases. Figure (6) shows the variation of pressure drop with n for different values of length of stenosis and a fixed value of height of stenosis. The expression for wall shear stress given by equation (16) has been plotted in figures (7)-(8). Figure (7) shows the variation of wall shear stress with axial distance for different values of n and a fixed value of height of stenosis. It is seen that the wall shear stress increases in the inlet zone of the artery and increases up to the maximum height of stenosis after which it decreases. Figure (8) shows the variation of wall shear stress with height of stenosis for different values of n at z=Lo/2. It has been shown that the wall shear stress increases as the height of stenosis increases for a fixed value of n. Also the wall shear stress increases as the value of n increases for a fixed value of stenosis height. Figure (9) shows the variation of wall shear stress with height of stenosis for different values of n at z=0. It has been shown that wall shear stress increases as the height of stenosis increases for a fixed value of n. Also wall shear stress increases as the value of n increases for a fixed value of stenosis height.

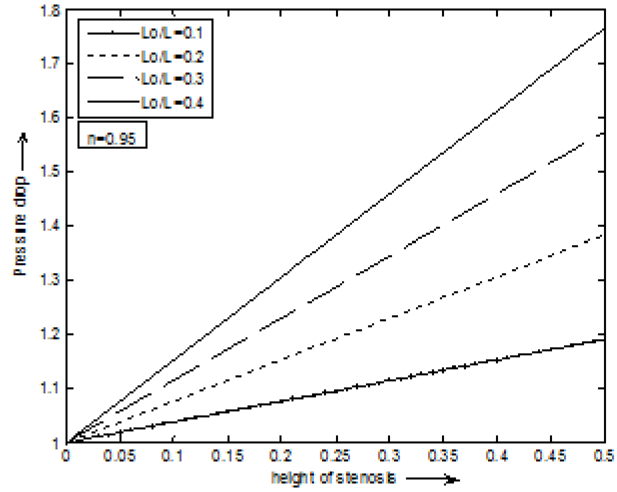

Figure 2: Variation of pressure drop with height of stenosis for different values of length of stenosis

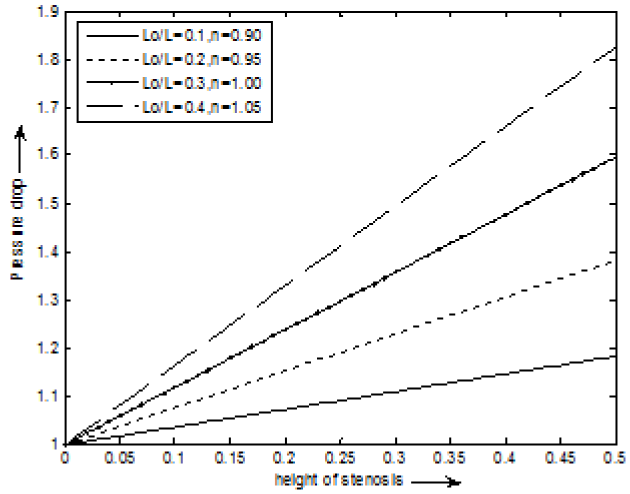

Figure 3: Variation of pressure drop with height of stenosis for different values of n and length of stenosis

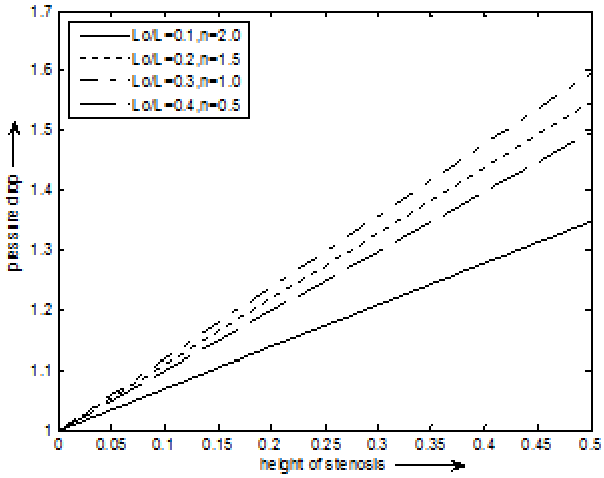

Figure 4: Variation of pressure drop with height of stenosis for different combinations of n and length of stenosis

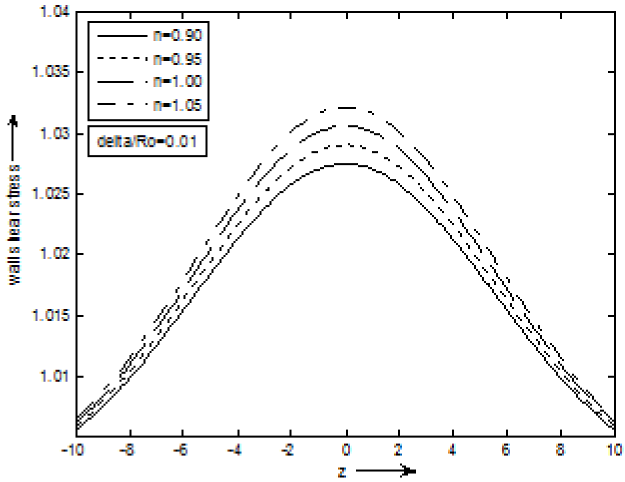

Figure 7: Variation of wall shear stress with axial distance z

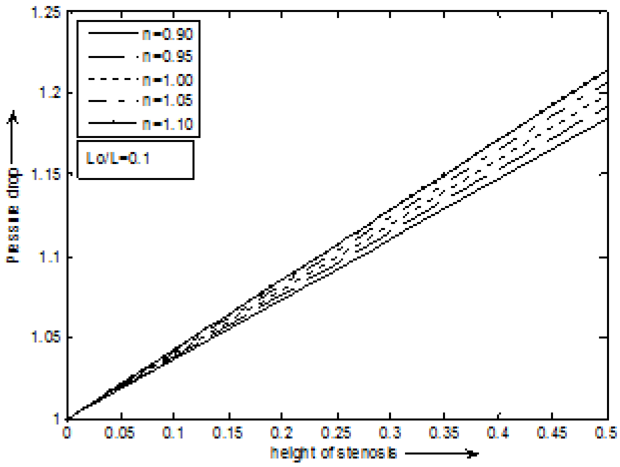

Figure 5: Variation of pressure drop with height of stenosis for different values of n

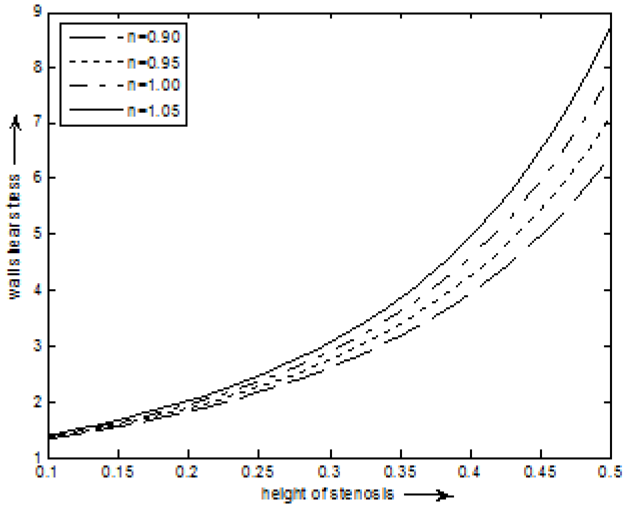

Figure 8: Variation of wall shear stress with height of stenosis for different values of n at $z=L_0/2$

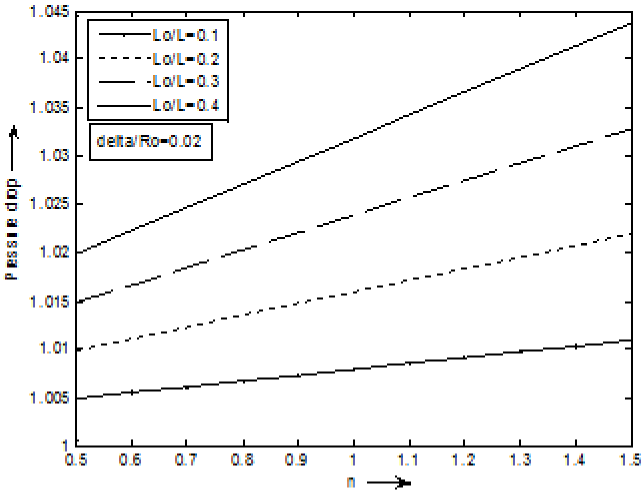

Figure 6: Variation of pressure drop with n for different values of length of stenosis

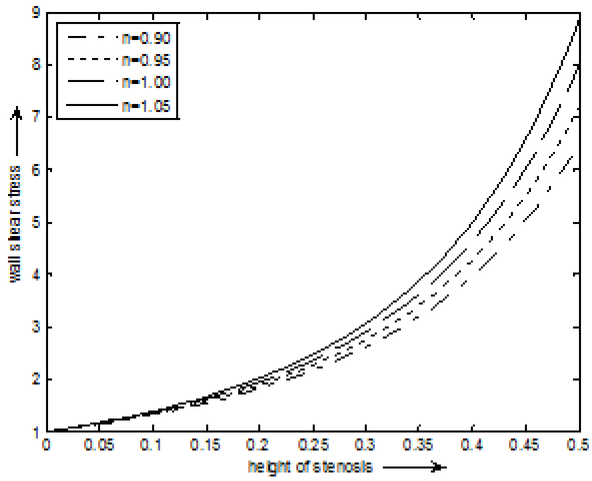

Figure 9: Variation of wall shear stress with height of stenosis for different values of n at z=0

6. Conclusion

In this paper a model has been proposed and analysed to study the effects of stenosis in an artery. It has been assumed that blood flowing in the artery is a power law non-Newtonian fluid. It has been shown that the pressure drop and wall shear increase, as the height and length of stenosis increases for a fixed value of n. Further it has been shown that if we decrease the value of n then the pressure drop may decrease even if the length of stenosis increases. Also the results for Pseudo plastic, Dilatant and Newtonian power law fluid are compared. It is observed that the wall shear stress and the pressure drop increase in the Dilatant power law fluid in comparison with corresponding Newtonian fluid.

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