

\mathcal{H}_1 : the PU is present.

And there are two important design parameters for spectrum sensing

- i) Probability of detection (P_D): the probability that SU accurately detects the presence of active primary signals
- ii) Probability of false alarm (P_F): the probability that SU falsely detects primary signals when PU is in fact absent.

Now the spectrum utilisation can be defined as

$$P(\mathcal{H}_0)(1 - P_F) + P(\mathcal{H}_1)P_D \quad (1)$$

and normalized SU throughput is

$$P(\mathcal{H}_0)(1 - P_F), \quad (2)$$

Here $P(\mathcal{H}_1)P_D$ is PU throughput when there are primary signals and the SUs detect the presence of the primary signals.

To determine whether the spectrum is being used by the primary user, the detection statistic T_D is compared with a predetermined threshold ϵ . Probability of false alarm P_F is the probability that the hypothesis test chooses \mathcal{H}_1 while it is in fact \mathcal{H}_0 :

$$P_F = P(T_D > \epsilon / \mathcal{H}_0). \quad (3)$$

Probability of detection P_D is the probability that the test correctly decides \mathcal{H}_1 when it is \mathcal{H}_1 :

$$P_D = P(T_D > \epsilon / \mathcal{H}_1). \quad (4)$$

A. Signal Model

Here time-slotted primary signals where N primary signal samples are used to detect the existence of PU signals is considered. The PU symbol duration is T which is known to the SU and the received signal $r(t)$ is sampled at a rate of $1/T$ at the secondary receiver. For MPSK modulated primary signals, the received signal of k -th symbol at the CR detector, $r(k)$ is given in [4].

$$r(k) = \begin{cases} n(k) \mathcal{H}_0 \\ h e^{j\varphi_n(k)} + n(k) \mathcal{H}_1 \end{cases} \quad (5)$$

Where $n(k) = n_c(k) + j n_s(k)$ is a complex AWGN signal with variance N_0 , $n_c(k)$ and $n_s(k)$ are respectively the real and imaginary part of $n(k)$, $\varphi_n(k) = 2n\pi/M$, $n = 0, 1, \dots, M-1$ with equi-probability, h is the propagation channel that is assumed to be constant within the sensing period. Denote $\mathbf{r} = [r(0) \ r(1) \ \dots \ r(N-1)]$. Assume that the SU receiver has no information with regards to the transmitted signals by the PU and $\varphi_n(k)$, $k = 0, 1, \dots, N-1$ are independent and identically distributed (i.i.d.) and independent of the Gaussian noise.

The detection statistics of energy detector (ED) can be defined as the average energy of observed samples as

$$T_{ED} = \frac{1}{N} \sum_{k=1}^N |r(k)|^2 \quad (6)$$

The likelihood ratio test (LRT) of the hypotheses \mathcal{H}_0 and \mathcal{H}_1 can be defined as

$$T_{LRT}(\mathbf{r}) = \frac{p(\mathbf{r}|\mathcal{H}_1)}{p(\mathbf{r}|\mathcal{H}_0)} \quad (7)$$

B. Optimal Detector Structure

The probability density function (PDF) of received signals over N symbol duration for hypothesis of \mathcal{H}_0 is denoted as $p(\mathbf{r}|\mathcal{H}_0)$ [5], which can be written as

$$p(\mathbf{r}|\mathcal{H}_0) = \prod_{k=0}^{N-1} \frac{e^{-|r(k)|^2/N_0}}{\pi N_0}$$

Since the noise signals $n(k)$, $k=0, \dots, N-1$ are independent. The PDF of received signals is

$$p(\mathbf{r}|\mathcal{H}_1) = \prod_{k=0}^{N-1} \sum_{\varphi_n(k)} p(r(k)|\mathcal{H}_1, \varphi_n(k)) p_{\varphi_n(k)}$$

the structure of the optimal detector (BD) for MPSK signals becomes:

$$T_{BD} = \frac{1}{N} \sum_{k=0}^{N-1} \ln \left(\sum_{n=0}^{M/2-1} \cosh(v_n(k)) \right) \geq \gamma + \ln \frac{M}{2} + \frac{\ln \epsilon}{N}$$

The above equation is complicated to use, we will simplify it in below section

C. Suboptimal Detector (ABD) Structure

The theoretical analysis (detection performance and threshold) for the suboptimal detector to detect complex MPSK ($M = 2$ and $M > 2$) in low SNR regime and comparison with the results for real BPSK primary signals.

a. Approximation in the Low SNR Regime

We study the approximation of our proposed detector for MPSK modulated primary signals in the low SNR regime.

When $x \rightarrow 0$, $\cosh(x) \approx 1 + \frac{x^2}{2}$ and $\ln(1+x) \approx x$ we can obtain:

$$\sum_{k=0}^{N-1} \ln \left(\sum_{n=0}^{M/2-1} \cosh(v_n(k)) \right)$$

Through approximation, the detector structure becomes:

$$T_{L-ABD-1} = \frac{1}{N} \sum_{k=0}^{N-1} |r(k)|^2 \geq \frac{N_0}{\gamma} \left(\gamma + \frac{\ln \epsilon}{N} \right)$$

b. Approximation in the High SNR Regime

We consider the high SNR regime in this section. When

$x \gg 0$, $\cosh(x) \approx \frac{e^x}{2}$ or when $x \ll 0$, $\cosh(x) \approx \frac{e^{-x}}{2}$

The detector structure becomes

$$T_{H-ABD} = \sum_{k=0}^{N-1} \left(\ln \left(\sum_{n=0}^{M/2-1} e^{\frac{2}{N_0} \Re[r(k)h^* e^{-j\varphi_n(k)}]} \right) \right) \geq \gamma + \ln M$$

It employs the sum of received signal magnitudes to detect the presence of primary signals in the high SNR regime, which shows that energy detector is not optimal in high SNR regime.

