



### 3. Development of the Model

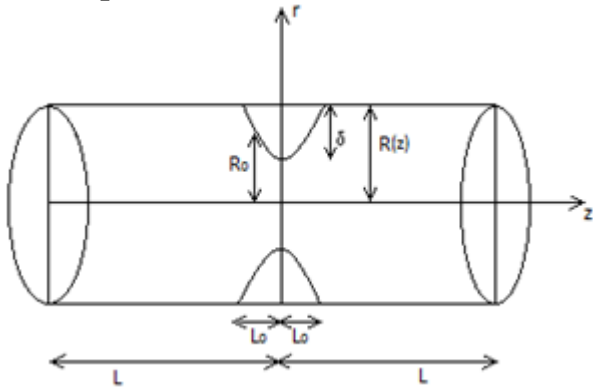


Figure 1: Geometry of a bell shaped stenosis in an artery

The radius of the artery depends upon the geometry of the stenosis and can be written as follows (Srivastava et. al., 2012)

$$R(z) = R_0 \left\{ 1 - \frac{\delta}{R_0} e^{-\alpha z^2} \right\} \quad (1)$$

where  $\alpha = \frac{M^2 \beta^2}{R_0^2}$

and  $\delta$  is the height of the stenosis assumed to be much smaller in comparison to the unobstructed radius  $R_0$  of the artery ( $\delta \ll R_0$ ).  $R(z)$  is the radius of artery in the stenosed region at the axial distance  $z$ .  $M$  is a parametric constant,  $\beta$  is the relative length of the constriction, defined as the ratio of the radius to the half-length of the stenosis, i.e.,

$$\beta = \frac{R_0}{L_0}$$

Now let us consider the laminar and steady flow of the fluid. When the inertial and entrance effects are neglected, the one dimensional flow equation is given by

$$0 = -\frac{dp}{dz} + \frac{\mu}{r} \frac{d}{dr} \left\{ r \left( -\frac{dw}{dr} \right)^n \right\} \quad (2)$$

Where  $w$  is the axial velocity,  $p$  is the fluid pressure. The boundary conditions associated with equation (2) are given as follows:

$$\begin{aligned} \frac{dw}{dr} &= 0 \text{ at } r=0 \\ w &= 0 \text{ at } r=R(z) \end{aligned} \quad (3)$$

### 4. Method of Solution

Following Shukla et. al. (1979) and solving equation (2) and using equation (3) we get

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left( \frac{R(z)}{R_0} \right)^{-(3n+1)} dz + \int_{L_0}^L dz \right\} \quad (11)$$

Using Equation (2) we have

$$w = \frac{n}{n+1} \left( \frac{P}{2\mu} \right)^{\frac{1}{n}} \left( \frac{R^{n+1}}{R^n} - \frac{r^{n+1}}{r^n} \right) \quad (4)$$

Also in case of no stenosis  $\delta = 0$

$$w' = \frac{n}{n+1} \left( \frac{P}{2\mu} \right)^{\frac{1}{n}} \left( \frac{R_0^{n+1}}{R_0^n} - \frac{r^{n+1}}{r^n} \right) \quad (5)$$

From the above two results we have

$$\bar{w} = \frac{w}{w'} = \frac{R^{n+1} - r^{n+1}}{R_0^{n+1} - r^{n+1}} \quad (6)$$

The constant flux  $Q$  is given by

$$Q = \int_0^R 2\pi r w dr = \pi \int_0^R r^2 \left( -\frac{dw}{dr} \right) dr$$

which on using equation (2) gives

$$Q = \frac{n\pi}{3n+1} \left( \frac{1}{2\mu} \frac{dP}{dz} \right)^{\frac{1}{n}} R^{\frac{3n+1}{n}} \quad (7)$$

The pressure gradient can be obtained from equation (7) as follows:

$$\frac{dp}{dz} = -\frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \quad (8)$$

Integrating equation (8) along with the condition  $p = p_0$  at  $z=-L$  and  $p = p_L$  at  $z=L$ , we have

$$p(z) = -\frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \int_{-L}^z \left( \frac{R(z)}{R_0} \right)^{-(3n+1)} dz \quad (9)$$

Or

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \int_{-L}^L \left( \frac{R(z)}{R_0} \right)^{-(3n+1)} dz \quad (10)$$

Now let us consider three axial zones of flow i.e., inlet ( $-L \leq z \leq -L_0$ ), stenotic ( $-L_0 \leq z \leq L_0$ ) and outlet region

( $L_0 \leq z \leq L$ ), then from equation (10) we have

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left( \frac{R(z)}{R_0} \right)^{-(3n+1)} dz + \int_{L_0}^L dz \right\} \quad (11)$$

Using Equation (2) we have

$$\Delta p = p_0 - p_L = \frac{2\mu}{R_0^{3n+1}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left\{ 1 - \frac{\delta}{R_0} e^{-\alpha z^2} \right\}^{-(3n+1)} dz + \int_{L_0}^L dz \right\} \quad (12)$$

Or

$$\Delta \bar{p} = \frac{\Delta p R_0^{3n+1}}{4\mu L} \left( \frac{n\pi}{(3n+1)Q} \right)^n = \frac{1}{2L} \left\{ \int_{-L}^{-L_0} dz + \int_{-L_0}^{L_0} \left\{ 1 - \frac{\delta}{R_0} e^{-\alpha z^2} \right\}^{-(3n+1)} dz + \int_{L_0}^L dz \right\} \quad (13)$$

The shearing stress at the wall is given by

$$\tau_w = \mu(r) \left( -\frac{dw}{dr} \Big|_{r=R(z)} \right)^n \quad (14)$$

which on using equation (2) and (3) gives

$$\tau_w = \frac{\mu}{R^{3n}} \left( \frac{(3n+1)Q}{n\pi} \right)^n \quad (15)$$

or

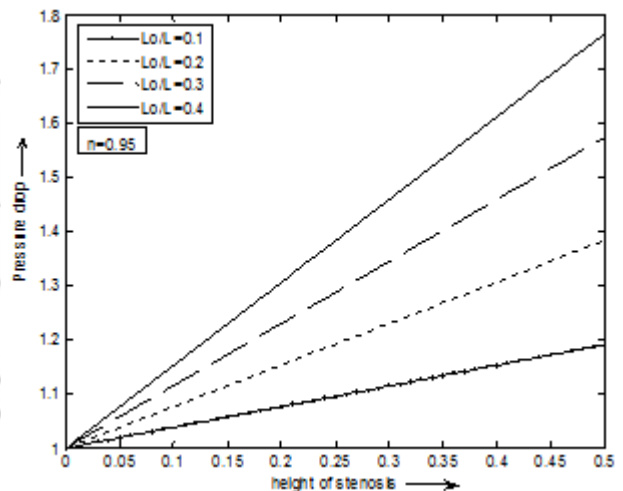
$$\bar{\tau}_w = \frac{\pi R_0^{3n} \tau_w}{\mu Q} \left( \frac{n}{(3n+1)} \right)^n = \left( \frac{R(z)}{R_0} \right)^{-3n} = \left\{ 1 - \frac{\delta}{R_0} e^{-\alpha z^2} \right\}^{-3n} \quad (16)$$

If  $n=1$ , results are for the Newtonian case.

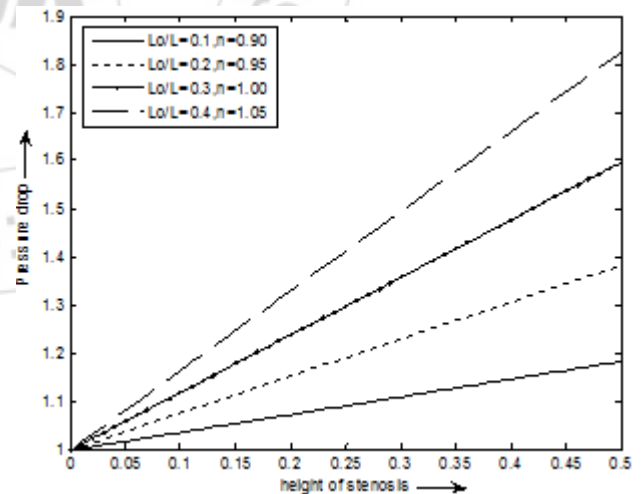
## 5. Discussion and Results

The expression for pressure drop given by equation (13) is plotted in figures (2) - (6). Figure (2) shows the variation of pressure drop with height of stenosis for different values of length of stenosis and a fixed value of  $n$ . Similarly figure (3) and (4) shows the variation of pressure drop with height of stenosis for different combinations of  $n$  and length of stenosis. It is seen that the pressure drop increases with height of stenosis as the value of  $n$  and length of stenosis increases. Further it has been shown that if the length of stenosis increases but the value of  $n$  decreases by a greater value then pressure drop decreases. Figure (5) shows the variation of pressure drop with height of stenosis for different values of  $n$  and a fixed value of length of stenosis. It has been shown that as the value of  $n$  increases the pressure drop increases. Figure (6) shows the variation of pressure drop with  $n$  for different values of length of stenosis and a fixed value of height of stenosis. The expression for wall shear stress given by equation (16) has been plotted in figures (7)-(8). Figure (7) shows the variation of wall shear stress with axial distance for different values of  $n$  and a fixed value of height of stenosis. It is seen that the wall shear stress increases in the inlet zone of the artery and increases up to the maximum height of stenosis after which it decreases. Figure (8) shows the variation of wall shear stress with height of stenosis for different values of  $n$  at  $z=L_0/2$ . It has been shown that the wall shear stress increases as the height of stenosis increases for a fixed value of  $n$ . Also the wall shear stress increases as the value of  $n$  increases for a fixed value of stenosis height. Figure (9) shows the variation of wall shear stress with height of stenosis for different values of  $n$  at  $z=0$ . It has been shown that wall shear stress

increases as the height of stenosis increases for a fixed value of  $n$ . Also wall shear stress increases as the value of  $n$  increases for a fixed value of stenosis height.



**Figure 2:** Variation of pressure drop with height of stenosis for different values of length of stenosis



**Figure 3:** Variation of pressure drop with height of stenosis for different values of  $n$  and length of stenosis

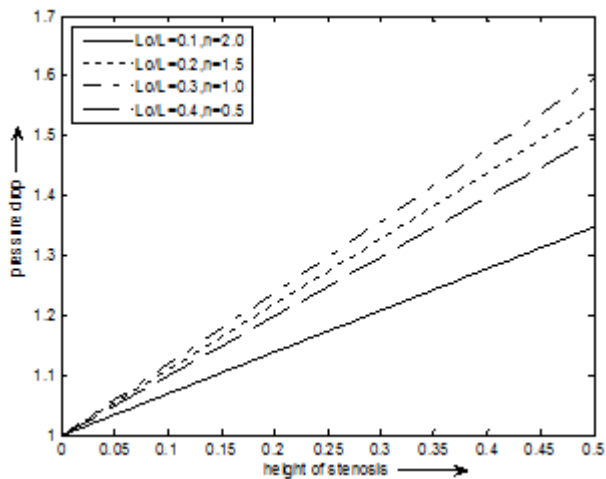


Figure 4: Variation of pressure drop with height of stenosis for different combinations of n and length of stenosis

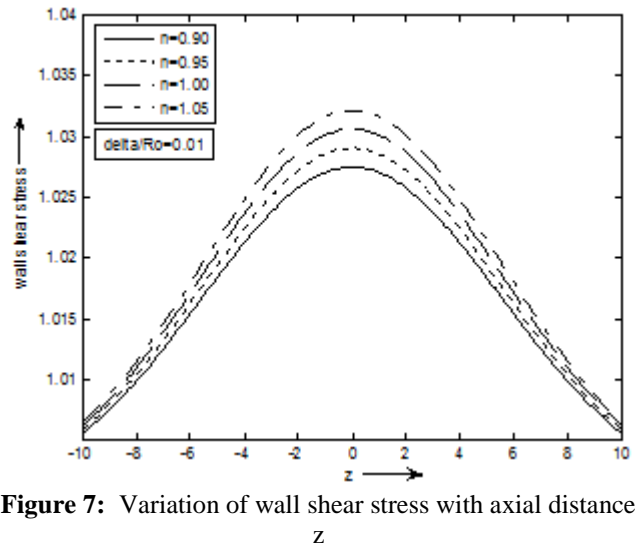


Figure 7: Variation of wall shear stress with axial distance z

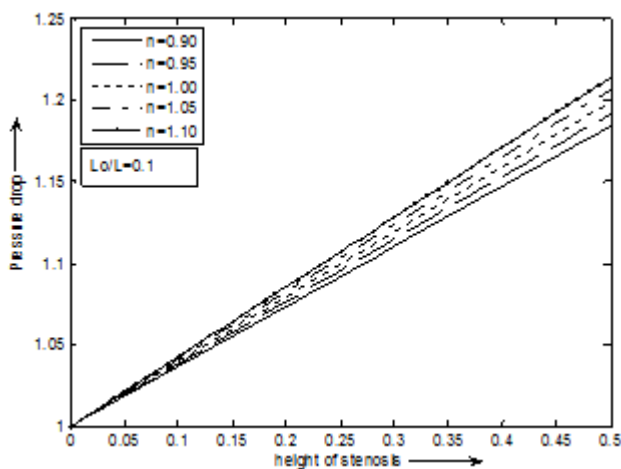


Figure 5: Variation of pressure drop with height of stenosis for different values of n

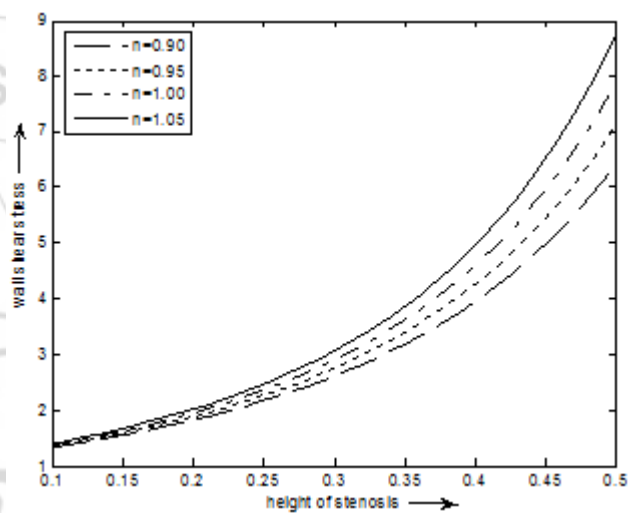


Figure 8: Variation of wall shear stress with height of stenosis for different values of n at  $z=L_0/2$

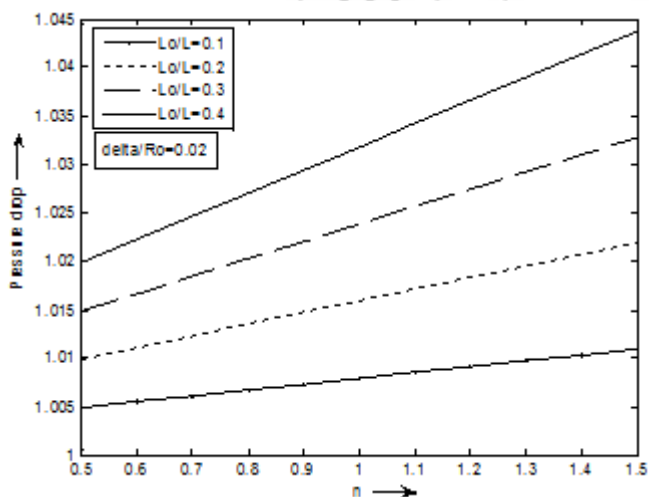


Figure 6: Variation of pressure drop with n for different values of length of stenosis

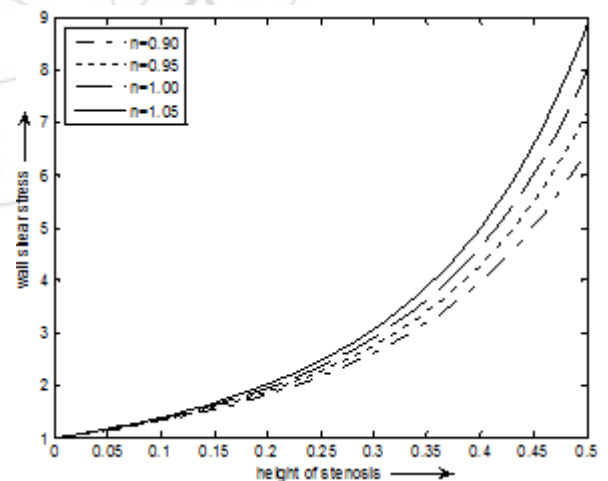


Figure 9: Variation of wall shear stress with height of stenosis for different values of n at  $z=0$

## 6. Conclusion

In this paper a model has been proposed and analysed to study the effects of stenosis in an artery. It has been assumed that blood flowing in the artery is a power law non-Newtonian fluid. It has been shown that the pressure drop

and wall shear increase, as the height and length of stenosis increases for a fixed value of  $n$ . Further it has been shown that if we decrease the value of  $n$  then the pressure drop may decrease even if the length of stenosis increases. Also the results for Pseudo plastic, Dilatant and Newtonian power law fluid are compared. It is observed that the wall shear stress and the pressure drop increase in the Dilatant power law fluid in comparison with corresponding Newtonian fluid.

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